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PREFACE.

THE determination of the best method of presenting a subject to a class of learners forms a difficult problem. A method of presentation which will appeal to one student will fail in the case of another. The problem is rendered more difficult by the fact that modern engineering courses, as taught in American colleges, are crowded. If it were not for this the desired end might be attained by the student's solution, in the case of surveying, for instance, of problems sufficiently numerous and varied to cover all cases likely to arise in practice. It is a fact recognized by all teachers of engineering subjects that this result must at least be approximately accomplished both in field-work and in computation.

The ideal engineering course would graduate its students practised draughtsmen, instrument men, computers, and designers in all the varied branches in which they may be called upon to operate. But the four years given to the completion of an engineering course does not permit of the entire realization of this ideal. Further, its *entire* realization during a course of study, no matter how severe and practical, is an impossibility. There is needed an experience more practical and more burdened with a sense of responsibility than a college course can ever give. But the consideration of this fact seems to have led some to reason, consciously or unconsciously, that since it is a fact, the attempt to change its degree is unnecessary.

The writer has known students who, having completed a course in surveying and use of instruments, were incompetent to handle or adjust a transit. Surely, by an intelligent conception of the needs of engineering education and arrangement of its work such a condition is susceptible of improvement. A long step in advance will be made when the engineering degree is placed where it belongs—on a level with the medical and legal degrees. The early American educators in engineering lines who coordinated the degree of Civil or Mechanical Engineer with that of Bachelor of Arts have much for which to be responsible.

There are two lines along which instruction may proceed. The student may be taught theorems whose proofs are left largely to his own intelligence and labor or the proofs may be taught along with the theorems, the work to be supplemented with practical applications in either case. The former method is undoubtedly the more valuable as far as regards its final results, but its complete application is precluded by the length of time in which the work has to be accomplished. The average engineering student has all that he can do without being burdened to a great extent with the *original* and laborious deduction of the formulæ of his science. Again, the bald statement of equations and theorems without the accompanying proofs is to be condemned. The idea is sheer nonsense—that, other things being equal, the engineer ignorant of mathematics is as capable or *more so* than if well grounded in the science.

The idea in the present book, therefore, has been to present the various theorems and formulæ accompanied, as far as possible, in each case by the proper deduction, attention being directed to special cases and to questions likely to arise in the mind of the student. In adopting this plan it is thought that, in general engineering education, sufficient stimulus to mental development is offered by the study of the completely given deductions of theorems *and the application of these latter to the solution of varied problems*. There is only one way to learn a mathematical subject—work examples.

The arrangement of the topics is in the order in which it has seemed easiest to teach them, due regard being had to completeness under each head, rather than by classification under the subdivisions of the subject. The term "longitude" has been used in place of "departure." The expressions "latitude difference" and "longitude difference" are indeed admirable, but it is thought that the simple terms "latitude" and "longitude" will prove more convenient for class-room use.

The use of the terms "error of closure" and "ratio of closure" will prevent ambiguity in the meaning of the single term "error of closure" heretofore used impartially for either of the two. Reference to the Method of Least Squares has been avoided as hardly necessary in connection with the ordinary practice of Plane Surveying and somewhat confusing to the student in his study of the latter subject. Before undertaking a survey of magnitude, he will, of course, have familiarized himself with the subject-matter of the former and of Geodesy.

The treatment of the adjustments of instruments by the methods of Descriptive Geometry, the chapter on telescopes together with its treatment of telescope adjustment as a problem apart from instrument adjustment in general, and the forms for systematic arrangement of computations at the end of the book are called to the especial attention of the reader. Appendices B and D treat of those modern developments in instrument construction and the theory and practice of surveying, the cyclotomic transit and phototopography. The valuable discussions of the planimeter and mining surveying have been written by Professors Crockett and Hall. The writer's appreciation of their cooperation in these matters, and of their and Professor Webster's assistance in reading much of the manuscript, is hereby expressed.

Tables XI*a* and XIV–XVII are published by kind permission from Professor Nagle's "Field Manual for Railroad Engineers"; Tables XIII and XVIII by that of Professor Carhart. The sources of other tables are acknowledged in connection with them. While many standard works have been consulted, the

author wishes to express especially his indebtedness to the "Engineers' Surveying Instruments" of Professor Baker.

Messrs. F. E. Brandis' Sons & Co. have furnished Figs. 131, 153, 181, 274; the A. Lietz Co., Figs. 152, 182, 205, 283–285; Keuffel & Esser, Figs. 175–178, 203, 228, 229, 231, 242, 273; Buff & Buff, Fig. 119; W. & L. E. Gurley, Figs. 4 (lower half), 27, 98, 116, 117, 126, 232–234, 259, 272, 275; C. L. Berger & Sons, Figs. 180, 187, 216, 217, 235, 277; Queen & Co., Figs. 3, 4 (upper half), 5, 6, 23, 44–46, 118, 120–122, 135, 142, 145–148, 150, 151, 155, 156, 159–164, 188–190, 206–208, 218, 221, 247, 253–257. The writer wishes to express his sincere appreciation of the courtesy shown by all of the above, and also, in the matter of supplying tables and general assistance, by the officials of the General Land Office and the Coast and Geodetic Survey.

PAUL C. NUGENT.

SYRACUSE, N. Y., December, 1901.

CONTENTS.

INTRODUCTION.

CHAPTER II.

CHAIN SURVEYING.

CHAPTER III.

COMPASS AND GENERAL SURVEYING.

CHAPTER IV.

The Telescopes of Surveying Instruments.

Section I.—Theory of Lenses and Construction of the Telescope.

CHAPTER V.

Leveling.

Section I.—Definitions and Divisions of the Subject.

CHAPTER VI.

TRANSIT SURVEYING.

CHAPTER VII.

The Planimeter and the Slide Rule.

Section I.—The Planimeter.

Section II.—The Slide Rule.

CHAPTER VIII.

Topographical Surveying.

CHAPTER IX.

Hydrographic Surveying.

CHAPTER X.

Mine Surveying.

Section I.—Definitions.

Section II.—Importance and Difficulties.

Section III.—Instruments.

Section IV.—Stations.

Section V.—Traversing.

Section VI.—Calculations and Mapping.

Section VII.—Contents of Beds and Veins.

Section VIII.—Connecting Surface and Underground Work.

Section IX.—Passageways.

Section X.—Measurement of Depth of Shaft.

Section XI.—Correction of Vertical Angles.

Section XII.—U. S. Mining Claims.

CHAPTER XI.

THE SOLAR INSTRUMENT.

CHAPTER XII.

THE U. S. PUBLIC LANDS.—RESURVEYS.

Section I.—The Public Lands.

Section II.—Resurveys.

APPENDIX A.

APPENDIX B.

APPENDIX C.

APPENDIX D.

TABLES.

FORMS.

PLANE SURVEYING.

1. **Definition.**—Surveying is the art of making such measurements of the earth's surface that a representation, called a map, of the portion measured, can be drawn on paper. By the surface of the earth is meant not only its visible portion, but also the bottoms of lakes and rivers, the interior of mines, etc. There are many different kinds of surveys and maps. These will be described more particularly later.

2. **Divisions of the Subject.**—Under the head of surveying in general, there are two subdivisions, plane surveying, and geodetic surveying, or geodesy. In plane surveying, the surface of the earth, or, rather, the surface which the world would assume were it entirely liquid, is treated as a plane. This is practically true up to areas of about one hundred square miles. Geodesy, on the other hand, takes into consideration the curvature of the earth's surface. The methods of plane surveying are applicable, therefore, to areas of one hundred square miles or less. They are sometimes used in connection with the surveys of larger areas, but only when a high degree of accuracy is not desired. This book will treat of plane surveying only.

CHAPTER I.

LINEAR MEASURING INSTRUMENTS AND RANGE POLES.

SECTION I.

Linear Measuring Instruments.

3. **Classes of Instruments.**—The instruments used for measuring distances directly are divided into three classes, chains, tapes, and rods. Of these, for the present, it will be necessary to treat only of chains and tapes.

4. **Surveyor's Chain.**—The ordinary surveyor's chain is made of iron or steel wire in the form of straight links with their ends bent into circles, adjacent links being joined to each other by one, two, or three rings. The length of each link is 7.92 inches. This includes half of the ring connection at each end of the link. The whole chain consists of 100 links, making the total length 66 ft. The end links are somewhat different from those intermediate, having attached to them brass handles. The total length of the chain should be measured from the outside of one handle to the inside of the other. The reason for this will be seen later. This instrument was invented in the early part of the seventeenth century by an Englishman named Gunter, and, on that account, is commonly known as "Gunter's chain." In old deeds and descriptions of property where the word "chain" is used, Gunter's chain is meant. An error right here, however, should be guarded against. The writer remembers a case where a survey was made with a half chain, and some perplexity caused by the line lengths being given in chains. Gunter's chain is also the one prescribed to be used in the United States public land surveys. The reason for making it 66 ft. long is that, with this length, 10 square chains equal one acre, so that if the area of a field is given in square

chains, to reduce it to acres it is necessary only to point off one decimal place. In this connection, note also that 80 chains equal one mile. To prevent their stretching apart and the consequent lengthening of the chain, the rings and the ends of the links should be strongly brazed. As there is more or less of this stretching going on whenever the chain is used, and on account of the change in length consequent on a change in temperature, the handles should be adjustable, so that the length of the chain may be made, from time to time, to agree with the standard. At every tenth link a brass tag is attached. The one at the centre is round. Those ten feet from each end have one point; those twenty feet from each end have two points, and so on. The idea is that these tags are more easily read than plain tags stamped with the number of the foot. A mistake is very liable to be made in using the chain by taking the forty-foot mark for the one at sixty feet, etc. Guard against this carefully. The writer thinks it would be better to have a round tag at each tenth link with the number of links, 10, 20, 30—60, 70, 80, and so on, stamped on it. Iron chains are rarely seen now. In fact, chains have pretty generally been replaced by steel tapes. Steel chains are best made of No. 12 wire about seven sixty-fourths of an inch in diameter.

5. **The Engineer's Chain.**—The engineer's chain is entirely similar to Gunter's chain, except that, in the former, the links are one foot each in length, and the total length of the chain is 100 feet.

6. **Metallic Tapes.**—The tapes used by engineers are of linen with brass wires woven in longitudinally, or of steel. The first kind are known as "metallic" tapes, and come, in lengths up to one hundred feet, in round leather cases. They are useful in measuring short offsets, the dimensions of buildings, etc., but, on account of their liability to stretch, cannot be depended upon for any degree of accuracy. These tapes are attached permanently to their cases, but it is much better to have them detachable, and when in use remove them entirely from the cases. This lessens the danger of breaking and of tangling inside the case. In winding up a metallic tape, let it slip between the first and second fingers of the hand holding the case. Otherwise it is liable to become twisted and slip off the roll inside. When this happens,

the center pin, generally, will have to be taken out and some time lost in straightening out the tape. After use in wet weather, any kind of a tape should be wiped dry before being rolled up.

7. **Steel Tapes.**—The steel tape is the most valuable linear measuring instrument known to the engineer. There are many varieties. In measuring long lines what is known as a "chain tape" is generally used. These are ribbons of steel about a quarter of an inch wide and perhaps two one-hundredths of an inch thick. For general work a length of 100 ft. is to be preferred. These tapes are usually divided into feet, with the foot at each end divided into tenths. The division-marks are indicated by small rivets. At every fifth foot a piece of metal, with the number of the division from 5 on to 95 stamped on it, is riveted on. This riveting weakens the tape, and it is better to have the divisions marked by pieces of metal soldered on, a fine line marking the exact division. They seldom are, but always should be, a foot longer than their list length, the extra foot being beyond the zero end of the tape, and divided into tenths. The reason for this will be discussed later on under the head of "Chaining." Other steel tapes of a hundred feet or under are divided into hundredths, the division-marks being etched on with acid. They are used in making the most exact measurements. The extremities of a chain-tape are often marked by brass points as in Fig. 1.

FIG. 1.

This method of placing the points is improper. They should be as shown in Fig. 2. (See under the head of "Chaining," Art. 9).

FIG. 2.

8. **Marking Pins.**—The pins used for marking chain lengths are what their name indicates, pieces of pointed steel or iron wire with the blunt end bent into a ring. There are eleven in a set. They should be kept strung on a wire ring, and pieces of red

cloth tied to the bent ends will assist in finding them when used in high grass. Steel pins are preferable. They should be made of No. 6 wire.

SECTION II.

Measurement of Lines.

9. Chaining.—Chaining requires two men, known respectively as fore and rear chainman. In measuring a line of more than one length most surveyors commence by placing a pin at the place of beginning. The writer does not prefer this method. A point some distance ahead on the line—its further extremity, if possible—being fixed, let the fore chainman start out, holding the front end of the chain or tape and carrying all the pins. If a chain is used, it should previously have been thrown out in a direction opposite to that in which the line is to be measured, so that the rear chainman may watch it as it is drawn past and straighten out any kinks which may occur. When the fore chainman has proceeded nearly a chain length, the rear chainman calls out "chain!" The fore chainman then halts, pulls the chain taut, and allows the rear chainman to line him in with the point fixed ahead. In doing this the fore chainman holds a pin vertically with the inside of the front handle against it, and is careful to place himself well to one side of the line, so that the rear chainman may see past. The rear chainman then lines in the pin carefully, the fore chainman inserts it in the ground, and the work is proceeded with as before. While the pin is being lined in, the fore chainman should keep his eyes fixed on the rear chainman, who indicates which way the pin is to be moved, not by calling out, but by motions to the right or left. The rear chainman should make all his motions distinctly. The amount that the pin is to be moved is indicated by the motion. Thus if the pin is to be moved some distance, the arm is held out in a horizontal position. If the pin is to be moved a less distance, the arm is held closer to the body, etc. While pulling the chain, the fore chainman should be careful always to keep on line. The only way to do this is, before starting, to fix a point ahead on the

line, and, in going forward, keep the eyes fixed on this point and walk directly towards it. Of course, in rough country, this rule cannot be followed. If made use of wherever practicable, much time will be saved. When the eleventh pin has been stuck in the ground the rear chainman delivers his ten pins to the fore chainman, both make note of a "tally," and the work goes on. Note that by this method every pin stuck in the ground counts for one hundred feet, and therefore the number of pins picked up by the rear chainman will give the length of the line in hundreds of feet. If the length of the chain is counted from the outside of one handle to the outside of the other and the measurements are made by holding the rear end of the chain against the back pin and the front end against the fore pin, each chain measurement will be wrong by the width of a pin. On this account the length of the chain should be estimated from the inside of one handle to the outside of the other. If the handle is dropped over the rear pin, there is danger of pulling this pin forward and out of place. The outside of the handle should, therefore, be placed against the front edge of the back pin, and the fore pin set just inside the handle. The case of a steel tape with its extremities marked as in Figs. 1 and 2 is similar. With the projections facing, as is usually the case, in opposite directions, we would bring the rear projection against the back edge of the rear pin and set the fore pin with its front edge against the projection at the front end of the chain. With this method, there is danger of pulling the rear pin out of place. Also every chain measurement is wrong by the thickness of a pin. With the projections attached as in Fig. 2, these difficulties are obviated. Note that the projections both face towards the rear or zero end of the tape. The rear chainman holds the face of the projection at his end of the tape against the front edge of the pin; the fore chainman sets his pin with its front edge against the face of the projection at the front end of the tape, and measurements in all cases are made to the same side of the pin.

With the ordinary chain tape the divisions are to feet only, except in the case of the extreme foot at each end. These are divided to tenths. With such an instrument as this, suppose it is desired to measure a line 657 and a fraction feet long. The

rear chainman holds his end of the chain at the 600-ft. pin, and the fore chainman notes that the front end of the line falls somewhere between 57 and 58 ft., but exactly what the fraction is he cannot know, unless he has with him a short rule or tape divided to tenths. It becomes necessary to hold the 58-ft. mark at the extremity of the line while the rear chainman reads off the extra tenths at his end. This difficulty is obviated by the use of a tape having at its zero end an extra foot divided to tenths. In the example cited, the fore chainman would hold the 57-ft. mark at the extremity of the line, the rear chainman reading off the extra tenths. Tapes are not commonly made in this way, but would be found very convenient.

10. **Erroneous Length of Chain or Tape.**—Note that if a chain or tape is too short, a line measured with it will be recorded as too long and *vice versa*. If a line has been measured with a chain or tape of incorrect length, the true length of the line may be found by the following method. Let $S =$ the standard length of the chain or tape, and T its true length. Also let $L =$ the true length of the line, and M its measured length. Then $\frac{T}{S} =$ length of tape in terms of standard, and

$$L = \frac{T}{S} M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Any linear measuring instrument is liable to change its length from time to time. This is due to stretching, change of temperature, etc. On this account, every surveyor should have a standard with which to compare his linear measuring instruments. Methods of obtaining such a standard will be discussed later.

11. **Horizontality of Measurements.**—As the distances desired are horizontal, it will be necessary in all cases either to hold the chain or tape horizontal, or to measure on a slope, afterwards making a correction for this. In ordinary work, the former method is the one generally adopted. In chaining down a hill, the rear chainman will hold his end of the chain or tape on the ground, while the fore chainman, holding his end on a level with the point occupied by the rear chainman, transfers the end division to the ground by means of a plumb-bob. Be sure that

this is done accurately. If the work is being done in high grass, it should be held down with the foot while the end of the chain or tape is being transferred to the ground. If the plumb-bob hangs in a pile of leaves, scrape these away, and make sure in all cases that the point in the ground at which the pin is placed is exactly under the end of the measuring instrument. In chaining up-hill the method is the reverse of that just given. It will readily be seen that chaining down-hill is the more accurate, and this method, therefore, should be adopted wherever practicable. The tendency of the down-hill man, generally, will be to hold his end too low. The horizontality of the chain is better estimated when viewed from the side. It then may be compared with the horizon, a horizontal line on a building, etc.

12. Winding up the Chain or Tape.—In winding up the chain, take the middle two links side by side in one hand. Then lay the two links adjoining these together and diagonally across the first two, and so on until the chain is wound up. It may then be strapped around the middle as in Fig. 3. Be careful to

FIG. 3.

lay the links diagonally across each other. Steel tapes are generally provided with reels. These latter are made in a variety of forms. Some of these are shown in Fig. 4.

In using the ordinary 100-ft. chain tape, it will be found more

convenient to loop it up in the form of a figure eight. The method is as follows. Take one end of the tape in the left hand, with the rest of the tape off to the right. Pass the right hand out to arm's length along the tape and bring the part then held by that hand across the body in front of the left hand, with which it may then be held, and so on until the whole tape is looped up

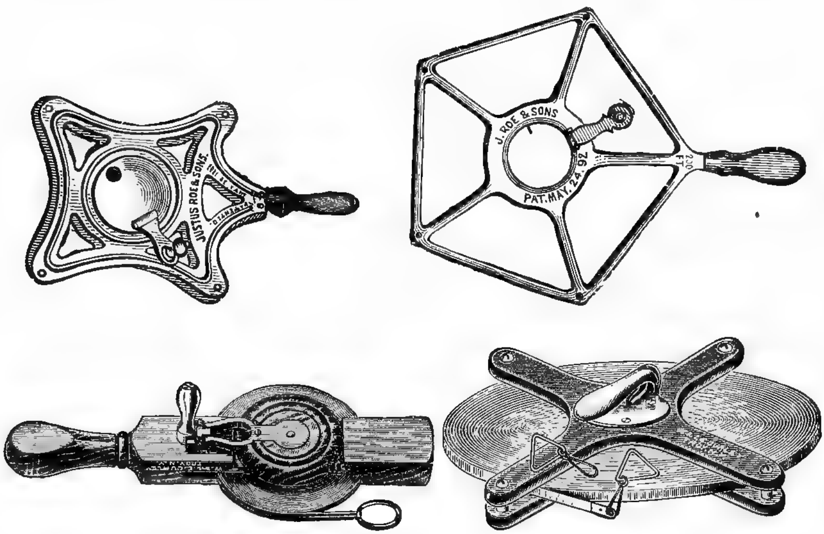


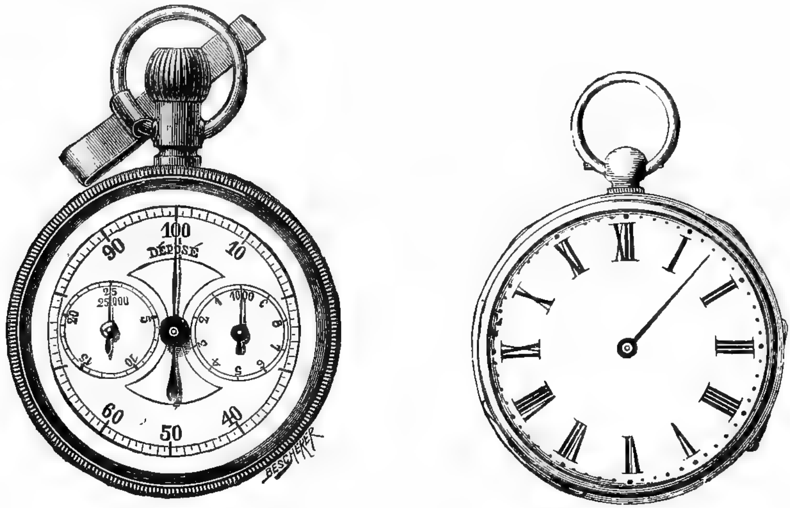
FIG. 4.

in the form of a figure eight. The tape may then be tied together in the middle. Throughout the whole proceeding keep the same side of the tape uppermost, and there will then be no twisting.

13. **Precision of Measurements.**—What has been said in regard to chaining applies to ordinary work where a precision of one in one thousand or better may be expected. That is, if a line has been measured and its length determined as about 1000 ft., a second or any other measurement should give a result varying from this by not more than one foot. Later on will be discussed the methods which obtain when higher precision is desired (Art. 107).

14. **Approximate Lengths of Lines.**—Where only approximate lengths are desired, they can be obtained by pacing. For

this purpose, the surveyor should know the average length of his step. The pedometer, Fig. 5*b*, is an instrument similar in appearance to a watch. It is adjustable for length of stride and registers the actual distance walked over. The passometer, Fig. 5*a*, is similar to the pedometer, except that it is not adjustable for



a FIG. 5. *b*

length of stride and records merely the number of steps taken. If neither of these instruments is to be had, the pacer will have to count his steps.

Another method of obtaining distances quickly is to tie a rag on the wheel of a vehicle, drive over the line, and count the number of revolutions. This method is in use in part of its work by the United States Geological Survey. In this connection an odometer (Fig. 6) is serviceable. This is an instrument used in connection with a vehicle, registering the number of revolutions of the wheel.

A time-honored device mentioned in some works on surveying is the holding of a pencil before the eye so that the length from its end to the hand just covers a known length at the point whose distance from the observer is desired. The distance may then be obtained from the following proportion: As the length of the

part of the pencil is to the length of the object observed on the distant point, so is the distance of the pencil from the eye to the distance of the point from the observer. This method is, of course, very inaccurate and useless except in short distances. It is related of Napoleon that he once demanded of an engineer,

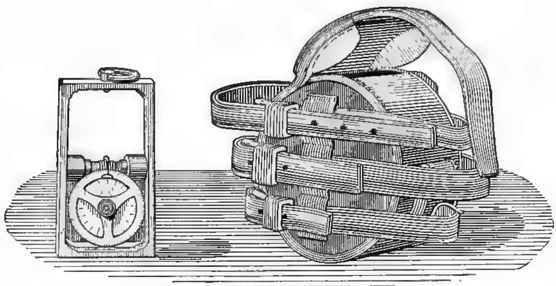


FIG. 6.

instantly, the distance across a river. The engineer, standing on the bank, and looking squarely across the stream, pulled his hat down until by sighting past the brim he could just see the opposite shore. Then, wheeling, he noted a point cut by his line of sight and on the same side of the stream as himself. Having measured the distance of this point from his first position, the result was reported as the width of the stream.

In connection with distances, memorize the following:

$$\left.\begin{array}{r}5280 \text{ ft.}\\ 1760 \text{ yds.}\\ 320 \text{ rods}\\ 80 \text{ chains}\end{array}\right\} = 1 \text{ mile}$$

The vara is the unit in which the old Spanish surveys in Texas are made. A vara equals 2.78 ft., nearly *

SECTION III.

Rods and Their Use.

15. **Range Poles, Rods, or Flags.**—These are either of wood or of metal. The wooden rods should be ten feet in length, painted red and white, every alternate foot, or, better, half red

* The value varies in other localities.

and half white every alternate foot. This latter arrangement presents lines running down the pole by means of which its verticality may be better estimated by the observer. For the same reason these poles generally have an octagonal cross-section. The iron range poles are smaller than the wooden ones, both in cross-section and in length. They are useful in fine work. It will be found convenient to have flags, preferably red, attached to these poles. By this means they are more easily discovered when some distance away. Poles are used to mark points to which lines are being measured, and stations on which an instrument is turned. Where great accuracy is not required they are also useful in prolonging lines. Under this last class of work two problems will be considered.

16. To Prolong a Line (*Fig.* 7).—(1) Having two points as A and B given on the line, and desiring to prolong the line in

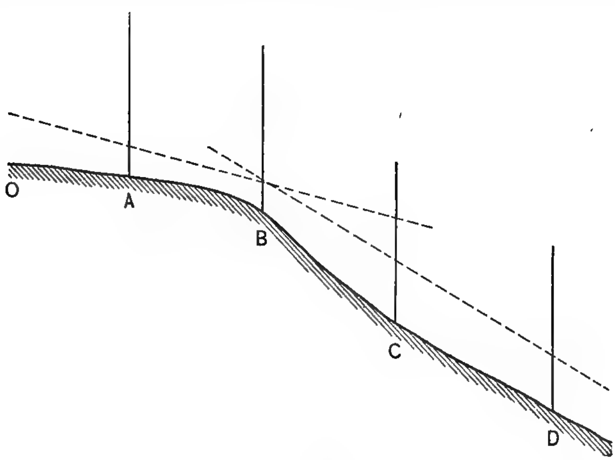


FIG. 7.

the direction AB, the observer stations himself at O, some distance from A. Rods being held at A and B, the observer sights past these and motions to the rodman at C until the rod at that point is held on line. The rod at A is then sent on to D, and the observer, moving up towards B, lines D in as before, and so on. The observer should be as far from the rods and the rods themselves as far apart as may be practicable.

(2) **To Run a Line Connecting Two Points on Opposite Sides of a Hill and Invisible from Each Other.**—Let *A* and *B*, Fig. 8, be two such points. Two rodmen station themselves at *C* and *D* approximately on line and in such positions that *B* and *D* are visible from *C*, and *C* and *A* visible from *D*. *C* lines in *D*

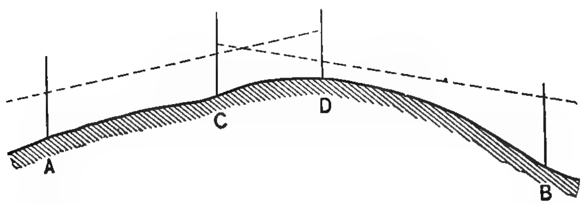


FIG. 8.

between *C* and *B*. Then *D* lines in *C* between *D* and *A*, and so on, turn about, until the points *C* and *D* are both in line between *A* and *B*.

17. Method of Holding the Rod.—In holding the rod be careful to hold it vertical. If it is desired to sight past the rod, the rodman must, of course, stand to the side of the line of sight. Otherwise he should stand directly behind the rod and facing the observer. In either case do not grasp the rod firmly, but balance it with the finger-tips, using both hands. Remember always that attention to such details will facilitate the work and save the temper of all parties concerned, especially on very hot and on very cold days.

CHAPTER II.

CHAIN SURVEYING.

18. Preliminary Steps.—The first step in a survey of any description is to go over the ground carefully and make an approximate sketch in the field-book (see Art. 19) of the area to be surveyed and mapped. The sizes of angles and lengths of lines can be estimated by the eye, and transferred approximately to paper. Where feasible it is desirable to pace the lengths of the boundaries. In this way can be formed a good general idea concerning the appearance of the tract, the difficulty of the work, and the time required for its completion. When the approximate sketch is made a station should be located at each corner and these numbered on the sketch. If desired for any reason to locate stations in the interior or on the exterior of the survey, these should also be shown on the sketch, and where there is any probability of difficulty in finding a station, a full and clear description of its location should be given. After having made a sketch of the boundaries, in the case of a chain-survey, the next proceeding is the division of the tract into a series of triangles. The considerations governing the location of these triangles are five in number:

(1) They should be as few as possible consistent with the measurement of check lines. It is evident that the more lines there are to be measured, the greater will be the opportunity for the occurrence of errors and the difficulty of detecting their whereabouts. In plotting the survey the length of each line, reduced to scale, will have to be spread on the dividers and, in some cases, this will cause the division of the tract into a larger number of triangles than would otherwise have been used.

(2) They should be as nearly equilateral as possible. As will appear later, this aids materially in the plotting and renders less laborious the calculation of the area.

(3) The lines to be measured should be located, as far as possible, on ground level and free from obstructions.

(4) For the purpose of a check and for the sake of the appearance of the map, the triangles should be arranged in a system of parallel lines, preferably parallel to some prominent side of the survey.

(5) If there are buildings, roads, etc., to be located, some of the sides of the triangles should be arranged to run parallel with these, thus saving much labor in the running of auxiliary lines.

It may be impossible to satisfy all of the above conditions, and in any particular survey other considerations will probably have weight.

19. **Notes.**—The notes of the survey are kept in the ordinary field-book of a convenient size to slip into the pocket, to be had of any dealer in engineer's supplies. The left-hand page is kept for "remarks." On this are entered, at the beginning of each day's work, the date and location of the survey, the temperature and state of the weather, the true length of the chain or tape, and the names of and the positions held by the different members of the party. Any further remarks as to the description of stations, etc., that may tend to make clear the meaning of the notes on the opposite page are also entered on the left-hand page. The right-hand page is ruled in rectangles in blue, with a red line down the middle. This line is taken to represent the line being run on the ground. Supposing that it is desired to keep the notes of a line running from station A to station B. A triangle or circle lettered "A" is drawn on the red line at its intersection with a blue line somewhere near the bottom of the page. On the blue line and at the left of the page is written O. The notes are kept up the page, and all distances along the line being run are entered on the blue lines at the left. All buildings, roads, etc., near to or crossed by the line AB are sketched in approximately in their true positions. Some engineers draw these notes to scale, but it is best, on account of the time required, not to attempt this. Care is taken to sketch directly on horizontal blue

lines all points to which the distances along AB are measured, and these distances are written directly on the same blue lines at the left of the page. For instance, if a road crosses the line, its centre is made to fall exactly on a blue line. If a house is parallel to the line, its sides are sketched on blue lines, etc. Offsets are measured to all points desired to be located, and the lengths of these are shown on the notes. So with the widths and names of streets and roads, the dimensions of buildings, etc. It is impossible to give explicit directions for the keeping of the notes in every case, and the ingenuity of the individual surveyor must be relied on to make a perfectly clear record. Be sure to take plenty of room and make everything very plain. Never put more than one line on a page. It may sometimes be necessary to take several pages for one line. In complicated places, it will sometimes be best to show simply the center lines of roads and streets, giving, of course, their names and widths. In sketching in the notes it is best to stand, or sit, directly on the line to be surveyed, facing in the direction in which it is to be run, with the note-book held in the same way.

Fig. 9 shows a specimen double page of a note-book like the one above described. This method may be called the method of "line notes." The "sketch method" referred to in the first part of Art. 18 is valuable, but becomes inadequate in the case of complicated work. In general, it is undoubtedly best to adopt a combination of the two methods, drawing in the first part of the field-book the sketch referred to in Art. 18, showing on this as many measurements, etc., as may be practicable, and, in the line notes, going more particularly into details. Too much stress cannot be laid on the necessity for keeping neat, accurate, clear, and copious notes. Before starting out, number the pages of the field-book. Never trust to the memory for anything, and when a measurement is taken, put it down at once. Carry a small ruler, pair of pencil dividers, and an eraser. The pencil used should not be too soft; 4 H is about right for this work. In sketching in the notes, sit down whenever practicable, resting the field-book on the knee. If a line or figure has to be rubbed out, do it thoroughly, so as not to give the page a smeared appearance. Rule all straight lines and draw all circles with the dividers.

When it comes to plotting the map, the surveyor will be amply repaid for any extra time spent in careful note-keeping.

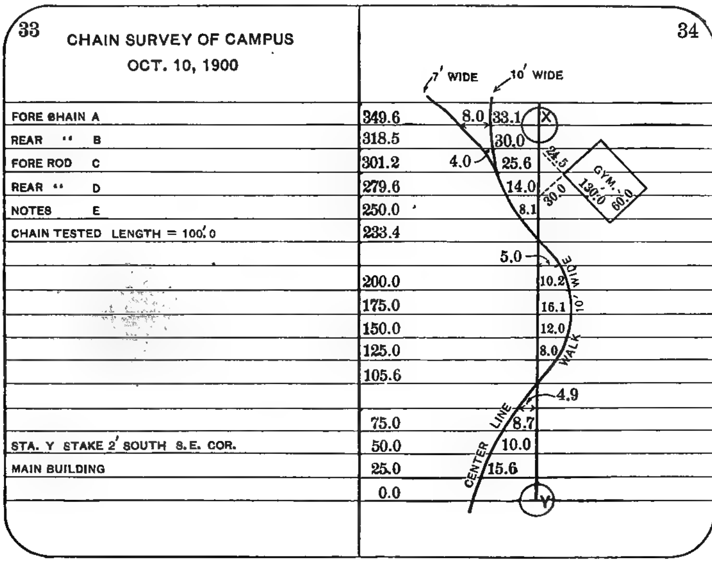


FIG. 9.

20. Field Problems.—For solving some of the more common of these, methods are here given.

(1) *To lay out a right angle.* First method, Fig. 10. Required to erect a perpendicular to the line AB at the point B.

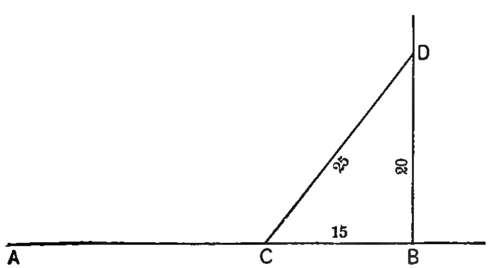


FIG. 10.

Hold, or fasten with a pin, the end of the chain and the end of the sixtieth link at B. Then if the end of the fifteenth

link is stretched out along the line AB towards A and held at C, and the end of the fortieth link is carried out towards D so as to make the chain taut along the lines DB and DC, the angle DBC will be a right angle, since $15^2 + 20^2 = 25^2$. Any other lengths of chain may be used provided they are in the proportion 3, 4, and 5.

Second method, Fig. 11. Measure equal distances, say 25 feet, on the line AB, and to each side of B, thus locating the points C and E. Fasten the two ends of the chain at C and E, or have them

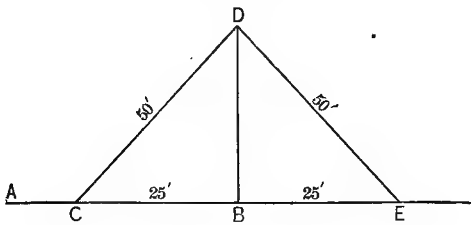


FIG. 11.

held at these points. Then draw the middle of the chain out to the point D as far as it will go from the line AB. ABD will be a right angle.

Third method. The following is a practical method of offsetting perpendicular to a fence or other marked line. First, select a point at about the required distance from the fence. Then, with the zero end of the tape held at this point, by trial find the point on the fence nearest to the point first selected. The line joining these two will be perpendicular to the fence. If now it is desired to locate another offset point at an equal distance from the fence, it may be found in a similar manner.

(2) *To run a line parallel to another given line.* Erect two perpendiculars to the given line, as far apart as may be practicable. Two points on these perpendiculars equidistant from the given line, will determine the line required. If, in addition to this, the parallel line is required to pass through some given point, have two rods carried out along the perpendiculars, keeping them equidistant from the given line, until the line of sight through them passes through the given point.

(3) *From an external point, to drop a perpendicular on a given line.* If the point is distant from the line less than the length of the chain, find two points on the line equidistant from the given external point. Midway between these will be the foot of the perpendicular from the external point on the line. If the external point is distant more than a chain's length from the given line, run out another line parallel to the one given and within a chain's length of the external point. A perpendicular dropped on this second line will be perpendicular to the first line also.

(4) *To lay out an angle of 60°.* The angles of an equilateral triangle are 60°. The method of applying this principle in laying out a 60° angle will occur to the student.

(5) *To measure any angle on the ground.* In Fig. 12, let it be desired to measure the angle BAC. On the lines AB and AC measure equal distances AB and AC, conveniently 100 ft. each. Measure BC. Then $BAC = 2 \sin^{-1} \dfrac{BC}{2AB}$.

(6) *To lay out any angle on the ground.* In Fig. 13, it is desired to lay out the angle $BAC = X°$. Measure any distance

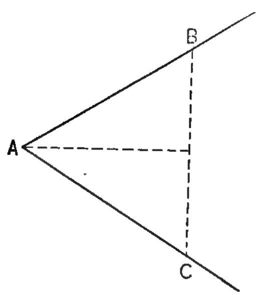


FIG. 12.

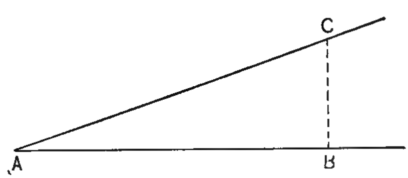


FIG. 13.

AB, most conveniently 100 ft. At B erect a perpendicular BC to the line AB and make $BC = AB \tan X$. BAC is the angle required.

(7) *To measure a line through any obstacle.* In Fig. 14, let it be required to continue the line AC on the other side of the building shown, and also to determine the distance CD. Run a line GJ parallel to AC and far enough away from it to miss the building. When once past this run another parallel to GJ,

making the perpendiculars DI and EJ equal to BG and CH. The line DF will be a continuation of AC, the distance CD being equal to HI.

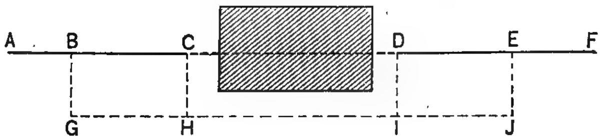


FIG. 14.

(8) *Another method.* At B, Fig. 15, lay out the angle $CBD = 60°$. At F and J lay out angles EFG and HJI each equal to $60°$, making $BF = FJ$. IK is the continuation of line AB and $BJ = BF = FJ$. Generally the first method is the better.

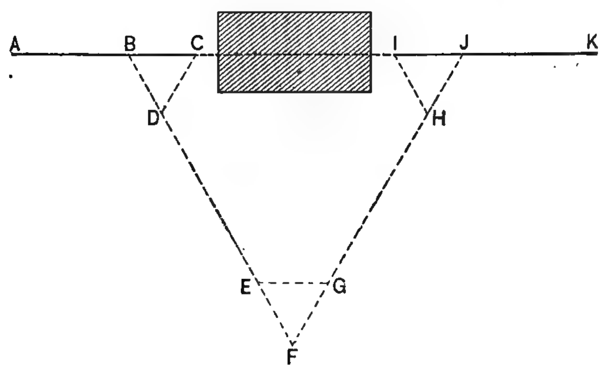


FIG. 15.

(9) Where it is desired to find the distance across a stream too wide to be spanned by the chain, one of the two methods shown in Figs. 16 and 17 may be used. The figures explain themselves.

(10) *To locate a rectangular building.* This problem is divided into two cases, as the line being run is or is not parallel with the side of the building. If the building is parallel with the line, note simply the point C, Fig. 18, where one of its sides intersects the line, and the distance CD. If the building is not parallel with the line, Fig. 19, note points C and D and measure distances CE and DE. If the building is nearly parallel to the

line, the last described method is not practicable. In this case proceed as follows: Note the points C and E, Fig. 20, where perpendiculars to the line will strike the corners of the building at the extremities of a side nearly parallel to the line. Note dis-

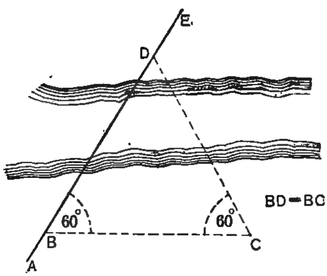


FIG. 16.

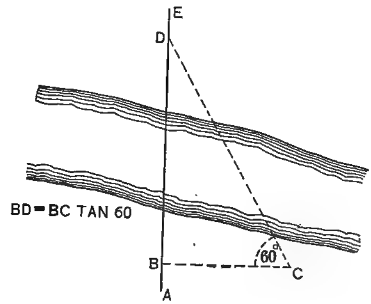


FIG. 17.

tances CD and EF. All the dimensions should, of course, be noted, and the lines run for location should be placed as close as may be practicable to the buildings.

(11) To locate a round building it is necessary to locate only its centre, taking note of the radius. In the case of a hexagonal

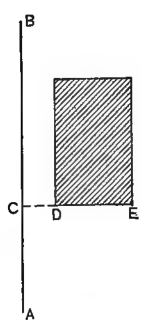


FIG. 18.

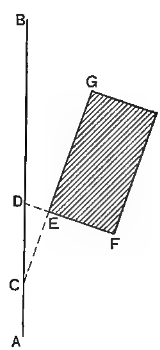


FIG. 19.

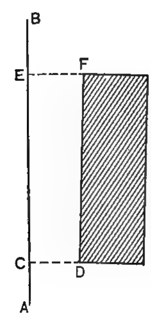


FIG. 20.

or octagonal building, locate one of the sides. If the building is a parallelogram, locate one each of the two pairs of parallel sides, etc. If the building is very irregular it may be necessary to locate each side separately. Remember always that to locate a

straight line, it is necessary and sufficient to locate two of its points.

(12) To locate an irregular line, as the bank of a stream, run a straight line, or a series of straight lines, parallel, as nearly as may be, to the direction of the irregular line. Measure offsets perpendicular to the straight line, to all salient points in the irregular line. In general, if an accurate representation is desired it will be found necessary to measure offsets every ten feet. In the case of a curving road which presents no angular points, be careful to measure offsets to those points which are nearest to or farthest from the line being run.

21. Plotting the Survey.—The method of plotting the survey is best shown by examples. In Fig. 21, having measured all

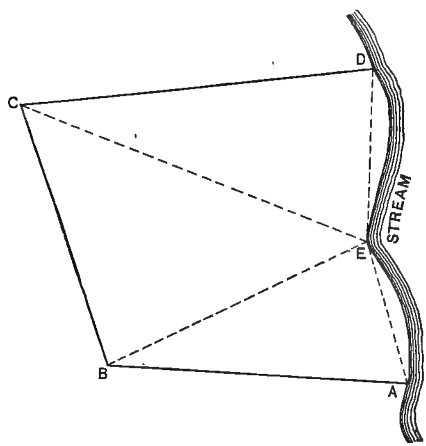


FIG. 21.

the lines, we could commence with any side of any triangle, for instance, *BA*. Laying this out on the paper and swinging arcs *AE* and *BE* from the points *A* and *B*, their intersection would give the point *E*. In like manner, locate the point *C* from the line *BE* and then the point *D* from the line *CE*. The bank of the stream *DEA* is located by perpendicular offsets from the lines *AE* and *ED*. Notice in this case that all the vertices of the triangles lie in the boundaries of the field, and there is no check on the work. To obtain such a check measure on the

ground either *DB* or *CA*. The measured length of this line should agree with its length as plotted on the paper to within one one-thousandth of the measured length.

In Fig. 22 a different state of affairs is shown. In this case it has been found advisable to divide the area into three tiers of triangles parallel with the line *AD*. This may have been because it was necessary to show the buildings in the interior on a large scale, and the corresponding lengths of the triangle sides would

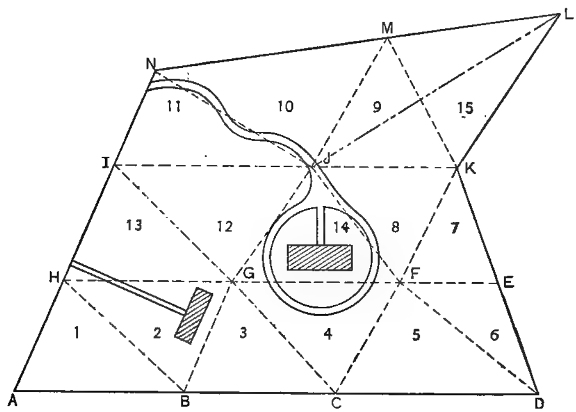


FIG. 22.

be awkward to handle if fewer triangles had been used. Or it may have been simply to obtain a check. The method of procedure is as follows: Beginning with the side *AB* lay out in succession around the edge of the field the triangles 1, 2, and 3, and so on around to 11. We now have several checks. The meassured and plotted lengths of the lines *HI*, *IG*, and *GJ* should agree. Also, on the ground several lines in series, as *AB*, *BC*, and *CD*, and *EF*, *FG*, and *GH*, are continuations of each other and should so appear on the plot. These conditions will never be exactly fulfilled. In chain-surveying the measured and plotted lengths of lines should agree to within one one-thousandth of their measured lengths, and lines which are continuations of, or parallel with, each other on the ground should so appear on the map. Surveying with the chain alone is the roughest kind of surveying, and highly accurate results cannot be expected ex-

cept in the case of small areas, and where the "chain" used is a steel tape.

22. General Remarks on Map Drawing.—The use to be made of a map determines the scale to which it shall be drawn. If distances are to be scaled, it becomes necessary to draw to a large scale. One hundred feet to the inch is convenient for general use. Some maps, notably in U. S. government work, are drawn to what is known as a "natural" scale. On such maps the scale will be indicated as $\frac{1}{1000}$, $\frac{1}{5000}$, etc. This means that a distance on the map is one one-thousandth or one five-thousandth of its length on the ground.

All lines should be drawn even. To insure this, the ink and ruling-pens used must be of good quality. The ink should flow freely and the pens will have to be cleaned frequently. Generally speaking, do not attempt to use very fine lines. It is difficult to draw such even. Auxiliary lines should be dotted, or shown in colored ink. A good dotting-pen would be useful here. The trouble is that a really good dotting-pen does not exist. The form in which the dotting-wheel receives its ink from an open reservoir is the best (Fig. 23). Avoid drawing all unnecessary lines, especially in pencilling in the work. It is difficult to erase pencil lines thoroughly, and, unless covered with ink-lines, they are liable to give a soiled appearance to the map. Make the principal line or system of lines, when the map is completed parallel with the edge of the paper. Surround the whole map with a neat border, and be sure to have, outside of this, a considerable margin of paper, never less than an inch and a half. Show the dimensions of all lines and buildings, the widths of roads, etc., but do not put an unnecessary line or figure on the map. Use plain letters and figures and guard against making these too large. On a map two and one-half feet square, the largest letters should not be more than a quarter of an inch high. These larger letters would occur only in the title, which should state the name of the tract surveyed, its area and the scale to which the map is drawn, and the name of the surveyor. Where a compass (Art. 26) has been used, the declination (Art. 28) should be noted. In college work it will be well to state also the kind of survey, as "Chain-survey," "Compass-survey," etc.,

and the class by whom the map has been drawn. Wherever practicable, make the lines of words and numbers run parallel with the bottom edge of the sheet on which the map is drawn. In cases where this cannot be done, arrange the lettering

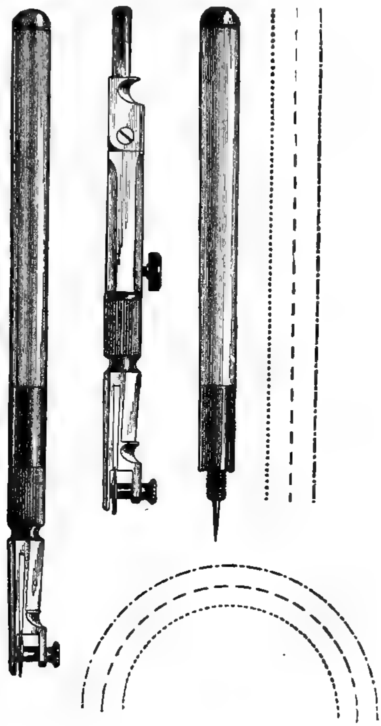


FIG. 23.

and figuring so that every word and number can be read by holding the map in one of two positions. Make all fours thus, 4, and never thus, *4*. The same remark applies to the figure nine, which should be written 9 instead of q. Make the top line of a five horizontal, and be careful to join it neatly to the rest of the figure. While making the figures too large is bad, on the other hand, to make them unnecessarily fine, cramped, and illegible is worse.

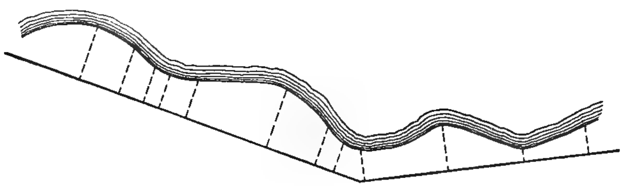
23. Calculating the Area.—Knowing the three sides, a, b, and c, of any triangle, we may calculate its area by means of the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad . \quad . \quad . \quad . \quad (2)$$

where $A =$ the area and

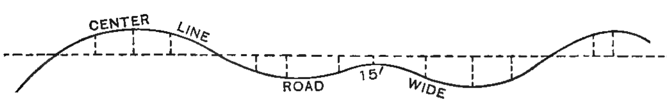
$$s = \frac{a+b+c}{2}.$$

The area of each triangle in the chain survey is thus calculated. and the sum of all these gives the area of the whole tract surveyed. Five-place logarithms are generally used in such computations. In the case of an area bounded by a curved line, as the part of the field between the stream and lines DE and EA, Fig. 21, and the similar areas in Figs. 24 and 25, the offsets divide the



LOCATION OF BANK OF STREAM

FIG. 24.



LOCATION OF ROAD

FIG. 25.

tract into a number of trapezoids, triangles, etc., each of which is calculated as such and their sums added to the total.

In connection with areas, note the following:

$$\left.\begin{array}{r} 10 \text{ sq. chains} \\ 43{,}560 \text{ sq. ft.} \\ 5{,}645 \text{ sq. varas} \\ 160 \text{ sq. rods, perches, or poles} \\ 4 \text{ roods} \end{array}\right\} = 1 \text{ acre.}$$

640 acres = 1 sq. mile.

A tract of land 1000 varas square contains 177 acres. Therefore, to reduce square varas to acres, multiply by 177 and point off six places.

24. Tabulations and Systematic Arrangement of Computations.—All mathematical computations should be tabulated, and systematized as far as possible. The importance of this cannot be overestimated. With the systematic arrangement, errors are much less liable to occur, and, when they do occur, can be much more readily detected and corrected. The systematic arrangement of the computations in the case of an area similar to that in Fig. 22 is here given (Form A).

With this tabulation of the calculations, they can easily be gone over and checked. The areas in the last line can be added together to give the total area A.

25. Correction of Area for Erroneous Length of Chain.—After a survey has been made and the area calculated, it is sometimes found that the chain or tape used has been too short or too long. In such a case the area can be corrected on the principle that the areas of similar figures are to each other as the squares on their homologous sides.

Let $L =$ true length of any side.
$M =$ measured length of any side.
$S =$ standard length of chain.
$T =$ true length of chain.
$A =$ true area.
$A' =$ area as given by chain.
$T/S =$ length of chain in terms of standard (Art. 10).

$$L = \frac{T}{S} M \text{ (Art. 10).}$$

$$\frac{A}{A'} = \frac{\frac{T^2}{S^2} M^2}{M^2} = \frac{T^2}{S^2}$$

and

$$A = \frac{T^2}{S^2} A'; \quad \ldots \ldots \ldots \ldots \quad (3)$$

or the true area is equal to the calculated area multiplied by the square of the length of the chain in terms of the standard.

CHAPTER III.

COMPASS AND GENERAL SURVEYING.

26. The Compass.—The compass consists essentially of a vertical plane of sight attached to, and including a diameter of, a graduated circle, which revolves, in a horizontal plane, around its center, where is pivoted a needle, horizontal and fixed in direction. It is thus seen to be an instrument for determining directions. The circle is divided into quadrants as shown in Fig. 26,

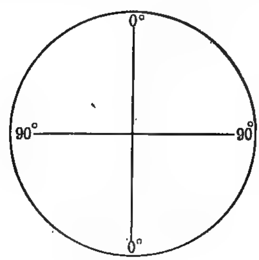


FIG. 26.

zeros being placed at each extremity of a diameter and the circle graduated ninety degrees to both sides of each of these. The needle is magnetized and therefore points to the north, or, rather, to the magnetic north, the distinction between the two being explained later. Consequently, if the plane of sight includes the zeros and is turned so as to contain any horizontal line, the needle will indicate the angle made by this line with the vertical plane through the magnetic north.

27. The Practical Surveyor's Compass.—The compass, as practically made, consists of a flat plate on which is placed the divided circle and two spirit-levels at right angles to each other.

The plate revolves around a central *vertical axis* to which it is perpendicular. The spirit-levels are so arranged that when the bubbles appear in the middle the vertical axis will be truly vertical and the plate, therefore, horizontal (see Art. 52). The plate holds, also, perpendicular to its surface, two standards with fine slits cut in them. These slits determine the plane of sight alluded to in Art. 26. In the best forms the circle can be revolved on the plate and around its centre, so that the plane of sight sometimes may and sometimes may not include the zeros of the circle. The amount of this revolution is measured by a vernier (see Art. 49). The circle itself is always graduated as described in Art 26. The divisions are half degrees, numbered at every tenth degree. The needle is made of steel with a piece of agate attached at its middle to bear on the pivot, also of steel. All the other parts of the compass are made of non-magnetic material. The magnetic force which attracts the north end of the needle being inclined so as to pull that end downward, a short piece of brass wire is wrapped around the south end. This is slipped in and out along the needle until it exactly balances the downward pull at the north end and thus allows the needle to assume a horizontal position. The circle is covered with glass, and on the plate is found a screw by which the needle, when not in use, may be lifted from the pivot and held against the glass. The pivot and needle should be subjected to as little wear as possible. Consequently, in moving the instrument from place to place, be sure that the needle is lifted from the pivot. When the compass is put away, first let the needle settle in the magnetic meridian (see Art. 28), and then screw it up against the glass. If kept in this position, the needle will retain its magnetism longer. Some compasses have an indicator which marks the number of chain-lengths. It is fixed to the plate outside of the circle. The whole instrument is mounted on a socket by which it may be attached to a tripod or to a plain staff shod with iron. This last is known as a Jacob's Staff. The compass as thus described is shown in Fig. 27 In some forms a telescopic line of sight (Chap. IV.) is substituted for the plain standard sights.

Occasionally the needle sticks to the compass-box glass. This is caused by the glass becoming electrified. In such cases, by

touching the glass with a moistened finger, the electricity will be discharged and the needle allowed to swing freely.

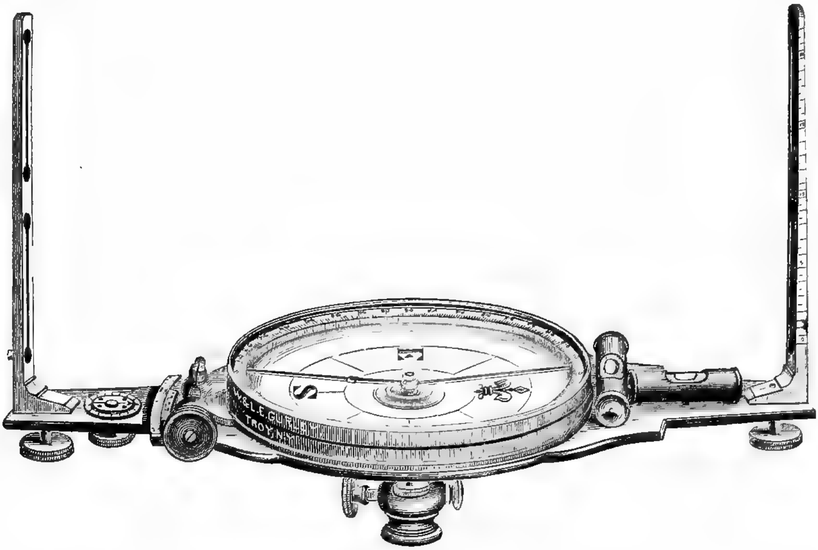


FIG. 27.

28. **Definitions.**—A few definitions and explanations here become necessary.

A vertical line is a straight line perpendicular to the surface which the earth would assume were it entirely liquid. At any point, a plumb line coincides practically, though not exactly, with a vertical line.

A horizontal line is a line lying in a plane perpendicular to a vertical line.

A level surface is a surface, as, for instance, the surface of a body of still water, parallel to the surface which the earth would assume were it entirely liquid.

A level line is a line lying in a level surface.

A meridian plane is a plane through the axis of the earth.

A meridian or true north and south line, is the intersection of a meridian plane with the surface of the earth. At any given point on the earth's surface, when "the meridian" is spoken of, by this expression is meant the particular meridian through the given point.

A parallel is the intersection with the surface of the earth of a plane perpendicular to the axis of the earth. Parallels are circles and are strictly parallel to each other all the way around the earth. Meridians, on the other hand, are ellipses and converge as we approach the north or the south pole. In plane surveying, this convergence is small. Consequently, in the discussion of any survey in this work, unless otherwise stated, the meridians will be assumed to be truly parallel to each other. That is, any two meridians taken with any two parallels will be assumed to bound a rectangle whose surface is plane.

A pivoted magnetic needle, when allowed to come to rest, will, generally speaking, not lie in a meridian plane. The horizontal angle between the needle and meridian plane through its pivot, is known as the declination. Methods of determining this will be given later (Art. 50).

A magnetic meridian is the intersection with the earth's surface, of a vertical plane through the compass-needle, when allowed to come to rest.

At any point on the earth's surface the horizontal angle, less than 90°, which any line makes with the meridian is known as the "true bearing" of the line. This is in distinction to the "magnetic bearing" of the line, which is the corresponding angle made

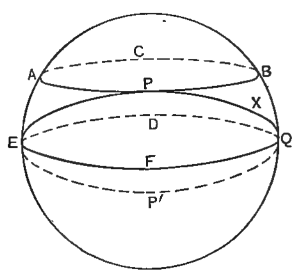


FIG. 28.

with the magnetic meridian. Evidently, the true bearing equals the magnetic bearing plus or minus the declination. Bearings are distinguished as north or south so many degrees, and fractions thereof east or west.

In this connection note that the same line may, if prolonged sufficiently, have more than one true bearing. Supposing that a

true east and west line is laid off at some point on the earth's furface as at P, Fig. 28. If this line should be prolonged, it would not coincide with the parallel $APBC$, but would describe the great circle $PEP'Q$, $EQDF$ being the equator, and, while running due east and west at the point P, at another point, as X, its bearing would be very different.

29. Method of Determining the Bearing of a Line.—The simplest method is to set the compass on the line, turn the plane of sight so as to include another point of the line, and read the angle indicated by the needle. If the plane of sight includes the zeros of the graduation, the angle read will be the magnetic bearing of the line. Use a hand magnifying glass to read the angle indicated by the needle. The graduation being to half degrees, with the aid of the glass the angle can be estimated to the nearest five minutes. The line will be north or south as the end of the compass which is being run ahead is nearer the north or the south end of the needle. If it is a north line and runs to the right of the needle, it is further known as a northeast line, and if to the left of the needle, northwest. If it is a south line and runs to the right of the needle, it is further known as a southwest line and, if to the left of the needle, southeast.

Another method of determining whether a line is northeast, northwest, southeast, or southwest, is by the marking of the compass-plate. On most compasses, the plate, inside the graduated circle, will be found marked as in Fig. 29.

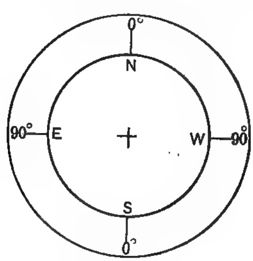


FIG. 29.

If we face the north, the east is on our right hand and the west on our left. Note that the marking of the compass-plate is the exact opposite of this. The N marks what is known as

the "north" end of the compass box. This distinction, of course, is arbitrary, and, when the compass is made, either end may be selected as the "north" end. However, with this arrangement of the letters, if the "north" end of the compass is always run ahead, the north end of the needle will always be found between the two letters which designate the direction of the line. Generally a *fleur de lis* is put in place of the letter N.

In Art. 27 it was stated that the graduated circle, in the best instruments, was movable about its center on the plate. This is to enable the declination to be set off. Suppose for instance that the needle points seven degrees to the west of north. The declination is then seven degrees west. The plane of sight being made to coincide with a meridian plane, let the circle be turned in a direction opposite to the motion of the hands of a watch through seven degrees or until the needle indicates zero degrees. The needle-reading will then be the "true bearing" of the line and will give the true bearing of any other line which the plane of sight is made to include. If the declination had been east, the circle would have been moved in the same direction as the motion of the hands of a watch, etc.

Where a line is so obstructed that the compass cannot be placed on it, find the bearing of a parallel line. For instance, if it is desired to determine the bearing of a fence, place the compass near the fence and measure the distance between them. Then have a rod held at the same distance from the fence and as far from the compass as may be practicable. The compass can then be turned on the rod and the bearing determined.

In setting up any instrument on a tripod, see that the legs are fixed firmly in the ground, though, to insure this, they need not be pushed in with all the operator's strength. If on a floor, set the points in cracks. On a hillside, put two legs down-hill and one up. Be careful always to make the head of the tripod as nearly level as may be. This will lessen the labor of the final leveling of the instrument.

30. Local Attraction.—In finding the bearing of any line, however, local attraction must be guarded against carefully. The presence of iron or steel near the compass will draw the needle from its normal position and cause it to indicate a wrong bearing.

Errors of this sort are caused by setting the instrument too near a wire fence, or an iron pipe buried in the ground, etc. On this account, as well as to insure the fact that no personal mistake has been made, the bearing of every line must be taken twice. The two readings are known technically as "foresight" and "backsight," and should agree. A knife or keys in the pocket may attract the needle. Consequently, remove these before reading a bearing. Be careful also to take away the chain, pins, axe, etc. Other sources of error may be an iron pin in the frame of a magnifying glass, metal buttons, a wire in the brim of the hat. By local attraction, however, is meant the attraction due to fixtures on the tract only.

31. Detection of Local Attraction.—The compass being in good order, and due care having been taken with the observations, supposing it is found that the foresight and the backsight on a line disagree. It is evident that local attraction exists at one of the places where the compass has been set up. There are two general methods of detecting the error. The first method applies when the bearings of a number of connected lines are being found, the lines themselves being open. Such a condition of affairs is represented in Fig. 30. In this case the most con-

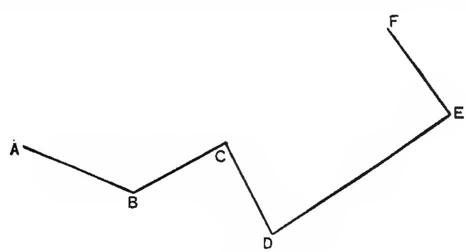


FIG. 30.

venient places to set up the compass will be at the extremities A, B, C, etc., of the different lines. A foresight on AB can be taken from A and a backsight on the same line from B. Then without moving the instrument a foresight can be taken on BC, and so on. Supposing that the foresight and backsight on AB do not agree. Making a note of this, a foresight is taken on BC and the com-

pass is moved to C, where a backsight on the same line can be had. If, now, the backsight and foresight on BC agree, it proves that there is no local attraction at B. Therefore the disagreement between the foresight and backsight on AB was caused by local attraction at A and the foresight on AB taken from A must be corrected to agree with the backsight on the same line, taken from B. If the foresight and backsight on BC do not agree, we proceed as before until we find a line whose foresight and backsight do agree. In Fig. 30 let EF be the first line whose foresight and backsight are found to agree. The backsight on DE will therefore be correct and the foresight changed to agree with it. From DE we can pass to CD, and so on back to the beginning. Example: In Fig. 30 let the foresights and backsights be as follows:

	Foresight.	Backsight.
AB	S 76° 15′ E	N 75° 00′ W
BC	N 65 10 E	S 66 15 W
CD	S 33 30 E	N 32 15 W
DE	N 40 00 E	S 39 25 W
EF	N 45 20 W	S 45 20 E

What is the correct bearing of AB? When the lines are obstructed so that the compass cannot be set up over these extremities, this method may be used by running off a series of random lines.

The second method is simpler and generally to be preferred, though when the compass can be set up over the ends of the lines it requires a little more time. It consists simply in setting the compass at a third point on the line whose foresight and backsight do not agree, reading the bearing from that point, and noting whether it agrees with one of the bearings determined before. If not, try a fourth point, and so proceed until two points are found, some distance apart on the line, from which points, the bearings of the lines agree. Guard against the existence at both points of the same conditions as to local attraction. For instance, if the line is an iron fence, equally near to the two points at which the readings agree, the two bearings, though agreeing, may be wrong by the same amount, owing to the equal attraction of the fence in each case.

32. Field-work.—The field-work of a compass-survey consists in finding the lengths and bearings of all the lines of the survey. When these are known, the plot can be drawn. For the location of buildings, etc., the same methods are used as in chain surveying.

The bearings of all lines are determined as described in Arts. 28, 29, 30, and 31. If the declination is set off, these are "true" bearings. Otherwise, they are "magnetic." The lengths of lines are measured with the steel chain tape. The chain is sometimes used, but this happens rarely now. The chain will stand rougher handling than the tape, but this is about the only advantage in its favor. The rods used to mark stations, etc., are the same as those described in the chapter on Chain Surveying.

33. Two Classes of Surveys.—Surveys, especially compass surveys, are known either as "original surveys" or as "re-surveys." The first class deals with tracts or lines which have never been surveyed before. The surveyor determines either (1) the size and shape of the tract to be laid out, together with the direction of its lines, and produces these upon the ground, or else (2) selects certain marks to determine his boundaries, and finds the directions and lengths of the lines joining these marks. The second class deals with the tracts which have been surveyed before. As under the first class, there are two subdivisions; (1) tracts whose boundaries are marked, and (2) tracts whose boundaries are not marked. Under the first subdivision will come, for instance, fields enclosed by a fence. The surveyor will determine the bearings of all fence lines together with their respective lengths. Where the boundaries are not marked the surveyor will have to take the field-notes of the first survey and produce the corresponding lines on the ground. In doing this, especial care must be taken in regard to the declination of the needle. The whole subject of re-surveys is much more complicated than might appear at first glance. (See Chapter XII, Sec. 2.)

34. Keeping the Notes.—The method of keeping the notes is entirely similar to that in use in chain surveying. In compass survey notes, however, the bearings of all lines are written on them and stated as true or magnetic. The backsights

are also written on the lines. On the left-hand page is noted the declination of the needle. In writing the bearings of lines, always give two figures to the minutes. Thus a bearing of north sixty-three degrees five minutes east, magnetic, would be written N 63° 05′ E Mag., and not N 63° 5′ E Mag.

35. Definitions and Formulæ.—In plane surveying, the latitude of a line is the perpendicular distance between the two parallels through its extremities. If the line is a northerly line, its latitude is north, or plus. If a southerly line, the latitude is south, or minus.

The longitude of a line is the perpendicular distance between the two meridians through its extremities. If the line is easterly, its longitude is east, or plus. If westerly, its longitude is west, or minus. The latitude of a *point* is its perpendicular distance from some assumed parallel. The longitude of a *point* is its perpendicular distance from some assumed meridian.

It will be noted that, having assumed a certain meridian and a certain parallel, they form a pair of rectangular coordinate axes, with reference to which the longitude of a point is its horizontal coordinate, x, or abscissa, and its latitude, its vertical coordinate, y, or ordinate. The latitude and longitude of a point are, therefore, together known as its coordinates.

The meridian distance of any point is the perpendicular distance of the point from some assumed meridian. If the point is east of the meridian, its meridian distance is plus. If west of the meridian, minus. The meridian distance of a point is seen to be the same as its longitude.

The meridian distance of a line is the meridian distance of its middle point.

The double meridian distance of a line is, what its name implies, twice its meridian distance.

In Fig. 31, NS represents the meridian. The angle DAB is the bearing of the line AB. AD is the latitude of AB and BD is its longitude. From an inspection of the figure, the following equations become evident:

$$AB = AB \cos DAB \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$DB = AB \sin DAB \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Consequently, we may write the following formulæ:

The latitude of a line equals its length multiplied by the cosine of its bearing, and

The longitude of a line equals its length multiplied by the sine of its bearing.

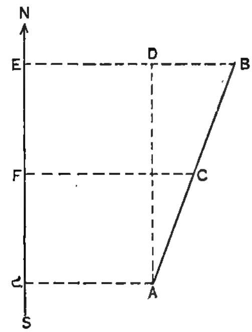


FIG. 31.

Throughout this discussion the length of a line will be denoted by L, its latitude by La, and its longitude by Lo.

The double meridian distance of AB, denoted generally by D. M. D., equals $2FC$, C being the middle point of AB. Note particularly that $2FC$ equals BE, AC, or, the double meridian distance of a line equals also the sum of the meridian distances of its extremities.

A traverse is any system of connected lines, the second starting from the end of the first, the third from the end of the second, and so on.

A closed traverse is a traverse the end of whose last line coincides with the beginning of its first line. The boundary of a field is an instance of a closed traverse.

To find a method of obtaining the D. M. D.'s of lines in any traverse, consider Fig. 32.

$$\begin{aligned} \text{D. M. D. of } BC &= BE + CF \\ &= BE + FH + HC \\ &= (BE + AD) + HC \\ &= (BE + AD) + [GC + (-GH)]. \end{aligned}$$

Now $BE + AD$ is the D. M. D. of AB, and $GC + (-GH)$ is the algebraic sum of the longitudes of AB and BC. Consequently, we have the following

RULE.—*In any traverse, the double meridian distance of any line equals the algebraic sum of the double meridian distance of the preceding line, its longitude, and the longitude of the line in question.*

The figure is not entirely general, as the lines might be made to run at other angles, etc., but, by drawing other figures, the rule can readily be seen to apply to any other arrangement

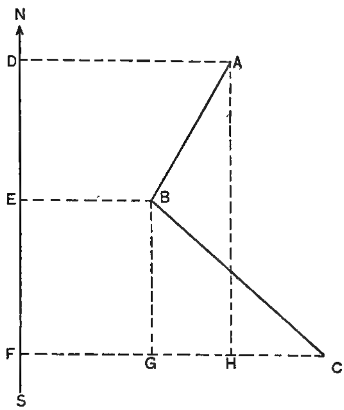


FIG. 32.

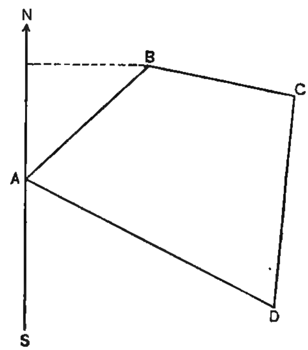


FIG. 33.

of the lines. The meridian can be assumed at any point, and the rule will still hold good. Assuming it to the west of all the lines in question makes the proof simpler, and this being perfectly allowable, it has been done. Fig. 33 shows a closed traverse with the meridian assumed through its most western station, which point is also taken as the beginning point of the first line. As before, the meridian could have been taken elsewhere, but the most western station is the most convenient point. Evidently, the double meridian distance of the first line equals its longitude; or it may be obtained by the rule, by considering the double meridian distance of the preceding line and its longitude as each equal to zero. Starting, then, with the D. M. D. of the first line, we may proceed around the traverse computing the D. M. D.'s of the successive lines. When we reach the last line

DA, its double meridian distance should equal, numerically, its longitude and this will test the correctness of the work.

If a traverse is not closed, we may still assume the meridian through any station, when the double meridian distances of lines running from this station will be equal to their longitudes. The D. M. D.'s of the other lines may then be calculated by the rule. In this case, however, there is no check on the work.

36. Summations of the Latitudes and the Longitudes.—It is evident that if we stand at any point of a closed traverse and proceed around it, stopping at the point of commencement, we will have gone as far east as west, and as far north as south. In other words, numerically, the sum of the north latitudes must equal the sum of the south latitudes and the sum of the east lougitudes must equal the sum of the west longitudes, Or, as north latitudes and east longitudes are plus, and south latitudes and west longitudes are minus, the sum of all the latitudes and the sum of all the longitudes must respectively equal zero. If we let Σ represent summation, these conditions can be stated analytically thus :

$$\Sigma La = 0, \qquad (6)$$

$$\Sigma Lo = 0, \qquad (7)$$

or, letting θ represent, in general, the bearing of a line,

$$\Sigma L \cos \theta = 0, \qquad (8)$$

$$\Sigma L \sin \theta = 0. \qquad (9)$$

37. Error of Closure.—If both the field and the computation work, therefore, have been entirely correct, the equations last stated will be true. In actual practice, however, these conditions are never realized. Supposing that, having made a survey of a closed traverse, we draw on a sheet of paper a line which we assume as the meridian through the most western station. Assuming any point on this meridian as the place of beginning, we lay off along the meridian the latitude of the first line, north or south as the case may be. If from the point thus determined, we lay off, perpendicular to the meridian, the longitude of the first line, another point will be found which is evidently the other end of the first line. Joining this point with the place of beginning, we will have plotted the first line.

We can then draw a new meridian through the point thus found, plot the second line, and so proceed around the traverse. If the latitudes and longitudes balance, and the drawing is done correctly, we will end at the place of beginning. But if, as is always the case in actual practice, the latitudes and longitudes do not balance, we will end at some point other than the place of beginning. Evidently the perpendicular distance from this point to the meridian through the place of beginning will equal the amount by which the longitudes fail to balance, or, as it is technically known, the error in longitude. Similarly, the perpendicular distance from the point last found to the parallel through the place of beginning will equal the error in latitude. Evidently the distance from the point last found to the place of beginning will equal the square root of the sum of the squares of the errors in latitude and in longitude. This distance, or the line which represents it, is known as the "error of closure." Its bearing is determined by the equation

$$\tan \theta = \frac{\text{The error in longitude,}}{\text{The error in latitude,}}$$

and will be northeast, southwest, etc., as the north or the south latitudes and the east or the west longitudes are in excess.

The ratio of the error of closure to the sum of the measured lengths of all the lines in the closed traverse is known as the "ratio of closure." The computations being correct, the ratio of closure is a measure of and in inverse proportion to the accuracy of the field work

38. Balancing the Survey.*—As stated above, owing to the impossibility of doing work with absolute correctness, in no set of field-notes of a closed traverse will equations (6) to (9) of Art. 36 be strictly true, and, therefore, the survey when plotted will not close. In good work, the amount by which it fails to close will be so small that it cannot be detected by the eye. However, it is manifestly better that the computations should be corrected so as to make it close. It is probable also, other things being equal, that the error of closure is due as much to one line,

* The discussion in this article is similar to that used by Professor Raymond in his "Plane Surveying."

in proportion to its length, as to another. Supposing, for instance, that, in a closed traverse of five sides, the lengths of these sides are 100, 200, 300, 400, and 500 ft. If the lines all lie on the same kind of ground and the same care has been exercised in the measurement of one as of another, it is highly probable that the total error of closure is due five times as much to errors made in running the last line as to errors made in running the first one. Also, owing to the fact that the compass is not an entirely reliable instrument, the error introduced by each line is due partly to incorrect determination of its length, and partly to incorrect determination of its bearing. On these suppositions is based the usual method of correcting the computed results, or, as it is generally called, " balancing the survey."

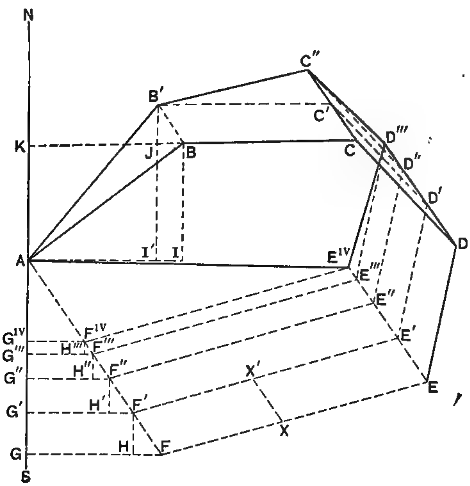


FIG. 34.

In Fig. 34 is shown the case of a five-sided field, whose boundary, though actually closed, does not so appear when plotted according to the field-notes. The ratio of closure is:

$$\frac{FA}{AB + BC + CD + DE + EF},$$

FA being the distance from the final extremity of the last line to the starting-point. Evidently the measured lengths and bearings

of the various lines have been such that, when transferred to paper, the field fails to close by a line whose length and direction are shown in the line FA. The problem then is to change the lengths and directions of the various lines in such a way that the point F will approach and finally coincide with the point A, the total change in any line being proportional to its measured length. By the "total change" in a line is meant the change of position consequent upon the change, in direction and in length, of the given line, undergone by every point beyond its extremity in the traverse. To illustrate this definition further, consider the line AB, Fig. 34. The line AB being drawn, supposing we draw the lines BC, CD, DE, and EF, making angles with AB, and having lengths as shown. Now supposing we move the point B to B' and then from B' draw lines $B'C'$, $C'D'$, etc., parallel to and of equal length with BC, CD, etc. It is evident that any point in the traverse, as X, will have moved to some point as X', and that the line XX' is equal and parallel to BB'. Therefore, in applying our correction in length and bearing to AB, that correction must be such, that B' the further extremity of the corrected line, and consequently every point, including the point F, beyond it in the traverse, will have moved in a direction parallel to FA, by a distance which is the same proportional part of FA as AB is of the sum of all the measured lengths. We therefore divide the line FA into five parts as follows, L denoting the sum of all the measured lengths:

$$FF' = \frac{AB}{L} \times AF,$$

$$F'F'' = \frac{BC}{L} \times AF,$$

$$F''F''' = \frac{CD}{L} \times AF,$$

$$F'''F^{iv} = \frac{DE}{L} \times AF,$$

and

$$F^{iv}A = \frac{EF}{L} \times AF,$$

Next draw BB' equal and parallel to FF' and join A and B', drawing lines $B'C'$, $C'D'$ etc., equal and parallel to BC, CD, etc. The point F will move to F' and the change applied to AB evidently has exactly accomplished its purpose. In like manner, move the point C' to C'' and proceed as before. Finally F will coincide with A and the survey will have been balanced by applying to each line a total change, proportional to its measured length. On examination of the figure, it will be noted that, in applying the corrections, lines whose directions are nearly perpendicular to the line FA have been changed in direction more than in length. The reverse is true of lines more nearly parallel with FA, such lines having been changed more in length than they have in direction. All of this is based on the further supposition that the errors made in running any particular line have been such as to displace its further extremity parallel to the line FA. This supposition, evidently, is approximately correct, and, such being the case, a line perpendicular to FA, to bring about its proportional part of the change necessary in FA, would have to be swung rather than lengthened or shortened. On the other hand, a line parallel with FA would have to be shortened or lengthened rather than swung.

To find a practical method of applying the corrections in accordance with the principles just demonstrated, let us examine Fig. 34 once more. The latitude of the line AB, as it would be computed according to the original field-notes, is represented by the line BI and its longitude by KB. In moving B to B', we have changed the latitude by the amount $B'J$, and the longitude by the amount BJ. Evidently $B'J = F'H$ and $BJ = HF$. Also AG is the total error in latitude, and GF the total error in longitude. The triangles AGF and $F'HF$ are similar. Consequently

$$B'J = F'H = \frac{AB}{L} \times AG$$

and

$$BJ = FH = \frac{AB}{L} \times GF.$$

In other words, in applying the corrections as above described, in the case of the line AB for instance, we have simply changed

its latitude by an amount which is the same proportional part of the total error of latitude as AB is of the sum of all the measured lengths. Its longitude has been changed in the same way. We therefore have the following

* RULE.—*Correct the latitude and longitude of each line by an amount which is the same proportional part of the total error in latitude or in longitude as the measured length of the line is of the sum of all the measured lengths. If the error in latitude is north, or plus, then the correction applied to each north latitude will be subtractive and the correction applied to each south latitude or longitude will be additive. If the error in latitude is south, or minus, the corrections will be applied in a manner the reverse of that just described. A similar statement applies to the error in longitude.*

39. **Balancing by Weighted Lines.**—The method described in Art. 38 assumes that the conditions under which each line has been measured are the same. In practice, this is often not the case. Supposing, for instance, that the field-notes report two lines each with a measured length of 1000 ft., and, further, that one of these lines has been measured along the level surface of a road, and the other over hills and rocks and through dense undergrowth. According to the method of Art. 38, we would apply the same correction to each line, but manifestly this would not be correct. The practice is to assume that line of the survey which has been run under the most favorable conditions as having a weight of 1. Another line of the survey run under such conditions that it is probable that, in the running of a certain distance on it, three times as much error has been introduced as in running the same distance on the line weighted 1, would have a weight of 3, etc. These weights are assumed according to the judgment of the surveyor, and are entered in the notes on each line. Generally, however, it is best to weight the lines after the field-work has been finished, as a more accurate judgment can then be had concerning the difficulties met with in the measurement of each. Note particularly that the weight of each line refers to the difficulty in both the angular and the linear measurement. Thus, suppose two lines run over ground offering, in each case, about the same difficulty in linear measurement.

* For proof of this rule by the methods of higher mathematics, see Wright's Adjustment of Observations, Art. 110, Ex. 5.

Suppose, further, the two ends of the first line situated on opposite hills, so that a clear sight could be had with the instrument from one point to the other, while in determining the bearing of the second it was found necessary to rely on a short sight only, with some uncertainty as to whether the point sighted on was exactly on line. Evidently, although in each case the difficulty in linear measurement was the same, the number expressing the weight of the second line would be greater than that expressing the weight of the first.

To illustrate the method of balancing the survey under these conditions, let us suppose that the field is the one shown in Fig. 34, and that the lines are weighted as follows:

AB	(3)
BC	(1)
CD	(4)
DE	(3)
EF	(1)

Further, for the sake of illustration, let us suppose that EF is twice as long as DE, that being the proportion shown in the figure. Then, as the weight of DE is 3, and the weight of EF is 1, the error due to DE is three times the error due to one half of EF, or, the error due to DE is to the error due to EF as 3 is to 2. That is, as $3DE$ is to EF. In other words, the error due to any line, and consequently the correction to be applied to it, is proportional to the length of the line multiplied by its weight.

To show this somewhat more analytically:

$$\text{error due to } DE = 3\left(\text{error due to } \frac{EF}{2}\right)$$

$$= \frac{3}{2}(\text{error due to } EF).$$

Hence

$$\frac{\text{error due to } DE}{\text{error due to } EF} = \frac{3}{2} = \frac{3DE}{EF}.$$

Or, more generally, let

$$w_1 = \text{weight of any line } l_1,$$
$$w_2 = \quad " \quad " \quad " \quad l_2,$$
$$l_1 = ml_2, \text{ and therefore } l_2 = \frac{l_1}{m}.$$

Then

$$\text{error due to } l_2 = \frac{w_2}{w_1}\left(\text{error due to } \frac{l_1}{m}\right)$$
$$= \frac{w_2}{w_1}\frac{1}{m}(\text{error due to } l_1)\ .$$

and

$$\frac{\text{error due to } l_2}{\text{error due to } l_1} = \frac{w_2}{w_1} \times \frac{1}{m}$$
$$= \frac{w_2}{w_1} \times \frac{1}{m} \times \frac{l_2}{l_2}$$
$$= \frac{w_2 l_2}{w_1 l_1},$$

the expression "error due to a line" being understood to mean the total change (Art. 38) which the line has undergone owing to the mistakes in its measurement, whether linear, angular, or both. We accordingly have the following

RULE.—*Multiply the length of each line by its weight, adding together all the results. Apply to each latitude or longitude a correction, additive or subtractive as the case may demand, which is the same proportional part of the total error in latitude or in longitude as the length of the line, of which that latitude or longitude is a function, multiplied by its weight, is of the sum of all the measured lengths, each multiplied by its weight.*

40. Balancing the Survey when the Error of Closure is Due to Errors in Linear Measurement Only.—When the interior angles of a closed traverse have been measured as with a transit (Chapter VI), their sum should equal $(n - 2)$ times two right angles, where n is the number of sides in the traverse. The algebraic representation of this condition is the equation

$$S = 2(n - 2)\ 90^\circ. \quad \ldots \ldots \quad (10)$$

If this is found to be true, or very nearly so, it becomes probable that the error of closure is due to errors in linear measurement only. The methods of balancing described in Arts. 38 and 39 assume that the error of closure is due partly to errors in linear measurement and partly to errors in angular measurement, and hence do not apply under these circumstances. If the field is rectangular, the case is as represented in Fig. 35. Consider one

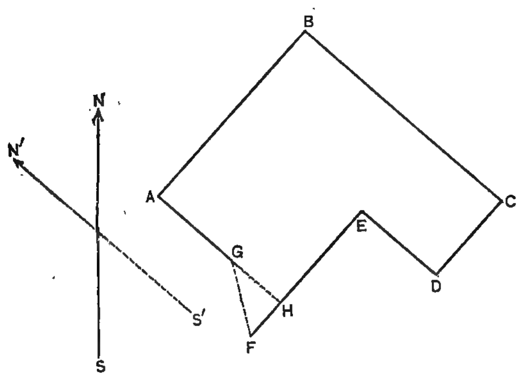


FIG. 35.

side of the field, for instance *GA*, as parallel with the meridian. The other bearings being corrected accordingly, all the lines will then run either due north and south or due east and west. The latitudes of *BC*, *DE*, and *GA* will equal their measured lengths, while their longitudes are zero. In like manner, the longitudes of *AB*, *CD*, and *EF* will equal their measured lengths, while their latitudes are zero. From this, it will be understood that the error in latitude is due to errors in the linear measurements of *BC*, *DE*, and *GA* only, and has not been affected in the least by any errors that may have occurred in the measurements of *AB*, *CD*, and *EF*. In like manner, the error in longitude has been caused entirely by errors in the measurements of *AB*, *CD*, and *EF*. We may therefore write the following

RULE.—*Correct the length of each north and south line by a quantity which is the same proportional part of the error in latitude as the line in question is of the sum of the north and south lines. Correct each east and west line by a quantity which is the*

same proportional part of the error in longitude as the line in question is of the sum of the east and west lines. If the north latitude is in excess, the correction to north lines is subtractive, and that to south lines additive, and vice versa. A similar statement applies to the corrections given to the east and west lines.

Evidently, the application of this rule will balance the survey. We will have taken, in each case, the difference between two sums of lines and divided it into two parts, adding one to the smaller sum and subtracting the other from the larger. The same rule is usually stated substantially as follows:

"*Correct each latitude by a quantity which is the same proportional part of the error in latitude as the latitude in question is of the sum of all the latitudes. Correct each longitude by a quantity which is the same proportional part of the error in longitude as the longitude in question is of the sum of all the longitudes. If the north latitude is in excess, the correction to north latitudes is subtractive, and that to south latitudes additive, and vice versa. A similar statement applies to the corrections given to the longitudes.*"

The student will note that the two rules are identical in final meaning. For the reason given in Art. 41, in practice, use the second. If the lines have been weighted as described in Art. 39, the rules will still apply by making the correction to a north and south or to an east and west line in the first, or to a latitude or to a longitude in the second, the same proportional part of the error in latitude or in longitude as, in the first, the line in question multiplied by its weight is of the sum of that line and the ones parallel to it multiplied each by its weight, or, in the second, the latitude or longitude in question, multiplied by the weight of the line of which it is a function is of the sum of all the latitudes or of all the longitudes multiplied each by the weight of the line of which it is a function. Note, however, that the weighting of the lines in this article refers to the difficulty in linear measurement only, while the weight of a line in Art. 39 referred to the difficulties both in linear and in angular or directional measurement.

If the field is not rectangular, some writers state that the survey should be balanced by the rule just given, though without

swinging the traverse from its original position in reference to the meridian as determined by the field-notes. Evidently this method would be incorrect. Suppose, for instance, the error of closure to be a northwest line; then, in the case of a northeast line among those to be corrected, its latitude would have to be increased and its longitude decreased. This would change its bearing, which is exactly what cannot be done under the given conditions. A similar statement would apply to southwest lines. The latitudes and longitudes of northwest lines would increase and decrease together, but, generally, not in the proportions necessary to retain the original bearings of the lines. This would apply also to southeast lines.

There seems to be no satisfactory method discovered for balancing such a survey. We may proceed on the supposition that all lines should be shortened, whose tendency, if lengthened, would be to lengthen the error of closure, and that all lines should be lengthened whose tendency, if shortened, would be in the opposite direction. The amount by which each line is to be lengthened or to be shortened will have to be determined by trial. Ordinarily, this is best done graphically. First determine from the original field-notes the length and bearing of the error of closure (Art. 37), and plot this line to a scale as large as may be

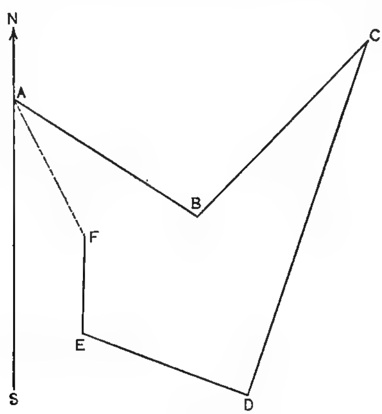


FIG. 36.

convenient. Fig. 36 represents the case of a five-sided field, the bearings of whose boundaries are correctly shown, but which,

when plotted, gives the error of closure FA. Fig. 37 shows the line FA plotted separately, and to a larger scale.

An inspection of the traverse, plotted according to the original field-notes as in Fig. 36, shows that lines AB and CD, if lengthened, would increase the error of closure. Therefore, these lines must be shortened. Similarly, BC, DE, and EF must be lengthened.

Starting from F, Fig. 37, lay off a line FF', parallel with AB, but in the direction BA. From F' draw a line $F'F''$ parallel

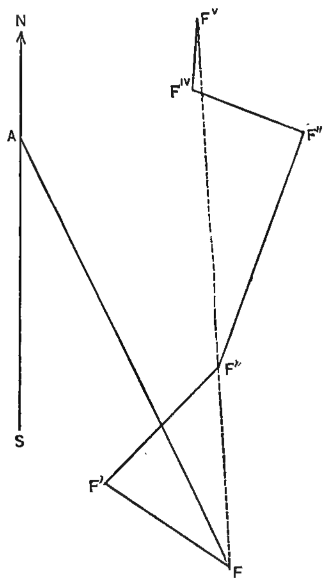


FIG. 37.

with BC, etc. So proceeding, we finally arrive at a point F^{V} and can then go over the work from F to F^{V}, adjusting the lines until F^{V} coincides with A. The lengths FF', $F'F''$, etc., can then be scaled off and applied as corrections to the original lines AB, BC, etc.

If so desired, the first trial corrections can be made proportional to the lengths of the lines to which they are to be applied. To do this, draw two lines making any angle as XY and XZ, Fig. 38. Lay off from X, and along XY, the lengths XB, XC,

XD, etc., equal, respectively, to any convenient scale, to AB, BC, CD, etc., of Fig. 36. From the points B, C, D, erect perpendiculars to XY, meeting XZ in B', C', D'. The lines BB', CC', DD', etc., will form a set of corrections proportional to the lengths

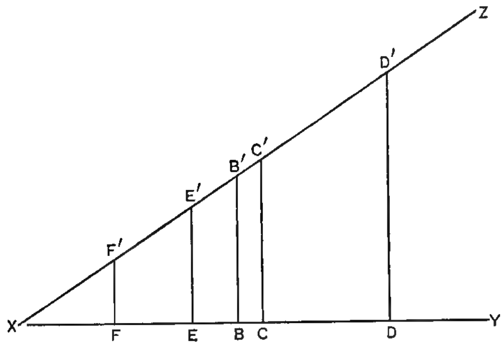


FIG. 38.

AB, BC, CD, etc., and may be applied as the first trial lengths for [FF]. Notice that, no matter what their absolute lengths may be, as long as FF', $F'F''$, etc., are proportional to AB, BC, etc., the point F^{V} will always lie on the straight line FT, Fig. 37.

Let l_1, l_2, etc., denote measured lengths AB, BC, etc.;
θ_1, θ_2, etc., " bearings of AB, BC, etc.;
L " total error in latitude;
D " " " " longitude;
x_1, x_2, etc., " length corrections to be applied to AB, BC, etc.

Then, if after the applications of the corrections x_1, x_2, etc., the survey is to close, the following equations will be true:

$$\pm x_1 \cos \theta_1 \pm x_2 \cos \theta_2 \ . \ . \ . \pm x_n \cos \theta_n = L, \qquad (11)$$
$$\pm x_1 \sin \theta_1 \pm x_2 \sin \theta_2 \ . \ . \ . \pm x_n \sin \theta_n = D, \qquad (12)$$

the signs of the terms being determined by considering N and E corrections + and S and W corrections —. In the case shown in Fig. 36, the sign of the first term in (11) would be plus, while the sign of the corresponding term in (12) would be minus, etc.

The corrections being proportional to the lengths of the sides to which they are to be applied, (11) and (12) may be expressed in terms of x_1 thus :

$$\pm x_1 \cos \theta_1 \pm \frac{l_2}{l_1} x_1 \cos \theta_2 \ldots \pm \frac{l_n}{l_1} x_1 \cos \theta_n = L, \quad (13)$$

$$\pm x_1 \sin \theta_1 \pm \frac{l_2}{l_1} x_1 \sin \theta_2 \ldots \pm \frac{l_n}{l_1} x_1 \sin \theta_n = D. \quad (14)$$

Dividing (14) by (13),

$$\frac{\pm l_1 \sin \theta_1 \pm l_2 \sin \theta_2 \ldots \pm l_n \sin \theta_n}{\pm l_1 \cos \theta_1 \pm l_2 \cos \theta_2 \ldots \pm l_n \cos \theta_n} = \frac{D}{L}. \quad . \quad . \quad (15)$$

Eq. (15) gives the condition under which the survey will close, or the line FT coincide with FA.

Analytically, the method of balancing by trial is unsatisfactory. When, however, the survey has been made with sufficient care, and with an instrument sufficiently accurate, to determine the angles correctly, the length measurements should be proportionately close, and the error of closure, therefore, so small as to render the length corrections inconsiderable.

41. Maximum Allowable Ratio of Closure. General Remarks on Balancing.—In general compass work, a ratio of closure of one in five hundred is fair, but, for good work, one in one thousand should not be exceeded.

It is not necessary that the corrections applied to the latitudes, etc., should be correct down to the last decimal. For instance, suppose, in a case under Art. 38, that the length of one line was 1005 ft., while that of another was 2035 ft. If we had determined the correction to the latitude of the former line to be 1 ft., we would at once call the correction to the latitude of the latter 2 ft., as the second line is about twice as long as the first. Suppose, also, in this same case, that the sum of all the lines was 10,100 ft., and that the error in latitude was 10.5 ft. The length of the first line, 1005 ft., is about one tenth of the sum of all the lengths, and we, therefore, apply to the latitude of that line a correction of one foot, which is about one tenth of the error in

latitude. Cases which come under Articles 39 and 40 are disposed of in a similar manner, the judgment of the computer being exercised in each instance. Be careful, however, to make all corrections very nearly equal to their exact values. Be sure, also, before attempting to plot the survey or compute the area, that the latitudes and longitudes balance *exactly*. If, after the corrections have been applied in the manner described above, there is still a small error remaining, it must be eliminated by a further slight change in the figures. Never alter the original field-notes unless they are found to be radically in error. The corrections to the latitudes are shown only on the sheet on which the final computation is made (Form C, Art. 43) and, as a general rule, the corrected lengths of lines are not computed except in cases under the general case of Art. 40, and then only to assist the computer in finding the latitude and longitude corrections. It is for this reason that the second rule under the case of the rectangular field, Art. 40, is preferable. If desired, the new lengths, and, in cases under Articles 38 and 39, the new bearings, of the lines can be computed. If this is done, enter them in red ink over the original lengths and bearings in the notes, but do not change these latter. The corrected length of a line is, evidently, the square root of the sum of the squares of the corrected latitude and the corrected longitude, while the tangent of the corrected bearing equals the corrected longitude divided by the corrected latitude. The following graphical method of computing corrections for latitudes and longitudes in cases under Articles 38 and 39 will faciltiate that work. A modified form of the method can be applied to cases under Article 40, first part. In cases under Articles 38 and 39 proceed as follows, Fig. 39: First, lay off, to some convenient scale, a line XY whose length is equal to the sum of the lengths, or of the lengths multiplied each by its respective weight, of all the sides of the traverse. At the point Y, erect a perpendicular YZ, drawn to scale to equal the total error in latitude or longitude as the case may be. The perpendicular need not be drawn to the same scale as the line XY, but must be drawn to the largest scale practicable, say five feet to the inch. Join X and Z. Next lay off from X towards Y, to the scale of the line XY, distances XB, HC, XD, etc., equal to the lengths

AB, *BC*, *CD*, etc., of the different sides of the traverse. From *B*, *C*, etc., erect perpendiculars *BB′*, *CC′*, etc. The length of any one of these perpendiculars scaled off to the scale of the line

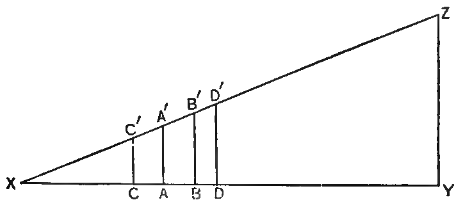


FIG. 39.

YZ will be the correction to be applied to the latitude, or the longitude, as the case may be, of the line at the extremity of whose equal in scale length the perpendicular has been erected.

42. Calculating the Area.—As defined in Art. 35, the "double meridian distance" of a line is twice the perpendicular distance of the middle point of the line from some assumed meridian. Throughout this work, in the case of a closed traverse, this meridian will always be assumed through the most western point of the traverse, and, therefore, all D. M. D.'s will be plus, that is, measured to the east, and all lines of the traverse will be entirely east of the meridian. Consider the case of the two lines *AB* and *FG*, shown in Fig. 40. *NS* being the meridian, the area *ABC*, included between it and the line *AB*, is a triangle and equals $\frac{1}{2}BC \times AC$. *ED* is the meridian distance of *AB*, that is, one half the D. M. D. of *AB*. Also *ED* equals one half of *BC*. Therefore *BC* equals the D. M. D. of *AB*. *AC* is the latitude of *AB*. Therefore the area included between the line *AB* and the meridian equals one half the product of the latitude of *AB* by its D. M. D.

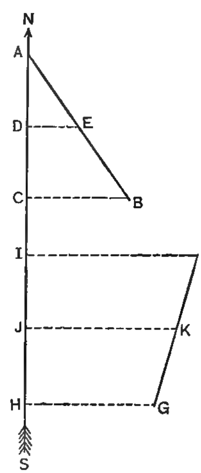


FIG. 40.

The area *FGHI*, included between the line *FG* and the meridian, is a trapezoid, and equals

$$\frac{GH + IF}{2} \times HI.$$

The first of these two factors is one half of the D. M. D. of the line FG, and the second is its latitude. Therefore, in this case also, the area between the line and the meridian equals one half the product of the D. M. D. of the line by its latitude. These two lines represent the only possible cases when the meridian is drawn through the most western point of a closed traverse, except when the line coincides with the meridian. The area between the line and the meridian is then zero, and the rule is seen to apply. We may say generally, then, that the area included between any line and the meridian equals one half the product of the D. M. D. of the line by its latitude. The D. M. D.'s all being plus, lines whose latitudes are north, or plus, give north, or plus, areas, while lines whose latitudes are south, or minus, give south, or minus, areas.

Fig. 41 shows the case of a closed traverse with the meridian, NS, drawn through its most western station, A. The problem before us is to determine a rule for finding the area included

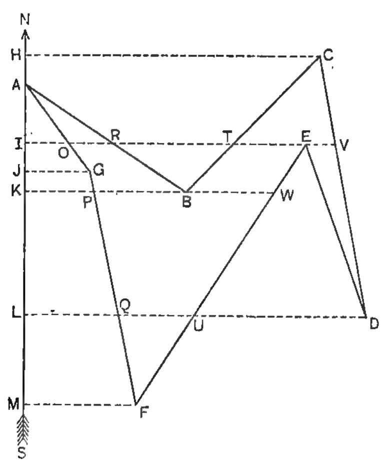


FIG. 41.

within the boundaries of this or any other closed traverse. Supposing that the lines of the survey have been run around in the direction of the hands of a watch. Consider the part CV of the line CD. The area between CV and the meridian is $HCVI$ and is a south area. In like manner the area $HCTI$ between CT

and the meridian is north. The algebraic sum of these two areas equals the area *VETC*, which is part of the area of the field. The area *EVD* is the algebraic sum of the south area *IVDL* and the north area *IEDL*. The area *PBWUQ* is the algebraic sum of the south area *KWUL* and the north area *KPQL*. And so, on examination of all the lines of the survey, the areas between these lines and the meridian will be found to to be arranged in pairs. For every north area there will be a corresponding south area, and the algebraic sum of these two will be a part of the total area to be found, the sum of all these parts giving the total area desired. Hence to find the area of any closed traverse it is necessary only to find the areas, north, or plus, and south, or minus, included between its lines and the meridian, and take the algebraic sum of all these. As the D. M. D. of a course multiplied by its latitude gives *twice* the area included between the course and the meridian, it will be more convenient to find the double area of the whole closed traverse and divide this by two. Accordingly, for finding the area bounded by any closed traverse, we have the following

RULE.—*Multiply the D. M. D. of each course by the latitude of that course, giving the product its proper algebraic sign. One half the algebraic sum of all these products equals the area desired.*

The method just described is the standard for the calculation of the area of any closed traverse. Other methods, more or less approximate, will suggest themselves to the student. Where convenient the tract may be divided into a number of triangles, the area of each of these found separately and their sum taken to give the whole. For fairly approximate work the dimensions of these triangles may be scaled from the finished map. If the tract has been plotted on cross-section paper (Art. 44), the area of each square is known from the scale and the number inside the traverse can be counted, fractional squares being estimated. For planimeter methods see Chapter VII.

In Art. 23 a method has been indicated for computing the area included between a straight line and a curve. A closer approximation to this last area is obtained by the application of "Simpson's Rule." Let it be desired to compute the area

ABCD, included between the straight line *AB*, the curved line *DC*, and the two lines *AD* and *BC*, perpendicular to *AB*, Fig. 42. Call the length *AD*, y_0, and *BC*, y_n. By means of the additional and equidistant ordinates y_1, y_2, etc., parallel to *AD* and *BC*, divide the area *ABCD* into n sub-areas, n being an *even* number. Call the perpendicular distance between any two ad-

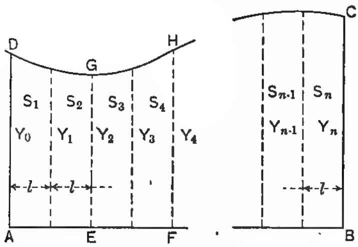


FIG. 42.

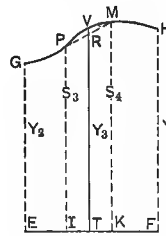


FIG. 43.

jacent ordinates, including the extreme ones *AD* and *BC*, l. the sub-areas are regarded as *trapezoids* the total area will be

$$S = \frac{y_0 + y_1}{2} \times l + \frac{y_1 + y_2}{2} \times l + \ldots + \frac{y_{n-1} + y_n}{2} \times l$$

$$= \tfrac{1}{2}l(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n).$$

This is known as the "Trapezoidal Rule," and is the same as that given in Art. 23. Where the curve *DC* is *concave* to the base line *AB*, the trapezoidal rule gives results which are too small. Similarly, with a convex curve, the computed results are too large. Consider the area between any two consecutive even ordinates as y_2 and y_4. Divide the portion *EF* of the base line into three equal parts, *EI*, *IK*, and *KF*, and erect the ordinates *IP* and *KM*, thus dividing the area *GHFE* into three sub-areas the width of each of which is two thirds that of the sub-areas in Fig. 42. Now regarding *these* three sub-areas as trapezoids,

$$GHFE = \left(\frac{y_2 + IP}{2}\right)\frac{2}{3}l + \left(\frac{IP + KM}{2}\right)\frac{2}{3}l + \left(\frac{KM + y_4}{2}\right)\frac{2}{3}l$$

$$= \frac{l}{3}(y_2 + 2IP + 2\,KM + y_4)$$

$$=\frac{l}{3}(y_2 + 4RT + y_4),$$

as $$RT = \frac{IP + KM}{2}.$$

This expression evidently represents a closer approximation to the true area of *GHFE* than would have been obtained by the application of the trapezoidal rule to the areas *GVTE* and *VHFT*. Now if in the above expression we substitute for RT, $TV = y_3$, it is evident that a still closer approximation to the true area will be reached. Thus

$$GHFE = S_3 + S_4 = \frac{l}{3}(y_2 + 4y_3 + y_4).$$

Applying this formula to each pair of consecutive sub-areas in Fig. 42, we have

$$S_1 + S_2 = \frac{l}{3}(y_0 + 4y_1 + y_2)$$

$$S_3 + S_4 = \frac{l}{3}(y_2 + 4y_3 + y_4)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$S_{n-1} + S_n = \frac{l}{3}(y_{n-2} + 4y_{n-1} + y_n).$$

And adding all of these we have

$$S = \frac{l}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 \ldots + 2y_{n-2} + 4y_{n-1} + y_n).$$

Consequently the following is

SIMPSON'S RULE.—*To obtain the area included between a straight base line, a curve, and two perpendiculars drawn to the base line at its extremities: Divide the base line into an even number of equal parts and, at the points of division, erect perpendicular ordinates to it, terminating in the curve. Number the ordinates, including those at the ends of the base,* 0, 1, 2, *etc., from left to right. Add together the first ordinate, the last ordinate, four times the sum of all the intermediate odd ordinates, and twice the sum of all the intermediate even ordinates; multiply the total*

sum by one third the common distance between adjacent ordinates. The result is the area required.

For another proof of Simpson's Rule, see Johnson's "Surveying," Appendix V. Note also under "Mensuration" in the "Pocket Companion" published by the Carnegie Steel Co., edition of 1900.

43. Computation Rules, and Systematic Arrangement of Computations.—In technical computations an important question is that concerning the number of figures necesssary to be retained at any step of a computation in order to insure the final result against a given percentage of error. The question is discussed in Professor Holman's "Computation Rules" and also more at length in his "Precision of Measurements."*

In general land survey computations, the work will be done with the aid of logarithms, and five-place tables will insure sufficient accuracy.

Form B is recommended for use in the computation of latitudes and longitudes, while Form C will be found suitable for the final balancing of the survey and the calculation of the area. Note that in Form B the log length is added upward to the log sin bearing and downward to the log cos bearing. The use of the various columns of these forms will be understood from their headings.

44. Plotting the Traverse.—The latitudes and longitudes of all the lines of the survey being carefully computed, corrected, and checked, and entered on Form C, we may proceed to plot the work. Find the difference in latitude between the most northern and the most southern points, and the difference in longitude between the most eastern and the most western points, laying off on the paper a rectangle whose dimensions are these two quantities. It is evident that the traverse, whether closed or not, will be inscribed in this rectangle, which should be laid off and checked with great care. Having found the difference in latitude between the most western and the most northern points of the traverse, lay this distance off from the northwest

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* "Computation Rules and Logarithms," The Macmillan Co., New York. "Precision of Measurements," John Wiley & Sons, New York.

corner of the rectangle south along its west line. This will give the location of the most western point. To locate any other point, find its total latitude south of the most northern point, and its total longitude east of the most western point. Lay off the total latitude of the point south of the most northern point, south from the north line of the rectangle along both its east and its west lines, joining, with a straight line, the two points thus found. In like manner, lay off the total longitude of the point east of the most western point, east from the west line of the rectangle along both its north and its south lines, joining the two points thus found. The intersection of the two lines thus drawn will give the location of the point desired. It is best not to draw these two lines entirely across the rectangle, but only for a few tenths of an inch on each side of their intersection, or, in fact, just enough to give that intersection. It will be found convenient, before commencing the plot, to make a table of the total latitudes of all stations south of the parallel through the most northern point of the traverse, and the total longitudes of all points east of the meridian through the most western station of the traverse, thus making the northwest corner of the rectangle the origin of coordinates. All the stations being located in the above manner, they can be joined by lines, and the survey will be plotted. Note that all measurements are made directly from the west line along the north and the south lines, or from the north line along the east and the west lines. Be careful of this, as it is not good practice, unless otherwise impracticable, to lay off part of a distance along a line and then the remainder starting from the point thus found. By such a method any error made in the first measurement will be introduced into the second at the start. In the case of a large map, a convenient modification of this scheme will be found in dividing the large rectangle into a series of smaller squares and fractional squares, the fractions lying on the east and south sides. The measurements can then be started from any interior line, the side of a square being taken at say 500 feet. Check the work of plotting by scaling off the plotted lengths of the lines, which should agree with the recorded lengths. If these recorded lengths have not been corrected, as is usually the case, in accordance with the

corrections applied to their latitudes and their longitudes, still, the agreement between the recorded and plotted lengths should be very close. Of course, if the field-work has been poor and the corrections applied accordingly large, the lengths themselves will have to be corrected before any agreement can be expected between their recorded and scaled values. The above is the best method of plotting the survey. The work can be facilitated further by the use of "cross-section paper." Fig. 44 shows a paper of this kind divided into sections, one tenth of an inch

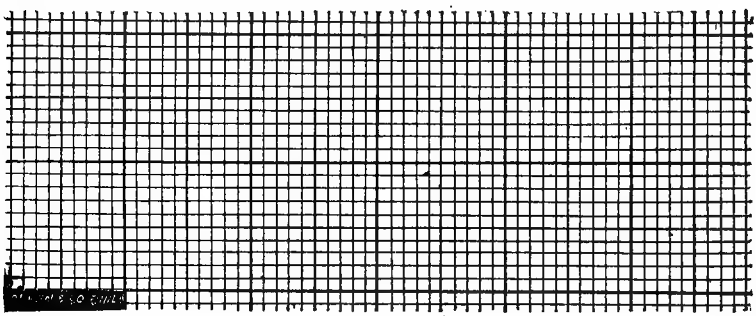


FIG. 44.

square, heavy lines, horizontal and vertical, being ruled at every even inch. With such paper as this, the use of an auxiliary scale is unnecessary. Its use, however, is generally confined to preliminary drawings.

If merely a sketch showing the general appearance of the traverse is desired, and there is not time to use the method described above, by the use of a protractor a plot can be made without calculating the latitudes and the longitudes. The protractor, as usually made, is simply a circle or semi-circle with the center marked, and the circumference divided in angular units. For plotting compass traverses, as the compass is read to the nearest five minutes, a protractor reading to five minutes should be used, though fairly good results can be had with the large paper protractor divided to quarter degrees. In any case, the method of procedure is as follows: Having selected some central spot on the paper, assume a meridian through this, and, by

means of the protractor, draw a series of lines, radiating from the central spot, and with the bearings of the lines of the survey. Then, starting from any point, draw the first line of the traverse parallel with the line which has its bearing, and scale off the proper length. Then, starting from the extremity of this first line, draw the second line in a similar manner, and so on around. If the traverse is closed, it can be checked by the distance the further extremity of the last line falls from the place of beginning.

Figs. 45 and 46 show forms of protractors.

The remarks in Art. 22 apply to the plotting of maps in general. On a compass survey map the bearing of each line should be

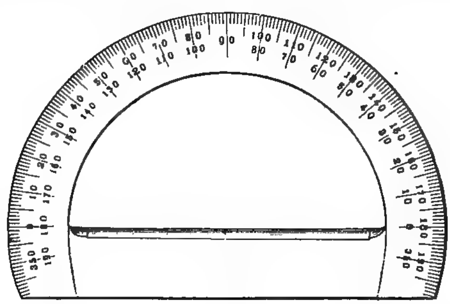


FIG. 45.

shown. The declination should be stated, as also whether the bearings given are true or magnetic. It is customary to draw two crossed arrows, one, with a full head and feather, indicating the true meridian, and the other, with a half head and feather, indicating the magnetic meridian.

45. **Supplying Omissions.**—If, for any reason, it has been found impossible to measure the length or bearing of one or more lines of a closed traverse, they can be supplied from the remaining notes, provided that there are not more than two unknown quantities, that is, that not more than two bearings, two lengths, or one bearing and one length are wanting. It is to be remarked, however, that the methods here given assume generally that the field-work has been exactly correct. That is, if two sides or bearings, or one side and one bearing have been omitted, the traverse, of course, will not balance, and the missing sides or bearings or side and bearing are supplied so as exactly to fill the gap thus

left. Now, this gap is due not alone to the omission of the sides in question, but also to the errors made in the measurement, linear and angular, of the other sides. Consequently, where there are two omissions to be supplied, no opportunity is offered for testing and balancing the survey. Where only one side or one bearing has been omitted, a method of balancing described below can

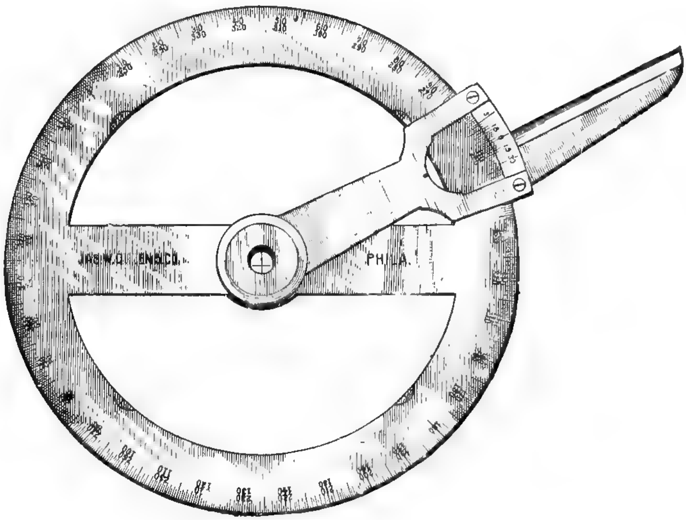


FIG. 46.

be used, but it is not entirely satisfactory. On account of these facts, the methods for supplying omissions will not be resorted to where the most accurate work is desired. They are useful when an approximate map of an area is desired in the shortest possible time and where the difficulties offered to measurement by one or two of the lines are such as unnecessarily to prolong the work. There are six possible cases:

1. Length } of one line wanting.
2. Bearing } of one line wanting.
3. Length and bearing } of one line wanting.
4. Lengths of two lines wanting.
5. Bearings of two lines wanting.
6. Length of one line and the bearing of another wanting.

CASES 1, 2, AND 3.—Note that in each of these cases, the missing data refer to one line only. The latitudes and longitudes of the other lines being computed, it is evident that the missing line is the hypothenuse of a right-angled triangle of which the errors in latitude and in longitude are the two sides. The tangent of the bearing of the missing side will be equal to the error in longitude divided by the error in latitude. If the error in latitude is north, then the missing line will be a southerly line, and *vice versa*. If the error in longitude is east, the missing line will be westerly, and *vice versa* (see Art. 37). In an example under case 3, after the omissions have been supplied as described above, the survey will balance exactly, but in examples under cases 1 and 2 the circumstances are somewhat different. Suppose, for instance, an example under case 1. The bearing of a given line has been observed, but its length is wanting. We would first calculate the length by means of the formula

$$\text{unknown length}^2 = (\text{error in lat.})^2 + (\text{error in long.})^2. \qquad (16)$$

Now, the unknown length found in this way when multiplied by the sine and cosine of the observed bearing of the line might not, and probably would not, give results which would balance the survey. Accordingly, having found the length of the line we would compute its latitude and longitude and balance the survey in the usual manner.

In an example under case 2, where the bearing is what is wanting, we would first find this bearing by the formula

$$\tan \text{unknown bearing} = \frac{\text{error in long.}}{\text{error in lat.}}; \quad . \quad . \quad . \quad (17)$$

then calculate the latitude and the longitude of the line and balance as usual.

CASES 4, 5, AND 6.—In these cases, note particularly that the omissions concern two lines, and that, if these two lines adjoin, generally speaking they, together with the line which would close the survey were they omitted entirely, form a triangle. If the two lines do not adjoin, by shifting some of the other sides of the traverse they may be made to adjoin. Thus in Fig. 47 suppose

CD and *DE* the wanting lines. They together with a line *CE*, which would close the survey were they omitted entirely, form the triangle *CDE*. If the two wanting lines were *AB* and *FE*,

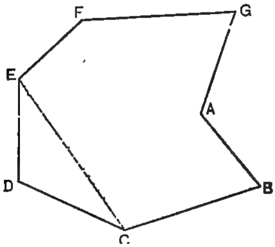


FIG. 47.

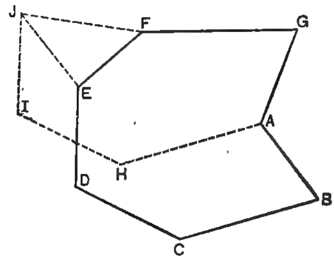


FIG. 48.

the sides could be shifted as shown in Fig. 48. *AH* being parallel and equal to *BC*, *HI* parallel and equal to *CD*, *IJ* parallel and equal to *DE*, and *JE* equal and parallel to *AB*. Evidently *JF* is a line parallel and equal to; or, in fact, is the line which would close the field were *AB* and *EF* omitted entirely. In what follows, then, the wanting lines will be assumed as adjoining each other.

CASE 4.—The lengths of two lines wanting. Find by case 3 the length and bearing of the line which would close the survey, were the two lines in question omitted entirely. Then in the resulting triangle there will be known all the angles and the length of one side. The triangle can be solved by the ordinary methods of trigonometry.

In case the two deficient sides are parallel, the situation is as shown in Figs. 49 and 50, *AB* and *DE* being the deficient lines.

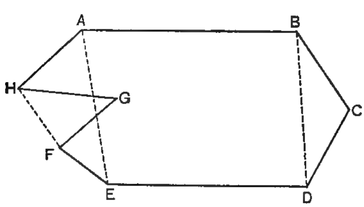


FIG. 49.

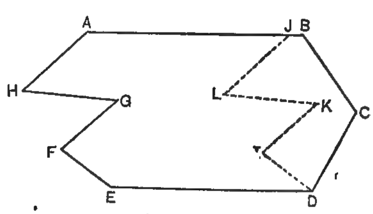


FIG. 50.

Fig. 49 shows the traverse in its proper shape, and Fig. 50 shows it with *AB* and *DE* shifted so as to adjoin. *JB* is the line which

would close the survey were AB and DE omitted. Note, however, that this assumes that the remaining lines balance exactly in a direction perpendicular to the deficient lines, and that, in general, the closing line BJ will not be found parallel with the two deficient lines. Evidently, in any event, there is no triangle found, that is, in the ordinary sense of the word, and the lengths generally speaking are indeterminate. If, however, the area of the whole survey is known the lengths can be determined as follows, Fig. 49: Swing the traverse so as to bring the deficient parallel sides into the meridian. Correct the bearings of the other lines accordingly and calculate their latitudes and longitudes. Calling the length of the first deficient side l_1 and that of the second l_2,

$$(l_1 - l_2) = \text{the error in latitude.} \quad . \quad . \quad . \quad . \qquad (18)$$

The longitudes being balanced by the ordinary methods, with l_1 and l_2, however, omitted, supply, as in case 3 of this article, the lines AE and BD, and compute the areas BCD and $AEFH$. In the particular case shown in Fig. 49, to compute the area $AEFH$, supply HF as a closing side to the triangle HGF and proceed as usual. The known area of the field plus the area HGF and minus the sum of the areas $AEFH$ and BCD equals the area $ABDE$. Calling this S,

$$S = \frac{l_1 + l_2}{2}(BD \sin BDE). \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

From equations (18) and (19) the unknown lengths may be computed.

CASE 5.—The bearings of two lines wanting. In this case we have a triangle in which are known the three sides and none of the angles. From this data the angles may be computed and the unknown bearings determined. In determining the bearings from the angles, care must be exercised. For instance, in Figs. 51 and 52 AB and DE are the lines whose bearings are supposed to be wanting in each case. All the lines in one figure are equal in length to the corresponding lines in the other. The bearings are also the same, except in the case of the lines AB and DE.

The angles which the deficient lines AB and DE make with the computed closing line are all that can be determined from the solution of the triangle. On which side of the closing line the

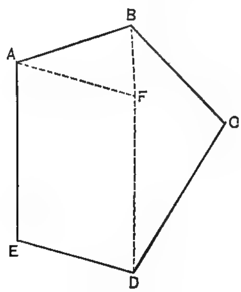


FIG. 51.

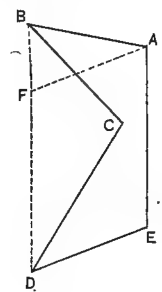


FIG. 52.

deficient lines lie and at which end of it each one adjoins it must be determined by the surveyor's general knowledge of the traverse and the bearings computed accordingly. In a case of this sort, it will be well to make an approximate sketch of the traverse before commencing the work of supplying the omissions. If the deficient lines are parallel as AB and DE, Fig. 53, it will be indicated by the length of the closing line's being just equal either to the difference or to the sum of the lengths of the two deficient lines, these lines being parallel with the closing line.

CASE 6.—The length of one line and the bearing of another wanting. The corresponding case in trigonometry is that of a triangle with two sides and the angle opposite one of them given. It will be remembered that, generally speaking, there are two solutions. Thus in Fig. 54, $ABCDE$ and $A'BCDE'$ are two closed traverses in the first of which all the data are known except the length of AB and the bearing of DE, and in the second of which all the data are known except the length of AB and the bearing of DE'. Also $DE = DE'$. The closing line in each case, with the two deficient lines omitted entirely, is FB, and the two solutions of the triangle are AFB and $A'FB$. It is evident that, in such a case as this, the proper solution would have to be determined by the surveyor's general knowledge of the traverse. In any example under cases 4, 5, and 6, first draw

an approximate sketch of the traverse and of the triangle to be solved. This will prevent mistakes and facilitate the work. In this last case, if the two deficient sides are perpendicular to each other, there will be only one solution. This will also be true if the side whose bearing is unknown is longer than the closing

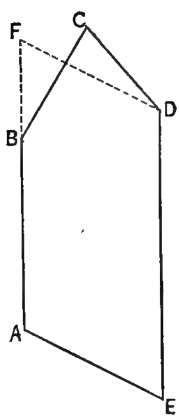


FIG. 53.

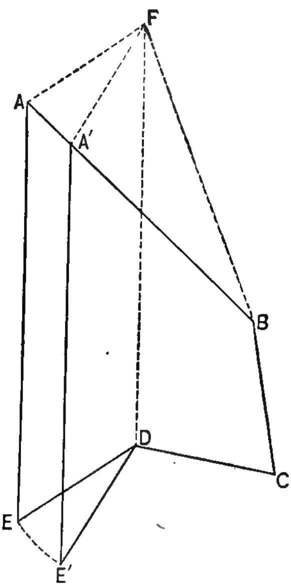


FIG. 54.

side. These are merely different forms of the same statements under the corresponding case in trigonometry. If the two deficient sides are parallel, it will be indicated by the bearing of the closing side's being found parallel with the deficient side whose bearing is known. As in case 5, the length of the closing side will equal the sum or the difference of the lengths of the two deficient sides.

46. Parting Off Land.—Under this head, numerous problems claim the attention of the surveyor. The solution of a few of these are given in this article. In any case where the length or direction of a line is *calculated*, be sure to run the line on the ground to see whether the measured and computed values agree.

1. To part off a given area from a known tract (Fig. 55). Supposing that the field in Fig. 55 has an area of S square

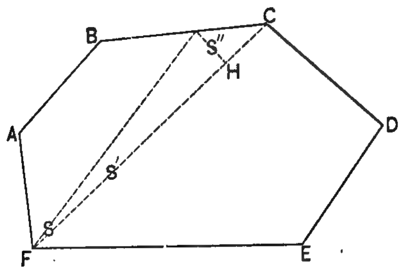


FIG. 55.

units, for instance, feet, and that it is desired to part off S' square feet. All the field notes being known, proceed as follows: Starting from any corner of the field, as F, draw a line to any other corner, as C, so that the area cut off, $FEDC$, will contain, approximately, S' square feet. This can be determined by inspection. By the method of case 3, Art. 45, supply the line FC. Calculate the area $FCDE$ and find the difference between this area and S'. In the Fig. $FCDE$ is supposed to be less than S'. The method of procedure is practically the same in either case. The problem now resolves itself into adding to $FCDE$ a triangle FGC whose area is the difference between S' and $FCDE$. Call this difference S''. Then, GH being perpendicular to FC,

$$S'' = \frac{FC \times GH}{2} = \frac{FC \times GC \sin GCF}{2}.$$

Hence
$$GC = \frac{2S''}{FC \sin GCF}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

All the quantities on the right side of (20) are known, the angle GCF being determined from the known bearings of BC and FC. Consequently the equation can be evaluated for GC. That distance being laid off from C towards B will give the point G, and a line joining G and F will cut off from the whole survey the area S'.

2. To part off, by a line parallel with a given line, an area S' square units from a tract containing S square units.

In Fig. 56 let $ABCDEF$ be the tract and LM the given line. By inspection of the plot, decide on some corner, as A,

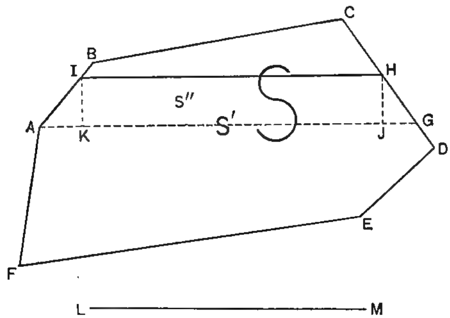


FIG. 56.

from which a line, running across the field parallel with LM, will cut off approximately an area equal to S'. The bearings of AG and DG are known, and we may, therefore, supply the lengths of these lines by case 4 of Art. 45. Having supplied these lengths, calculate the area $AGDEF$, and find the difference between this and S'. The problem now resolves itself into adding to, or subtracting from, $AGDEF$ a trapezoid, in this particular case $IHGA$, whose area equals the difference between S' and $AGDEF$. Calling this difference S'', we have

$$S'' = \frac{IH + AG}{2} \times HJ. \quad \quad (21)$$

$$IH = AG - HJ\,(\cot IAK + \cot HGJ). \quad \quad (22)$$

$$HJ = \frac{AG - IH}{\cot IAK + \cot HGJ}. \quad \quad (23)$$

Substituting (23) in (21)

$$S'' = \frac{IH + AG}{2} \times \frac{AG - IH}{\cot IAK + \cot HGJ}$$

and

$$IH = \sqrt{AG^2 - 2S''\,(\cot IAK + \cot HGJ)}. \quad \quad (24)$$

Having found from this the length IH, substitute it in (21) and

solve ior HJ. Then

$$GH = \frac{HJ}{\sin HGJ}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

and we may lay off this distance from D towards C, thus finding a point H which is one of the extremities of the line required.

3. To part off from a tract a given area in such a way that a spring on the original tract shall be situated on the boundary between the two subdivisions, Fig. 57. Let W represent the location of the spring. Run a line from W to any convenient

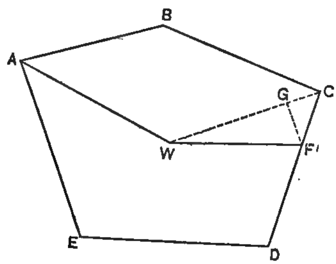


FIG. 57.

corner of the tract as A. This will have to be done on the ground. Then supply, from the field-notes, a closing line WC to some other corner, such that the area cut off, $WABC$, will approximate the area desired. The problem then becomes the same as problem 1 of this article. If the desired area is to be cut by a line passing through the spring and which shall be straight between its intersections with the boundary of the original tract, the problem becomes more complicated. A solution may be found in Gillespie's or in Carhart's "Surveying."

47. Other Field Problems.—1. To run a straight line between two points not visible from each other, Fig. 58. Let A and B be the two points, separated in this case by woods. Starting from A run along a random line AC approximately in the direction of B until a point C is reached from which B is visible. Measure CB and take its bearing. Having previously marked off regular distances AD, DE, EF, etc., along AC, return to these points and

FIG. 58.

lay off lines parallel with CD and of lengths proportional to the distances of the points from A. For instance, $FJ = \frac{AF}{AC} \times CB$, etc. The points H, I, etc., will be on the line AB.

Or, knowing the lengths and bearings of the lines AC and CB, we can calculate the angle ACB. Then in the triangle ACB there will be known the two sides AC and CB and their included angle ACB. From these the angle BAC; and hence the bearing of AB, can be determined. Starting from A and laying off this bearing, the line AB may be run out. In running a compass line through woods, if a tree is found to be exactly on the line, it will be sufficiently accurate to place the compass on the further side of the tree, as nearly as may be on the line, and proceed with the work. The compass, though a very valuable instrument, is not highly accurate, and, consequently, with it, it is a waste of time to attempt refinements of measuring.

2. To straighten a crooked boundary (Fig. 59). Supposing that the boundary between two fields, between the

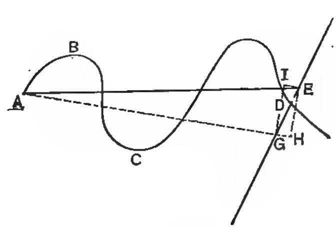


FIG. 59.

points A and D, Fig. 59, is the stream $ABCD$, that at D the stream intersects another common boundary of the two fields, the straight line FD, and that it is desired to substitute for the stream a straight line running from A to some point on FD, as E, in such a way that the area of each field shall remain the same as before. Run a random line AG, measuring offsets from it on both sides to the stream, and calculate the areas between the line and the stream, on both sides of the line. If these areas are equal, the random line is the one required. If not, find the difference between them. In the figure, the area

on the lower side of the random line is the smaller, and, consequently, the point G must be moved up towards F. Calling the area on the upper side of the random line S_1, and that on the lower S_2, if EH is perpendicular to AG, the following equation must be made true:

$$S_1 - S_2 = \frac{AG \times EH}{2}. \quad \ldots \ldots \quad (26)$$

From which

$$EH = \frac{2\,(S_1 - S_2)}{AC}. \quad \ldots \ldots \quad (27)$$

From G, run out the line GI perpendicular to AG and of the length given by (27). From I run out a line parallel to AG. E, the intersection of this line with the line GF, will be the end of the line required.

3. To obtain the length of any inaccessible line. (See also 9 of Art. 20.) This problem, in general, is solved by the methods of trigonometry. It becomes necessary to include the line in a triangle, three of whose parts, one of which is a side, can be measured directly. If possible, simplify the work by making the triangle right-angled. Trigonometries, notably Crockett's, written for the use of engineering students, contain numerous examples of this kind. Gillespie's "Surveying" will also be found a valuable work.

48. Network of Traverses.—In Fig. 60 is shown a case of a field, in the interior of which it has been necessary to run out a number of closed traverses tying on to each other at various points. Such a case would occur in the survey of a park containing numerous winding paths. Manifestly, it would be difficult to balance the survey. The following is a practical method: First balance the outside boundary. Then take some principal traverse, as 5–6–7–8, intersecting the outside boundary in two points, 5 and 8. This traverse, together with the lines 5–1, 1–2, and 2–8, will form a closed traverse which can be balanced as such. Only, care must be taken not to change the lines 1–2, 5–1, or 2–8, which have been balanced already. Next we might balance the closed traverse, 6–12, 12–

13, 13–14, 14–7, 7–6, taking care not to alter 6–7, which has been balanced before. And so on, throwing the balancing of each traverse entirely into the new lines, leaving the old ones undisturbed. In a case, as in the second traverse, where any one of the lines, as 2–8, is *part* of a line already balanced, we would allot to the line 2–8 the same proportional part of the total *balanced* latitude and longtitude of 2–3, as the line 2–8 is of 2–3, etc. In making a survey of this sort it is best to locate stations as we go along, wherever they may be needed. For

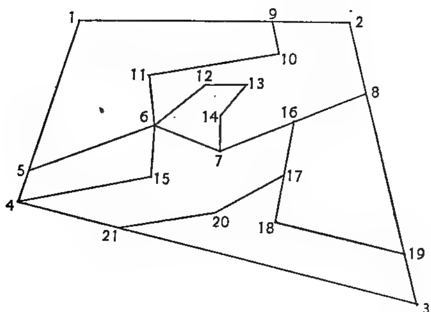


FIG. 60.

instance, in running the line 16–18, it is noticed that a path leads off in the direction 17–20. We accordingly locate a station at 17, and, having finished the traverse 16–18–19, can return to 17 and run off the traverse 17–20, 20–21. Such work as this requires careful note keeping.

49. **The Vernier.**—The vernier, alluded to in Art. 27, is a device for measuring smaller distances and angles than could otherwise be distinguished. It is named after its inventor, Vernier, and is an accessory of nearly all surveying instruments. Its use in connection with the compass is to set off the magnetic declination, and, as this is to be discussed in the next article, a thorough understanding of the vernier becomes necessary.

Suppose that two bars of equal length are divided, the one into N parts and the other into $N + 1$ parts, and that these bars are placed adjoining each other throughout their entire

length, as in Fig. 61. Now, suppose that one of these bars remains stationary and the other slides along it. It does not

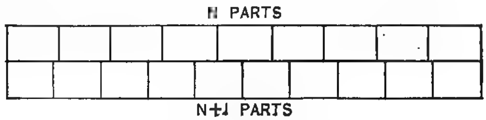


FIG. 61.

matter which moves. For the sake of illustration let the upper one in Fig. 61 move to the right. Just after it starts to move, no line on one bar will coincide with a line on the other, but after the upper bar has moved a certain distance, the first line to the left of its right end will come into coincidence with the first line to the left of the right end of the lower one. It is evident that these two lines will be the first to come into coincidence and that this will happen when the upper bar has moved to the right a distance just equal in length to the difference between the length of a division on the upper bar and that of a division on the lower. Such a *combination* of divided bars is known as a "vernier and scale." Either may be the vernier and either the scale. The difference between the length of a division on one bar and that of one division on the other is known as the "least count" or "least reading" of the vernier.

In Fig. 62 let us consider the upper divided bar as the vernier, and that it moves to the right as before. The line

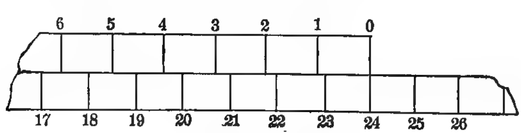


FIG. 62.

marked 0 on the vernier coincides with a line on the scale. The scale is shown extending on to the right and the numbers on the vernier run in an opposite direction to those on the scale. When the vernier has moved one least count to the right, the line marked 1 on the vernier coincides with a line on the scale, and the line marked 2 on the vernier is still one

least count to the left of the nearest line on the scale. When the line marked 2 on the vernier comes into coincidence with a line on the scale, the vernier will have moved two least counts to the right, etc. That is, if the zero line on the vernier coincides with any line on the scale, and the vernier is moved along the scale any distance less than a whole scale division, the number of the line on the vernier which coincides with a line on the scale will indicate the number of least counts through which the vernier has moved. When the zero of the vernier has moved over exactly one scale division, the line at the other extremity of the vernier will also coincide with a line on the scale, and the measurement is commenced over again. From which note that the vernier is used for measuring only distances smaller than the smallest division on the scale. For instance, in the figure the zero of the vernier is shown coinciding with the 24 line on the scale. Supposing that the vernier is moved along the scale to a point between the 28 line and the 29 line. We would say that the vernier had moved four scale divisions and so many least counts, the number of least counts being indicated by the number of the line on the vernier which is found in coincidence with a line on the scale. In Fig. 62, the divisions on the vernier are larger than those on the scale and the numbers on the vernier run in an opposite direction to those on the scale. On this last

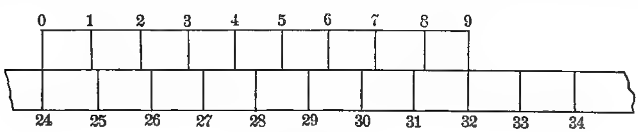


FIG. 63.

account, it is known as a "retrograde" vernier. In Fig. 63, in which the upper bar is still considered as the vernier, the divisions on the vernier are shorter than those on the scale, and the numbers all run in the same direction. This is known as a "direct" vernier, and is the one almost always used. Note that, in the direct vernier, its length lies on ahead in the direction in which the scale numbers run, and that, in the retrograde vernier, the reverse is the case. In practice, the

scale divisions are in some standard unit, as tenths of a foot, and the least counts in some smaller standard unit, as hundredths or thousandths. On surveying instruments, the vernier and scale are circular, the scale divisions being half degrees, twenty-minute spaces, etc., and the counts, single minutes, ten-second spaces, etc. To "read" a vernier, then, we first note the numbers of the scale lines between which the zero of the vernier is found and then the number of the vernier line which coincides with any line on the scale. If, for instance, the vernier zero falls between lines 25 and 26 on the scale, while line 11 of the vernier coincides with some line of the scale, the reading would be 25 scale divisions and 11 least counts. If the scale and vernier are circular and we know further, as is the case on many surveying instruments, that the scale divisions are half degrees and the least counts single minutes, the reading would be 25 half degrees and 11 minutes, that is, 12 degrees and 41 minutes.

To determine the least count of any vernier. Generally, the length of the scale divisions is known. Note the total number of spaces on the vernier from its zero line to its highest numbered line.

Let c = the least count;

s = length of space on scale;

v = length of space on vernier;

n = number of spaces on vernier.

Then for the direct vernier

$$nv = (n - 1)s;$$

$$s = \frac{nv}{n-1}; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (a)$$

$$c = s - v = \frac{nv}{n-1} - v = \frac{nv - nv + v}{n-1}$$

$$= \frac{v}{n-1} = \frac{nv}{\frac{n-1}{n}} = \frac{1}{n}s \text{ from } (a).$$

For the retrograde vernier

$$nv = (n+1)s.$$

$$s = \frac{nv}{n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$c = v - s = v - \frac{nv}{n+1} = \frac{nv + v - nv}{n+1}$$

$$= \frac{v}{n+1} = \frac{nv}{\frac{n+1}{n}} = \frac{1}{n}s.$$

So that in both cases

$$c = \frac{1}{n}s, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

that is, the least count of any vernier, direct or retrograde, equals one nth of the length of a space on the scale. Equation (28) may be used to determine the least count of any vernier with which the surveyor is not familiar; or any two of its three unknown quantities being assumed at will, it may be used for designing a vernier. Some examples of its application will here be given.

1. To design a vernier to read to thousandths of a foot, the scale being graduated to hundredths.

Substituting in (28)

$$.001 = \frac{1}{n}.01$$

$$n = \frac{.01}{.001} = 10.$$

Therefore the number of divisions on the vernier must be 10. If the vernier is to be direct, these must cover a length equal

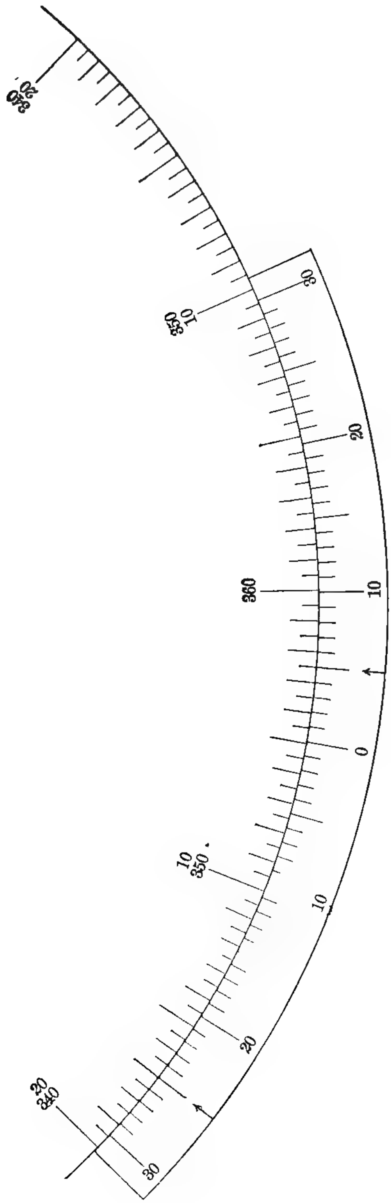


FIG. 64.

to that of nine spaces on the scale. If retrograde, eleven spaces on the scale.

2. To design a vernier to read to ten seconds, the divisions on the scale being ten minutes each.

$$10 = \frac{1}{n}(10 \times 60) = \frac{1}{n}600,$$

$$n = 60.$$

3. It is noted that thirty spaces on a circular vernier cover twenty-nine on the scale, and that the scale divisions are one half degree each. What is the least reading of the vernier?

$$c = \frac{1}{n}s = \frac{1}{30}30' = 1'.$$

The vernier usually met with is that described in 3. An illustration of it is here given, Fig. 64. Note that the vernier is double, that is, extends both ways from the zero line, and that there are two sets of figures on the scale; one running one way and one the other. This is to enable angles to be read in either direction. Care must be exercised to use the proper one of the two verniers. Each being a direct vernier, note that the vernier to be read is that which has been moved on ahead in the direction in which the angle has been turned off. The following is another way of deciding which half of a double vernier to read: Supposing that the vernier has been moved from right to left, and, consequently, we are reading the inner row of figures. The divisions on the scale are half degrees and the reading, by estimation, is about 4° 50′. The right-hand vernier reads five minutes, and therefore cannot be the proper one to use. The left-hand vernier's 25 line coincides with a line on the scale, and therefore is the one to read. Some instruments indicate, by the inclination of the figures, the scale and vernier that are to be used together. Fig. 65 shows this arrangement. In this figure the scale is divided into twenty-minute spaces. Forty spaces on the vernier cover thirty-nine on the scale, and the least reading, therefore, is one fortieth of twenty minutes, or thirty seconds. Using the upper row

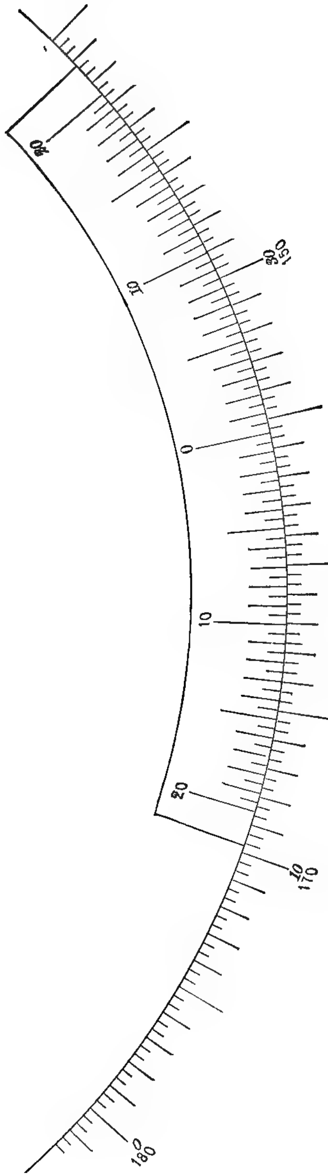


FIG. 65.

of figures on the scale, we would, of course, read the right-hand vernier, and the total angle would be 24 degrees 20 minutes and 15 least counts; that is, 24° 27′ 30″. Note, however, that the numbering on the vernier gives the minutes, so that we do not have to calculate the number of minutes in the number of least counts given by the vernier reading. For instance, if the vernier reading had been twenty least counts, since, in this case, one least count equals thirty seconds, and, therefore, twenty least counts equals ten minutes, the line marked 10 on the vernier would have coincided with a line on the scale, etc. In other words, the numbering on the vernier mechanically multiplies the number of least counts in the reading by the value of one least count, and divides the result by the number, contained in one minute, of the units in which the least count is expressed. A similar statement applies to the vernier shown in Fig. 67. Fig. 66 shows a vernier found on some instruments

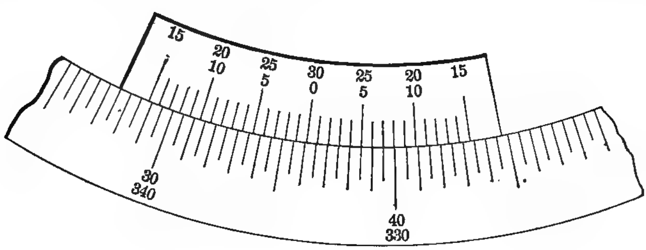


FIG. 66.

where space must be economized. The lower right-hand and upper left-hand verniers form one vernier, and the same thing is true of those on the lower left hand and the upper right hand. Thus supposing the angle is being measured to the left, and that the vernier reading is more than fifteen least counts. As the vernier shown in the figure is direct, we would first look along the lower left-hand vernier. Finding that no line there, from zero to fifteen, coincides with a line on the scale, we would next turn to the upper right-hand vernier. Looking along this from 15 to 30, we find the 23 line in coincidence with a line on the scale, and the reading of the vernier, therefore, is twenty-three least counts.

Fig. 67 shows a vernier somewhat different from those previously described.

This vernier is sometimes used on an instrument called a sextant, to be described later on (Chap. IX). Note that sixty

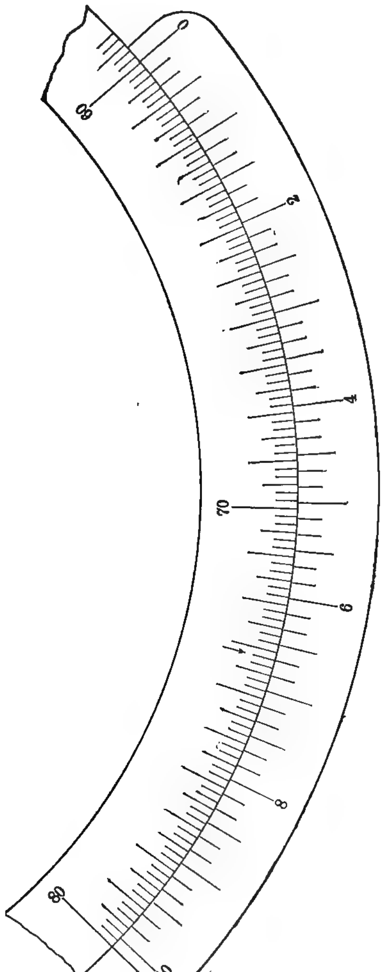


FIG. 67.

spaces on the vernier cover one hundred and nineteen on the scale. That is

$$nv = (2n - 1)\, s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

The vernier is direct, but a space on it, instead of being a little shorter than one space on the scale, is a little shorter than two spaces on the scale, and the least count, instead of being $s - v$, is $2s - v$. However,

$$nv = (2n - 1)s, \quad \ldots \ldots \quad (a)$$

$$\frac{s}{n} = \frac{v}{2n - 1}, \quad \ldots \ldots \quad (b)$$

$$2s = \frac{2nv}{2n - 1}, \quad \ldots \ldots \quad (c)$$

$$c = 2s - v = \frac{2nv}{2n - 1} - v$$

$$= \frac{2nv - 2nv + v}{2n - 1} = \frac{v}{2n - 1}$$

$$= \frac{s}{n} \text{ from } (b). \quad \ldots \quad (d) \text{ and } (28)$$

The least reading is seen to be determined in the same way as in the ordinary vernier.

The vernier is a most useful device. It would be impracticable, generally, to divide a scale into the small "least counts," yet by means of the vernier these are read with the greatest ease.

50. The Magnetic Declination (Art. 28).—This often causes trouble in compass work. The difficulties are increased by the fact that the declination varies with the place on the earth's surface at which the compass is being used. Moreover, at any particular place the declination is changing continually. This change is known as the "variation of the declination." The earth is a huge magnet whose north pole is situated at about 68 degrees north latitude and longitude 95 degrees west of Greenwich. But the needle points only approximately to this pole. The point for the surveyor to

remember, however, is that he possesses a means of determining the declination for any particular time, place, and instrument. This method of determination is described a little further on. We first proceed to discuss

The Variations of the Declination.

The position of the needle varies from a variety of causes. At the time of magnetic storms certain irregular changes, sometimes quite large, are noted. There is a regular change, known as the "annual variation," and others classified as "lunar inequalities." These changes, however, the annual variation and the lunar inequalities, are so slight that, in practical work, they may be neglected. The only kinds of regular variation with which the practical surveyor is concerned are two in number, and these are known as the "secular" variation and the "diurnal" or "daily" variation, the secular variation being much the more important.

1. *The Secular Variation.*—Observations have shown that, year in and year out, the north end of the needle is moving either towards the east or towards the west. For instance, in the city of Paris, France, in the year 1540, the north end of the needle pointed seven degrees to the east of the true north. In the succeeding years it pointed more and more towards the east of north, until in 1580 the declination was more than eleven degrees east. The needle then began moving west year after year until about 1800, when the declination at Paris was over twenty-two degrees west. Since that time the needle has been moving east, and at present points about fifteen degrees west of north. This phenomenon of a long continued movement of the needle in one direction is observed all over the world. In the eastern part of the United States, since about 1800, the needle has been moving west, and this westward movement still continues. From the fact that it continues through so many years, this change in the direction of the needle is known as the "secular variation." No satisfactory explanation of it has ever been given. Supposing that the traverse shown in Fig. 68 was run out fifty years ago, and that during that time, and at the place of survey, the secular variation has been 3° 20′ west.

If the bearings in the original field-notes were magnetic, and we now attempt to run out the survey with the same bearings, the result will be the dotted lines shown in the figure. From which becomes evident the importance of stating in any set of compass field-notes, and on any map of a compass sur-

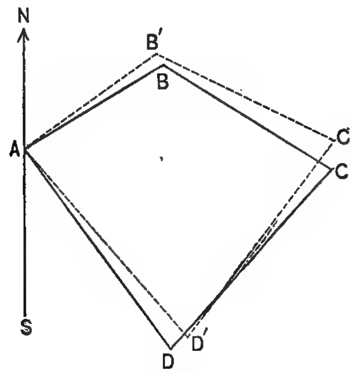


FIG. 68.

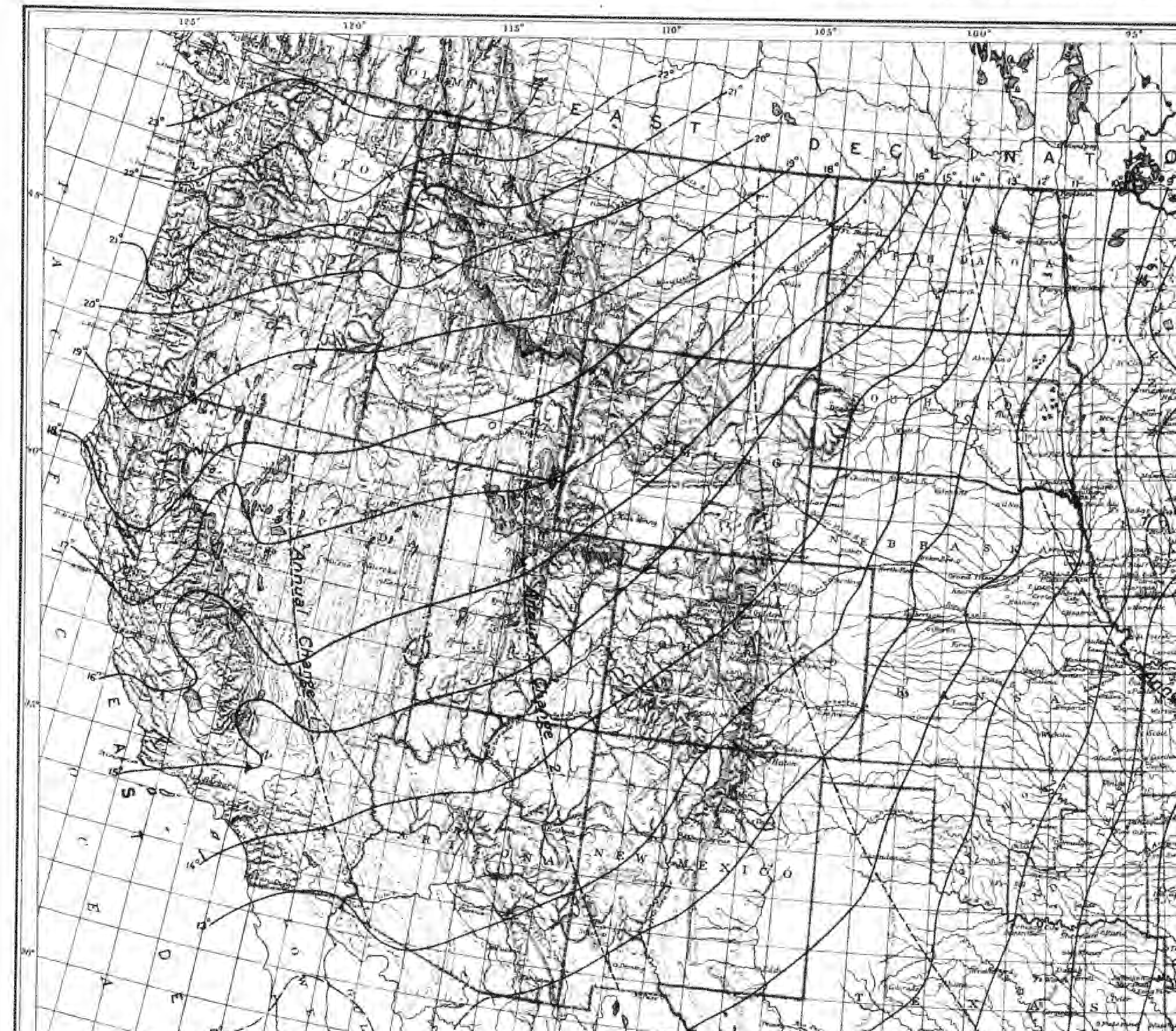
vey, whether the bearings given are true or magnetic, and, if the latter, what the amount and direction of the declination were at the time and place of the survey. Knowing the declination at the time of the original survey, and also the declination at the present time, we can correct the magnetic bearings of any set of field-notes, and, using the corrected bearings, retrace the lines run by the original surveyor. If the amount of the declination is not stated in the original notes, or if the declination at the present time is unknown, the change in declination may be computed approximately. This is done by means of a map similar to that shown in Plate I taken from the report of the U. S. Coast and Geodetic Survey for 1902. In Plate I lines are shown joining points at which, at the time of the publication of the map, the declination was an integral number of degrees, and at the ends of the lines is shown the amount of the declination at the points which they join. The map states also whether the declination is west or east. These lines are known as "isogonic," from two Greek words meaning equal and angle. The particular isogonic line

joining points at which the declination is zero is known as an "agonic" line. The term "agonic" is also from the Greek, and means "without angle." At all points east of the agonic line the declination is west, and *vice versa.* The declination, for the year of the map, for any point not on an isogonic line may be obtained by interpolation. Thus if a point is between the five- and the six-degree west declination lines and one-quarter of the perpendicular distance between them from the six-degree line, its declination is five and three-quarters degrees west, etc. To obtain the declination at any point for a year other than that of the publication of the map, obtain first the declination at the point for that year. At numerous points on the map of Plate I will be found that yearly change in the declination given at those points, dotted lines being shown through points of equal yearly change. (See explanation on map.) If the point in question is not one of those for which the yearly change is stated, it may be obtained, approximately, by interpolation by the method described above. Knowing the yearly change, it will be an easy matter to figure the declination for any year other than that of the publication of the map, and knowing the declination at the present time, and also at the time of the original survey, the change of declination can be found and the field-notes corrected accordingly.

Another approximate method of determining the secular variation at any point between given dates is by the use of the tables compiled by Mr. Charles Schott of the U. S. Coast Survey. The formulæ found in these tables are deduced from an extended series of observations, and will give the approximate declination at a number of points for some time to come. The tables may be found in Appendix 7, U. S. Coast and Geodetic Survey Report for 1888.

However, whether or not the declination at the time of the original survey is known, the only good way for the surveyor to determine the declination at present is by running out a true north and south line himself and finding the magnetic bearing of this line. The method of running out a true meridian is by observations on the North Star, or Polaris, by which name it is known in astronomy. The North Star is

LINES OF EQUAL MAGNETIC DECLINATION AND OF EQUAL AN



L ANNUAL CHANGE IN THE UNITED STATES FOR 1902.

PLATE 1

JULIUS BIEN & CO PHOTO LITH N Y

easily discoverable in the northern heavens on account of being approximately in line with the two stars known as the pointers in the bowl of the Dipper, as shown in Fig. 69. It is not situated exactly at the North Pole, but revolves around it, or appears to do so, in a direction contrary to that of the hands of a watch. If it were situated exactly at the pole, to obtain the true meridian we would merely have to sight at the star. However, in its apparent revolution around the pole, twice in a little less than twenty-four hours, it appears in the vertical plane including that point, being once directly above it and once directly below it. The upper position is called "upper culmination" and the lower "lower culmination." If then we sight at the star when at either upper or lower culmination the line of sight will be in the true meridian. Or, if we sight at the star in any other position, and know the horizontal angle through which the plane of sight must then be turned in order to include the pole, that angle can be turned off, giving the true meridian. It is convenient to sight at the star either when at upper or lower culmination, or when it is at its farthest distance east or west of the pole. These two last positions are known as eastern and western "elongation." Further, when the star is at culmination its motion is horizontal. Consequently, if we do not sight it at the exact time when it comes into the meridian plane, our observation is liable to be appreciably in error, as the star is moving at right angles to that plane. On the other hand, when at elongation, the star is moving directly up or down, that is, in a vertical plane, and an observation a little earlier or later than the exact time at which it reaches elongation will not introduce an appreciable error.

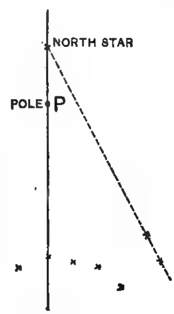


FIG 69.

The time of the North Star's coming into culmination or elongation can be estimated from the fact that that star and the star in the crook of the handle of the Dipper come into culmination and elongation at the about the same time. Thus, in Fig. 69 a straight line through the North Star and the pole

is seen to include the star in the crook of the Dipper handle also. These two stars are on opposite sides of the pole, and appear to revolve about that point as a center. A total revolution takes place in, approximately, twenty-four hours, really in about four minutes less than that time.

If, on examination of the stars, it is noted, for instance, that the line joining the North Star and the star in the Dipper handle makes an angle of about 45 degrees with the vertical, then that line will become vertical or horizontal, as the case may be, after about another 45 degrees revolution, that is, after about one eighth of a whole revolution, or in three hours.

However, by the methods of astronomy, tables have been calculated which state the exact time of culmination and elongation for the North Star. (See Table 1.)

The best instrument for making an observation on the North Star is a "transit." The use of this will be explained later (Chap. VI). If a compass, or, rather, the compass-sight, is to be used, proceed as follows: Having determined, from the table, the time of culmination or elongation, whichever is to be observed, go out during the day and drive two stakes, with their tops level, on about an east and west line, nailing a smooth board on top of them. Next suspend a plumb-bob by a fine line from the limb of a tree or other support about twenty feet or more from the ground. Drive a stake flush with the ground directly under this plumb-bob, marking the exact point over which the bob hangs. Be careful that there are no obstructions between the plumb line and the north. The stakes will have been placed south of the plumb-line and driven down so that the straight line from a point about half way between them on the board through the upper end of the plumb-line will pierce the sky above the star when it is to be observed. In latitude forty-five degrees north, for instance, the stakes will have to be south of the plumb-line by a distance shorter than its length. Prepare a bucket of water to steady the swinging plumb-bob and also a lantern to illuminate the upper part of the line. Take a sight from the compass and

fit it into a rather heavy block of wood, so that the face of the sight will be flush with the end of the block. Take another plumb-bob and line and pass the line through the sight so that, when the block is placed on the plank with the end holding the sight projecting a little over the south side of the plank, the bob will hang free, and the line will be a continuation of the slit in the sight. The other end of this line may be wrapped around a nail driven in the block. Obtain another stake short enough to be easily driven flush with the ground and have an axe ready for driving it. An extra lantern will also be convenient. Having made these preparations, go out, a little before the time at which the star is be observed, place the compass sight on the board, and bring the sight in line with the star and the plumb-line. As the star moves, keep the sight in line with it and the plumb-line until the exact time arrives at which the observation is to be made, when a stake driven under the plumb-bob hanging free from the sight, together with the stake driven before, will determine the line required, if the star has been observed at culmination. Be careful that the sight is not moved while the stake is being driven, and when it is driven mark the exact point on it over which the plumb-bob hangs. If, however, the star has been observed at elongation, the line determined by the two stakes will not be a meridian, but will make with the meridian an angle equal to what is known as the "azimuth" of Polaris, at elongation in the latitude of the place at which the observation has been made. For a short time before and after reaching elongation, the star will appear to move vertically. This furnishes a check on the time of the observation. Supposing, for instance, that the observation is to be made on the star at western elongation. Going out a little before the calculated time, and bringing the sight in line with the plumb-line and the star, after a little, on looking through the sight, the star will be seen to have moved west of the plumb-line. By watching the star, and keeping the sight continually in line, finally the star, which was seen on the line a minute before, will be observed to have moved east. The sight will then be in the position desired, and can be held there while the stake is being driven under it.

The time at which the star appears to be moving vertically should agree with the time as calculated from the table.

The "azimuth" of a star at any particular time, and at any particular place, as understood in surveying, is the angle through which the vertical plane including the star must be turned in order to coincide with the meridian plane. For a clear understanding of the method of determining the azimuth of the star when at elongation, the meanings of a few terms must be explained. For the purposes of such a discussion as this, the earth may be considered as a perfect sphere at the center of an infinitely larger one, known as the "celestial sphere," to whose surface the fixed stars are attached. The axis of the earth pierces the celestial sphere in two points known as the poles. It is understood, here, that when the "pole" is referred to, the *North* Pole is meant. An angle subtended by an arc on the celestial sphere will be the same whether its vertex is at the center of the earth or at any point of its surface. This is because the celestial sphere is vastly larger than the earth.

The terms "latitude" and "zenith" are supposed to be understood. The latitude of any point on the earth is equal to the latitude of its zenith on the celestial sphere.

The "pole distance" of a star is its angular distance from the pole; that is, it is the angle at the center of the earth, or at any point on the earth's surface, subtended by the arc on the celestial sphere joining the star with the pole.

The "colatitude" of a place is its angular distance from the pole (of the earth) and is equal, therefore, to the angular distance of the zenith of the place from the pole of the celestial sphere. Evidently the colatitude equals ninety degrees minus the latitude.

The "horizon" for any point on the earth's surface is the great circle cut from the celestial sphere by a plane passing through that point perpendicular to a vertical line. This, of course, is the same circle that would be cut from the celestial sphere by a plane parallel to the one described above, but passing through the center of the earth.

The celestial equator is the intersection with the celestial sphere of the plane of the terrestrial equator.

In all figures illustrating relations between the earth and the celestial sphere, the earth may be regarded as a point. In

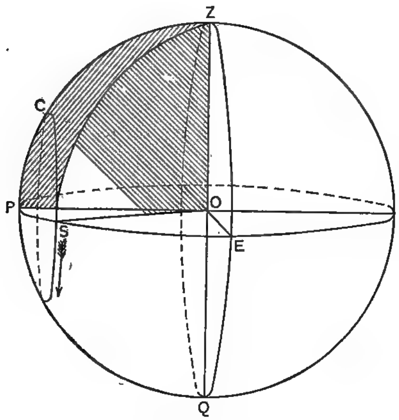


FIG. 70.

Fig. 70 *O* represents the earth, and the observer is supposed to be at the equator. *Z* represents the zenith, *P* the pole, *S* the star at elongation, *ZP* the meridian, *ZE* the celestial equator, and *PEP′* the horizon. The circle *SC* represents the apparent path of the star around the pole. The plane of this path being perpendicular to the line of sight of the observer at the equator, it will appear to him in its true shape; that is, as a circle. Evidently the two vertical planes at *O*, one through the pole and the other through the star, cut from the celestial sphere two arcs *ZP* and *ZS*, and these, together with the pole distance *PS* of the star, form a birectangular spherical triangle, the two right angles being at *P* and *S*. Also, the vertical plane through *S* is tangent to the circle of the apparent revolution of the star at the intersection of that circle with the horizon. In this case the angle between the two vertical planes at *O* is equal to the arc *PS*; that is, the azimuth of the star at elongation is equal to its pole distance. In Fig. 71 the observer is supposed to be at some point north of the equator. *P*, *Z*, and

S represent the same points as before, EQ represents the celestial equator, PEP' the horizon at the equator, and S' the intersection of the circle of apparent revolution of the star with the horizon at the equator. It is seen that the vertical plane including the star is tangent to the circle of apparent revolution of the star at S instead of at S', as in Fig. 70. The dihedral angle, shown in Fig. 70, has been tilted forward in Fig. 71, its edge moving from the position OZ' (Fig. 71) to the position OZ. During this movement the vertical plane through the pole has not moved out of the meridian, the plane of the paper in each figure. But, as the dihedral has been tilted, the vertical plane through the star has been forced away from the vertical plane through the pole, and, consequently, in Fig. 71 the angle between the two vertical planes is greater than in Fig. 70. In other words, as we go north from the equator the azimuth of the north star at elongation, at the equator equal to its pole distance, becomes greater and greater. The arcs cut from the celestial sphere by the two vertical planes and the pole distance of the star still form a right-angled spherical triangle, but in this case there is only one right angle, and that is at S. From the principles of spherical trigonometry,

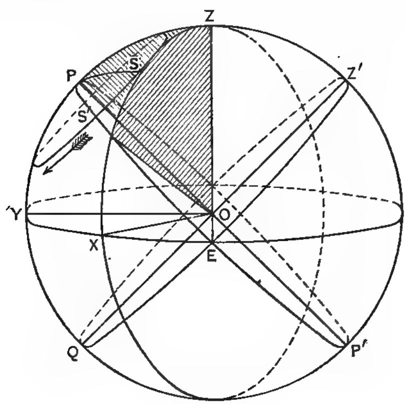


FIG. 71.

$$\sin PZS = \frac{\sin PS}{\sin PZ};$$

or, sine of azimuth at elongation

$$= \frac{\text{sine of pole distance}}{\text{sine of colatitude}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Any good map will show the latitude of the place of observation sufficiently accurately, or the latitude is equal to the elevation above the horizon of the North Star at culmination corrected for pole distance and refraction as in Tables II and III. The angle of elevation can be measured with a transit (Chapter VI).

From the map shown in Plate I, the latitude can easily be determined to the nearest degree, and an error of a degree in latitude will not introduce an error of more than two and a half minutes in the azimuth determined from equation (29) anywhere in the United States. Further north the error becomes greater. The pole distance of the star can be taken from Table II.

Knowing the pole distance and the colatitude, by making the proper substitutions in equation (29) the azimuth of the star at elongation may be computed and the line run out on the ground swung by that amount to bring it into the meridian. Having run out a true north and south line, its bearing, as given by the compass with its declination vernier set at zero, equals the magnetic declination with its sign changed. Having run out a line towards the star at elongation, it will be more accurate, instead of laying out a meridian by turning off the azimuth, simply to take the bearing of the line itself. This, corrected by the azimuth, will give the magnetic bearing of the true north and south line, and hence the declination. If a true north and south line has been *accurately* laid out, by an observation on the star at culmination or otherwise, to determine the declination more precisely than can be done by ordinary needle readings, sight the compass, with the vernier set at zero, over the true north-and-south line. Then move the vernier until the needle reads due north. The reading of the vernier will equal the declination.

In connection with the determination of azimuth, see also Tables IV, V, and VI.

Every surveyor should have a carefully laid out and permanently marked meridian line. It should be determined not by one, but by many observations on the North Star, and should be the mean of them all. Before commencing any particular piece of work, if the declination or the true bearings are desired, determine the declination at the time and place at which the work is to be done. Sometimes the declinations will differ appreciably in opposite ends of the same county at the same time. The bearing of the meridian should be taken at about ten o'clock in the morning. The reason for this will be given in the discussion of

2. *The Diurnal or Daily Variation.*—It has been observed that the needle does not remain stationary throughout the day, but points farthest east in the morning at about eight o'clock and farthest west in the afternoon at about half-past one. It then moves eastward again, the mean position each day being reached at about 10.30 A.M. and 8 P.M. It is on this account that an observation for declination should be taken at about ten or eleven o'clock in the morning. The total daily range in the United States is from five to ten minutes of arc. In ordinary compass work it is customary to neglect the daily variation. The compass can not be read closer than to five minutes, and the error introduced by the daily variation will probably be smaller than this. For determining the declination or important bearings the compass should be read at about half-past ten in the morning. For a table giving corrections to bearings taken at other hours see the U. S. C. and G. Survey Report for 1881, p. 136.

51. Tests of the Compass.—By these are meant the tests of those qualities in the compass the failure of which can be corrected by the maker only. In the next article, on "Adjustments of the Compass," several tests will be described, but they will be supposed to precede adjustments which can be made by the surveyor personally.

1. The test of coincidence of the axes of magnetism and of figure in the needle.—The axes of magnetism and of figure in a magnetic needle may not coincide. In such a case the needle itself will not point to the magnetic north, but its mag-

netic axis will. A method of testing this would be to turn the needle upside down on the pivot, the compass-box remaining stationary. If the needle continues to point to the same division mark the axes of magnetism and of figure coincide. Otherwise they do not. The case is illustrated in Fig. 72.

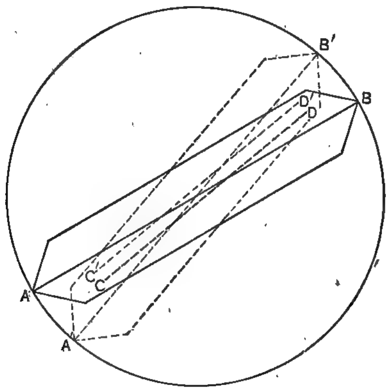


FIG. 72.

The two axes being supposed not to coincide, AB represents the axis of figure of the needle and CD its magnetic axis, both in the first position of the needle. When the needle is turned upside down its position is shown by the dotted lines. The magnetic axis assumes the position $C'D'$ parallel to its former position CD, and the needle, evidently, will not point to the same division mark as before. Generally, this test cannot be made, as there is no recess on the top of the needle to fit on the pivot. However, if the two axes do not coincide it will cause no error in the work except that all bearings will be recorded wrong by the same amount, and if the compass is set up over a line whose bearing is known, and the vernier turned until the needle reads that bearing, this last error will also be eliminated. Usually, the compass-needle is so narrow that the axes of figure and of magnetism practically coincide. If this test is made, it should be made after the adjustments 3, 4, and 6, described in the next article.

2. The test for the plane of sight's including the center of graduation.—Pass a thread through the sights and note

whether it cuts divisions 180 degrees apart on the graduated circle. It should do so. If not, the error introduced is illustrated in Fig. 73. Suppose that, with the compass at C,

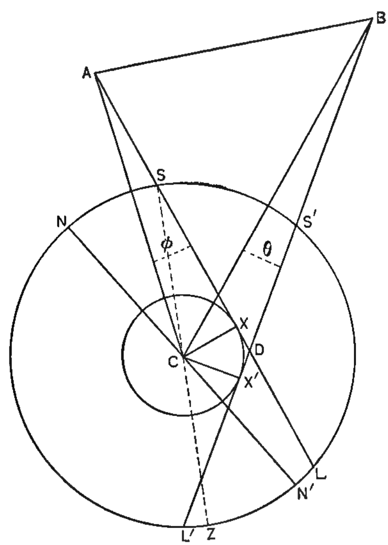


FIG. 73.

it is desired to measure the angle between the points A and B in this figure at unequal distances from C. The needle being in any position as NN', and the line of sight LS directed to the point A, suppose that LS misses the center C by a distance CX. Next direct the line of sight LS to B, when it revolves into the position $L'S'$. The angle read by the needle will be that of revolution of the box. This will equal XCX', which in turn equals ADB, their sides being mutally perpendicular. Now, the angle desired to be measured is ACB. In the triangles ADB and ACB,

$$ADB = 180 - (DAB + DBA), \quad . \quad . \quad . \quad . \quad (30)$$
$$ACB = 180 - (CAB + CBA), \quad . \quad . \quad . \quad . \quad (31)$$
$$CAB = DAB + \sin^{-1} CX/CA, \quad . \quad . \quad . \quad . \quad (32)$$
$$CBA = DBA - \sin^{-1} CX/CB, \quad . \quad . \quad . \quad . \quad (33)$$

CA and CB are unequal. Therefore, CAX and CBX' are un-

equal. Call CAX, ϕ, and CBX', θ. Substituting these expressions in (32) and (33) and then substituting (32) and (33) in (31).

$$ACB = 180 - (DAB + DBA + \phi - \theta) \quad (34)$$
$$ADB = 180 - (DAB + DBA). \quad . \quad . \quad . \quad . \quad (30)$$

Comparing (34) and (30) the difference between the angles ACB and ADB is seen to be $(\theta - \phi)$. Therefore, if the line of sight does not pass through the center of graduation, the angular distance between the two objects at unequal distances from the compass will not be correctly given by that instrument. If the two points are at equal distances from the instrument, $\phi = \theta$, and the angular distance will be corrrectly given.

In the figure ZS represents the line connecting the zeros of the graduation, and the plane of sight is shown as passing through one of these. This can easily be made true, of course, as the graduated circle can be revolved around the central plate. It is evident, whether or not the plane of sight passes through the center of graduation, if it be directed at any object, the compass can, by revolving the circle and keeping the plate and sights stationary, be made to read the correct bearing. The next bearing, however, with the plane of sight turned on another object, would be read wrong. This test should follow the second adjustment, described in the next article. Supposing, Fig. 74, that the line of sight LS is parallel to the line joining the zeros of the graduation and passes one tenth of an inch from the center of the compass-box. It is desired to find the bearing of line CA. With the needle at N an zeros directed to A, the bearing is NCO. As the line of sight swings around towards A the reading changes by an angle "theta," equal to OCO'. If the distance CA is 10 feet, "theta" equals the arc whose sine is one tenth divided by 120, equals .00083, equals sine of three minutes, about. If the line were longer, the angle of error

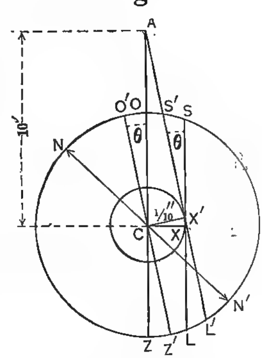


FIG. 74.

would obviously be smaller. If the line of sight in the figure passed to the left of ZO, the error would have a different sign.

3. The test to determine whether the plane of sight includes the zero of the vernier, Fig. 75.—This test is made in the same way as the last one: by running a thread through the sights and noting whether it includes the zero of the vernier. It should also follow the second adjustment, described in the next article. In the figure the plane of sight LS is turned on the magnetic north, and therefore includes the needle. The zeros of the circle and of the vernier coincide, but the reading is two degrees. If this test is not satisfied, no error will be caused in the work except that all bearings will be given wrong by the same amount. If, however, the bearing, true or magnetic, is known, the instrument sighted over this line and the vernier made to read the correct bearing, the bearings of all other lines will be given correctly also.

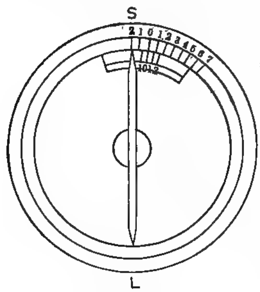


FIG. 75.

4. The test to determine whether the plate is perpendicular to the vertical axis.—If this test is not fulfilled an error will be introduced, as illustrated in Fig. 76. $NSPL$ represents the compass plate, shown in vertical projection at $P'P'$, and making an oblique angle with the vertical axis VV. Supposing the bearing of the line CA is desired, A being any point and C the point where the compass is set up. The needle being at NN, the bearing indicated would be SCN, shown in its true size at S_1CN_1, while the correct bearing of CA is ACN_2.

In the discussion of this test the plane of sight, which includes the line of sight LS, has been assumed to remain vertical. This will be true, because whatever may be the inclination of the plate to the vertical axis, the plane of sight, determined by the two standards shown in vertical projection at $S'S''$ and $L'L''$, can still be made truly vertical by the method of the second adjustment in the next article. Then, as the standards will be parallel to the vertical axis as they revolve

around it, they will, in any position, determine a truly vertical plane. In the figure, to simplify the construction the needle is shown as if passing through the plate.

To test whether the plate is perpendicular, to the vertical axis, first, by the method of the first adjustment in the next article, make the vertical axis truly vertical. Next place an

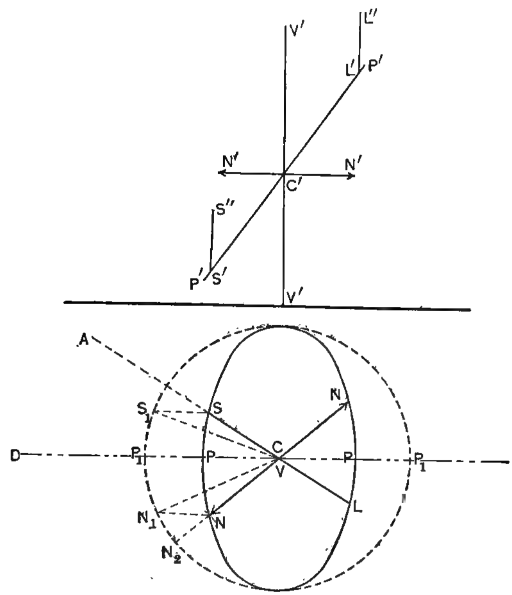


FIG. 76.

ordinary level on the plate and note the position of the bubble. Turn the level end for end and note whether the bubble remains in the same part of the tube as before. If so, the plate is perpendicular to the vertical axis. If not, the plate is inclined to the vertical axis. Note particularly that this test cannot be made with the aid of the plate levels, but that an entirely separate level, which can be raised off the plate and turned end for end, must be used. The test is illustrated in Figs. 77 and 78. Fig. 77 shows the case where the plate is inclined to the vertical axis, and Fig. 78 the case where it is perpendicular. In each figure, *VV* represents the vertical axis made truly vertical by the method of adjustment 1 in the next

article; *PP* the plate, and *LBE* the bubble tube. The primed letters show the positions, after the tube has been turned end for end, of the points which they, without the primes, represent in the first position of the tube. *B* and *B'* represent the positions of the bubble in the tube. The bubble tubes are arcs of circles, and the position of the bubble in the tube may always be determined by the intersection with the tube of a vertical line through the center of the circle of which the tube is an arc. The two supports at the ends of the tube are shown of unequal length. This represents the general case.

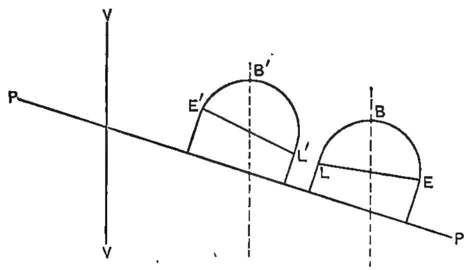


FIG. 77.

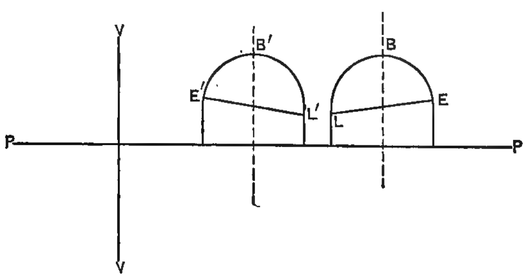


FIG. 78.

In Fig. 77, when the tube is turned end for end, the bubble moves to *B'*, the highest point in the circle. In Fig. 78, on the other hand, when the tube is turned end for end, the bubble remains in the same part of the tube as before. That is, in Fig. 77, *L'B'* is unequal to *LB*, while in Fig. 78, *L'B'* equals *LB*, etc. This test would fail, in the case where the plate was inclined to the vertical axis and the tube happened to be

placed exactly perpendicular to the line of steepest declivity of the plate. To guard against this, make the test on two parts of the plate, the tube in the second test not being parallel to its first position.

5. The test of the metal in the compass box. Sometimes iron gets mixed in with the brass of the compass box. To test this, having set off a given bearing, turn the circle by means of the tangent screw through a few integral degrees—four or five—determining by the vernier when this amount has been turned off. Then note whether the reading of the needle on the graduated circle indicates the same amount of rotation. If it does not the needle is being pulled out of place by the attraction of metal in the box. Make the test all around the circle. It should follow adjustments 3, 4, and 7 described in the next article.

The foregoing tests are interesting theoretically and a discussion of them is necessary for a clear understanding of the compass. However, with an instrument made by a reliable firm, they may generally be neglected, there being no probability of the introduced error's being appreciable.

52. Adjustments of the Compass.—Under this head will be described tests of certain conditions and arrangements of various parts of the instrument which can be altered and "adjusted" by the operator. The general distinction between "tests" and "adjustments" will be carried out through the remainder of this work.

1. The adjustment to make the tangents of the plate levels perpendicular to the vertical axis (Art. 27) of the compass.

Test. Having set up the compass, by means of turning the whole instrument in its ball and socket joint, bring the bubble to the middle of one of the level tubes. Revolve the plate 180 degrees around the vertical axis and note whether the bubble is then, at the conclusion of the revolution, found in the center of the tube as at first. If so, the tangent of the level tube is perpendicular to the vertical axis of the instrument. If not, it must be made so. Note that the bubble does not necessarily remain in the center during the whole of the revolution.

Adjustment. If, at the conclusion of the revolution, the

bubble is not found in the center of the tube as at first, by means of the capstan headed screws to be found at the ends of the brass case inclosing the level tube, raise or lower one end, or raise one end and lower the other, until the bubble has moved half way back to its original position in the middle of the tube. Then, by moving the whole instrument in the ball and socket joint, bring the bubble to the middle of the tube as at first, test again, repeat the adjustment if necessary, and so on until the test is satisfied. The other level tube can be tested and adjusted in the same way.

Explanation. It must be clearly understood that it is impossible, by means of the plate levels, to decide whether or not the compass plate is horizontal. All that we can tell from the plate levels is whether or not the vertical axis is truly vertical. Of course, as a matter of fact, the compass is so constructed by the maker that the plate is perpendicular to the vertical axis. Consequently, when we show that the vertical axis is truly vertical, it amounts to showing that the plate is truly horizontal. But whether or not the plate is horizontal can never be shown, *directly*, by the plate-levels. Some writers state that it can, but this is incorrect. Whether or not the plate is perpendicular to the vertical axis can be determined by the method of test 4 of Art. 51. In order to do correct work with the compass, however, the plate must be horizontal. The error introduced by the non-horizontality of the plate is illustrated in Fig. 79. This figure, it will be noticed, is very similar to Fig. 76, the difference being that in Fig. 76 the vertical axis VV is shown truly vertical with the plate PP inclined to it, while in Fig. 79 the plate is shown perpendicular to the vertical axis while this last is inclined to the true vertical. Also, in the discussion under Fig. 76, the plane of sight remains always vertical. In Fig. 79, however, the plane of sight is vertical only when parallel with the vertical projection plane. The standards, parallel with the vertical axis, are shown in vertical projection at $L'L''$ and $S'S''$. The bearings are subject to the same errors as in Fig. 76, and to the further errors caused by the plane of sight's not being vertical. When the *plane* of sight is not vertical, by holding the

eye at any point on one standard and sighting past any point on the other, many lines of sight are formed, each included in a different vertical plane, and giving as many different bearings for the line sighted over. However, in ordinary work the plate will be so nearly horizontal that errors owing to the cause just described will not be appreciable. So that the main

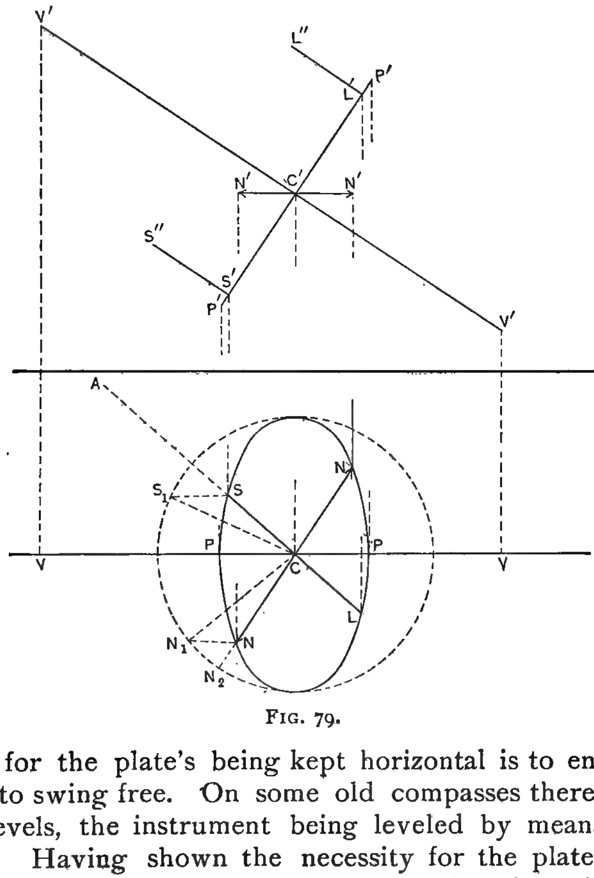


FIG. 79.

reason for the plate's being kept horizontal is to enable the needle to swing free. On some old compasses there are no plate levels, the instrument being leveled by means of the needle. Having shown the necessity for the plate's being made level, we proceed to a further explanation of the test and the adjustment.

The plate level bubble tubes are arcs of tubular circles, and are attached to the vertical axes of surveying instruments either radially or tangentially—generally the latter—and in

such a manner that when the vertical axis is truly vertical, the axes of figure of the tubes lie in vertical planes. The case in which the tubes are attached tangentially will here be considered. By the expression "tangent of the bubble tube" is meant the straight line tangent to the axis of figure of the tube at its middle point. In the accompanying Figs. 80 to 82, *VV* represents a truly vertical line, *B* the position of the bubble assumed in Fig. 82 to move on the axis of figure of the tube, *AA* the vertical axis of the instrument, *TBL* the axis of figure of the bubble tube, and *DB* the tangent of the bubble

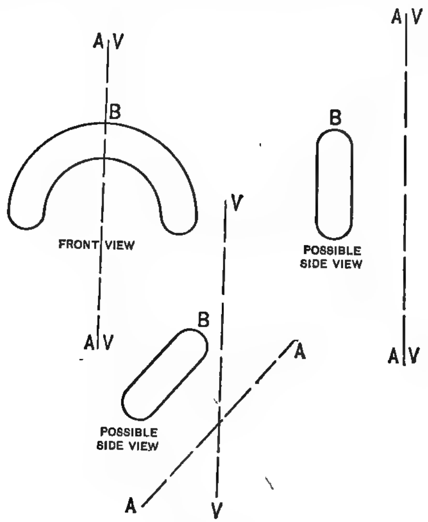


FIG. 80.

tube. An examination of Fig. 80 will show that when the tangent of the bubble tube is perpendicular to the vertical axis of the instrument, and the bubble is brought to the center of the tube, the middle point of the tube and the vertical axis determine a truly vertical plane.

Now, supposing that we have two bubble tubes rigidly attached to the vertical axis and at right angles to each other, and that the tangent of each is perpendicular to the vertical axis. If the two bubbles are brought simultaneously to the center, the vertical axis will determine with the center of

each bubble a truly vertical plane, that is, the vertical axis will be the intersection of two vertical planes and therefore a truly vertical line. Now, this is the state of affairs which we wish to bring about, because if the vertical axis is truly vertical and the instrument well made, the plate will be horizontal. If the bubble tubes are at right angles to each other, it will be an easy matter to bring the bubbles to the center simultaneously. Therefore, what we desire to find is a method of making the tangent of a tangential bubble tube perpendicular to the vertical axis of the instrument to which it is attached. Consider Fig. 81. It shows the case where the tangent of the bubble-tube is perpendicular to the vertical axis of the instrument, the vertical axis being not truly vertical. The tube is supposed to be revolved around the vertical axis, and five different positions during the revolution are shown. B_1 is the first position of the bubble in the center of the tube. Then, as the tube is revolved, the bubble moves to B_2, B_3, etc., always seeking the highest point in the tube; but when 180 degrees has been moved through, the bubble is found at B_5, back in the center of the tube. If the vertical axis had been truly vertical, the bubble would have remained in the center

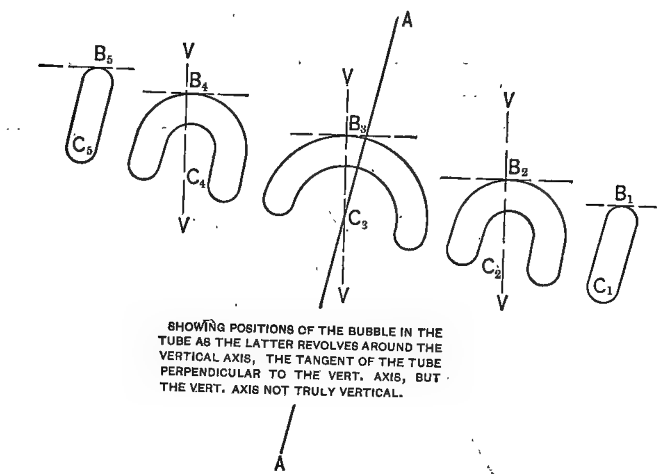


FIG. 81.

all the way round. It thus becomes evident that, a bubble tube being tangentially attached to a vertical axis and the bubble being brought to the center, if the tube is revolved around the vertical axis 180 degrees the bubble will still be found in the center. Thus to test whether the axis of a plate level bubble tube on a compass is perpendicular to the vertical axis of the instrument, by moving the instrument in its ball and socket joint, we bring the bubble to the center of the tube, then revolve the tube around the axis 180 degrees and note whether the bubble is still in the center. If so, the axis of the tube is perpendicular to the vertical axis of the instrument. If not, it must be made so. We next discuss how to make it so.

Supposing, then, that the tangent of the tube is not perpendicular to the vertical axis, that the bubble has been brought to the center, the tube revolved, and the bubble, then, not found at the center. The situation is as represented in Fig. 82. On the left are shown, in vertical projection, the

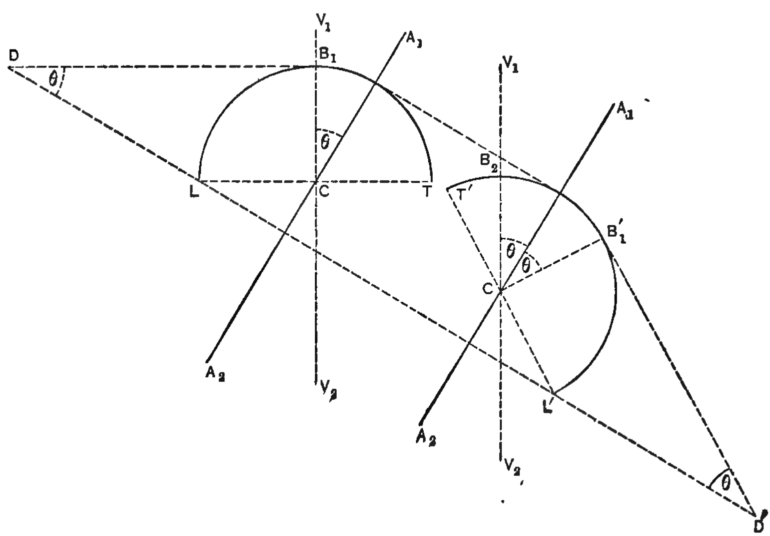


FIG. 82.

positions of the geometrical axis of the tube, etc., at first, and on the right the positions of the same lines after the revolu-

tion through 180 degrees has been completed. The vertical axis and the axis of figure of the tube in each case are shown parallel to the vertical projection plane. The bubble being brought to the center of the tube appears at B_1, the tangent DB_1 of the tube of course being horizontal. When the tube is revolved 180 degrees the bubble, seeking the highest point in the tube, moves to B_2. Now, during the revolution the vertical axis remains stationary. Therefore the angle between the vertical axis and a truly vertical line is constant. Let this be denoted by θ as in the figure. Also, the vertical axis and the tube are rigidly attached to each other, except that the tube may be rotated about the axis. Therefore, the angle between the vertical axis and a line from the centre of the bubble tube to its middle point is a constant. But at first, as shown on the left of the figure, the line from the center of the bubble tube to its middle point coincides with a truly vertical line. Therefore, throughout the whole revolution through 180 degrees and at its conclusion the angle between the vertical axis and a truly vertical line and the angle between the vertical axis and the line joining the center of the bubble tube with its middle point are each equal to θ. That is, on the right of the figure, $V_1CA_1 = A_1CB'_1$. All the angles are shown in their true sizes, the lines which form them being parallel to the projection plane. The angle through which the tangent of the bubble tube must be swung in order to bring it perpendicular to the vertical axis is $B'_1D'L' = A_1CB'_1$, because their sides are mutually perpendicular. That is, to bring it perpendicular to the vertical axis the tangent of the tube must be swung through the angle θ. Also, the bubble has moved through angle $B'_1CB_2 = 2\theta$, or, to become perpendicular to the vertical axis, the tangent of the tube must be swung through one half the angle through which the bubble has moved. Now, if the tangent of a bubble tube is swung through a given angle the bubble will move through the same angle. Consequently, if we move the tube, as directed in the adjustment, until the bubble travels half way back to its first position, the tangent of the tube will have been made perpendicular to the vertical axis of the in

strument. A similar proof will apply in the case of the tube's being attached radially to the vertical axis. The fact that the arcs through which the bubble moves and the tangent of the tubes wings are equal, is shown in Fig. 83.

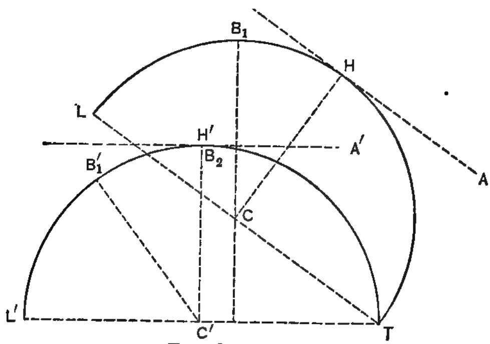


FIG. 83.

Let THL be the bubble tube's axis of figure in a position in which the tangent HA of the tube, parallel to TL, is not horizontal. The bubble is found at B_1. As the tube is revolved around T into a position in which its tangent is horizontal, the bubble moves from B_1 to H', and the angle B_1CH equals angle LTL' because their sides are mutually perpendicular. Had the revolution been about some point other than T, the statement could be proved in a similar manner.

2. The adjustment to make the sights parallel with the vertical axis.

Test. By the method of the first adjustment, make the vertical axis truly vertical. Look through the sights at some vertical line, as a plumb line or the edge of a building. If the sight farther from the eye coincides with the vertical line, no adjustment is needed. Otherwise a correction must be made. Swing the instrument through 180 degrees, and test the other sight in the same way.

Adjustment. If the above test is not satisfied, correct the sight by filing its bottom where it comes into contact with the plate, or by putting paper under one side until it can be made to coincide with the vertical line.

Explanation. If the sights are not parallel with the vertical axis, when the latter is made truly vertical, the situation will be the same in one respect as that described in the explanation under the first adjustment. That is, by placing the eye at any point on one sight and looking past any point on the other, many different lines of sight will be found lying in as many different vertical planes. Each different line of sight will cause the needle to indicate a different bearing for the same line on the ground. When the sights are parallel with the vertical axis and this last truly vertical, the compass can give only one bearing, and that the correct one, for any given line. The method of making the adjustment needs no explanation.

3. To sharpen the pivot.

Test. Let the needle settle several times, and note whether it quivers as it does so each time. If it does, the pivot is sharp and the jewel in the needle which rests on the pivot is in good condition.

Adjustment. If the needle fails to quiver as it settles, unscrew the pivot with the small wrench which comes for that purpose and have a jeweler sharpen it down to a conical point. If the needle still fails to quiver, the jewel is out of order and the needle must be sent to the maker to be repaired. Never attempt any work with a compass whose needle is sluggish. The time will be worse than wasted.

4. To magnetize the needle.

Test. Turn the graduated circle until one end of the needle coincides exactly with some division mark. Then raise the needle from the pivot and let it settle again. Do this several times, keeping the plane of sight stationary, and note whether the needle comes to rest each time at the original division mark. If not, it must be remagnetized.

Adjustment. To do this place the needle for a short time in the magnetic field of a dynamo. Do not forget the caution in the last adjustment. It is impossible to do satisfactory work if the needle is sluggish and unreliable.

5. The adjustment to make the center of the needle and its ends included in the same horizontal plane.

Test. As the needle quivers, if its ends swing noticeably

from side to side the needle is bent in a vertical direction and may be difficult to read.

Adjustment. Bend the needle in a vertical direction until the swinging motion disappears.

6. The adjustment to make the center of the needle and its ends included in the same vertical plane.

Test. Having set up the compass, note what points on the graduated circle are cut by both ends of the needle. Revolve the instrument on its vertical axis until the point on the circle cut at first by one end of the needle is now cut by the other end. Then if *both* the points cut by the opposite ends of the needle are the same as the two points cut by the opposite ends at first, the center of the needle and its two ends are included in the same vertical plane. One of the points cut on the circle, as indicated above, has, of course, been *made* the same as at first. If the other one is not, however, the needle is bent horizontally and must be straightened.

Adjustment. Bend horizontally the end of the needle which does not cut the same point as at first to a position half way between the position in which it is found after the revolution, and the position it would occupy did it cut, after the revolution, the point cut by the other end at first. The ends of the needle and its center will then be included in the vertical plane. Repeat the test, and, if necessary, also the adjustment.

Explanation. Fig. 84 shows a much exaggerated case of the needle's being bent horizontally. Note particularly that the pivot P is not shown in the center of the graduated circle $A_1B_1A_2B_2$. This test and adjustment do not depend on the pivot's being in the center and can be made whether or not it is in that position. The needle is shown in its first position at $N_1P_1S_1$, the points on the circle cut by its two ends being A_1 and B_1. The circle is next supposed to be revolved around C until S, the south end of the needle, is found at S_2 and cuts A_1, the point cut at first by the north end. During the revolution the pivot moves from P_1 to P_2, A_1 moves to A_2, while B_1 is now found at B_2 at a distance along the circle from A_2 equal to A_1B_1. The new position, parallel to its first position, of

the needle, is $N_2P_2S_2$. The angle of deflection of the needle, the meaning of which expression will be readily understood by a reference to Fig. 84, is DP_2N_2. To prove that the method of adjustment above described is correct, it is necessary only to show that DP_2B_2 equals DP_2N_2, which is the same thing

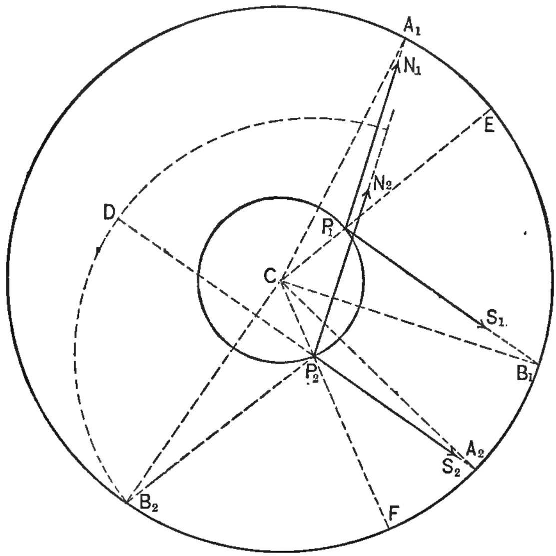


FIG. 84.

as showing that $B_2P_2A_2$ is equal to $N_2P_2A_2$ or to its parallel angle $A_1P_1B_1$.

Consider the lines A_1P_1 and B_1P_1 *on the compass box and directly under the needle* in the first position of it and the box. The *relative* position of these lines has not changed while A_1P_1 has been revolved to A_2P_2 and B_1P_1 to B_2P_2. That is, angle $B_2P_2A_2$ equals angle $A_1P_1B_1$, equals angle $N_2P_2S_2$, which was to be proved. To make this a little plainer, during the revolution, every point in the compass box has revolved through the same angle around C. That is

$$A_1CA_2 = B_1CB_2 = P_1CP_2.$$

Let this angle of revolution be represented by θ;

$$A_1CP_1 + P_1CA_2 = \theta;$$
$$A_2CP_2 + P_1CA_2 = \theta.$$

Therefore

$$A_1CP_1 = A_2CP_2.$$

Then in the triangles A_1CP_1 and A_2CP_2,

$$A_1C = A_2C;$$
$$P_1C = P_2C;$$
$$A_1CP_1 = A_2CP_2.$$

Therefore the triangles are equal and the external angle A_1P_1E equals the external angle A_2P_2F. We may therefore write

$$A_1P_1E = A_2P_2F. \quad \ldots \ldots \ldots \quad (31)$$

In like manner

$$EP_1B_1 = FP_2B_2. \quad \ldots \ldots \ldots \quad (32)$$

Adding (31) and (32) we have

$$A_1P_1B_1 = A_2P_2B_2, \quad \ldots \ldots \ldots \quad (33)$$

as before.

If the needle is bent horizontally, it will cause no relative error in the bearings and angles of a survey, provided the same end of the needle is read always, and the pivot is in the center. The bearings will probably all be wrong by the same amount owing to the non-coincidence of the axes of magnetism and of figure as explained under test 1. In practice always read the north end of the needle and run the "north" end of the box ahead.

7. The adjustment to bring the point of the pivot in a perpendicular to the plane of the graduated circle at its center.

Test. The needle having been found to be straight, or having been made so by the method of the last adjustment, set up the instrument and note whether, in two positions of the circle, 90 degrees apart, the needle cuts divisions 180 degrees apart. If not, the pivot is out of center. Note that even if the pivot is not in the center there will be two positions of the circle, 180 degrees apart, in which the needle will cut points on the graduation 180 degrees apart. This is illustrated in Fig. 85.

If, on first examination, the needle is noted as cutting divi-

sions 180 degrees apart, it may be true that the point is out of center, but that the box is in the position shown in Fig. 85. Making this test with the box in two positions 90 degrees apart will eliminate the possibility of error from this cause.

Adjustment. Revolve the instrument on its vertical axis until the line joining the pivot with its center is perpendicular to the needle. This will be when the angular distance on the circle, on one side of the needle and between its two ends, is a maximum while on the other side it is a minimum, as illus-

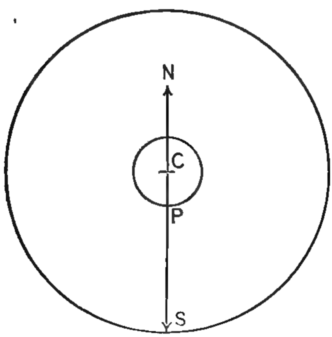


FIG. 85.

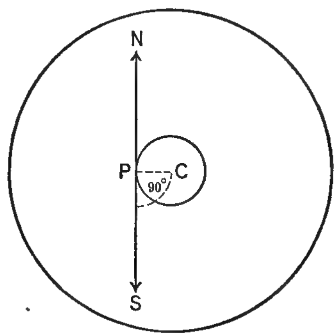


FIG. 86.

trated in Fig. 86. Then, with a small pair of pliers, carefully bend the pivot perpendicular to the needle until the opposite ends cut divisions 180 degrees apart.

53. General Remarks on the Tests, Adjustments, and Use of the Compass.—Unless the instrument has been damaged in some way, it will seldom be necessary to make any of the tests or adjustments just described. The tests may not be entirely satisfied, but, unless the instrument has fallen down or met with some like accident, the lack of adjustment will generally be so slight that the error introduced will be inappreciable. The greatest trouble is apt to be a sluggish needle, and, as stated before, no work should be undertaken with the instrument in this condition. Handle the compass carefully. Whenever the instrument is being moved, be sure that the needle is held against the glass, and when put away, that the needle has been allowed to come to rest in the magnetic meridian and then

lifted against the glass. Great accuracy cannot be expected in compass work, but the instrument has no equal for general work where the highest accuracy is not required, especially in a wooded region. For country work, generally speaking, unless the ground is entirely clear it is a waste of time to use the more accurate instruments.

CHAPTER IV.

THE TELESCOPES OF SURVEYING INSTRUMENTS.

SECTION I.

Theory of Lenses and Construction of the Telescope.

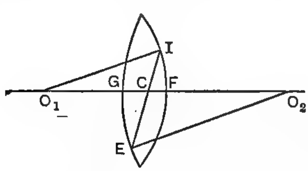
54. Preliminary to a discussion of the telescopes of surveying instruments, some of the facts in connection with convex lenses will here be given. The deduction of optical formulæ in general or a complete statement of the theory of convex lenses being beyond the scope of this work, no attempt in either direction will be made, it being the desire to state only such facts as will be necessary to aid the student in a clear understanding of the practical construction of the surveyor's telescope. In general, in order to make more apparent the facts desired to be brought out, the figures are distorted.

55. **Kinds of Lenses.**—Convex lenses are of three kinds, double convex, plano-convex, and concavo-convex. The three forms are illustrated in Fig. 87. The nature of the refraction

FIG. 87.

during the passage of light through any convex lens is such that any two rays of light proceeding from a point and pass-

ing through the lens are more convergent, or less divergent, at emergence than at incidence. On this account, convex lenses are also known as "converging" lenses. The principles which govern the refraction of light in convex lenses are, in general, the same in the three forms. For the purpose of illustration the simplest form, the double convex, will be used. The double-convex lens, then, consists of a piece of glass bounded by parts of the surfaces of two spheres with their centers on opposite sides of the lens. In other words, it is the volume of glass common to two intersecting glass spheres. The radii of these two spheres may or may not be equal. The line joining the centers of the two spheres is known as the "principal axis" (O_1O_2, Fig. 88). On this "principal axis" and in the interior of the lens, are two points known as the "principal points." Any ray of light incident on one side of the lens and passing through one of the principal points will pass through the other also and emerge parallel to its direction at incidence. In thin lenses, such as are used in surveyors' telescopes, the two principal points practically coincide at



$$\frac{R_1}{R_2} = \frac{O_1I}{O_2E} = \frac{O_1C}{O_2C} = \frac{O_1I - O_1C}{O_2E - O_2C} = \frac{CF}{CG}$$

FIG. 88.

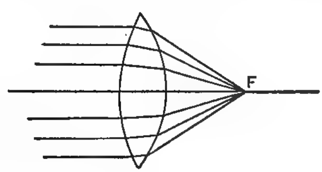


FIG. 89.

a point situated between them on the principal axis and known as the "optical center." If the two spheres, parts of whose surfaces bound the lens, are of equal radii, the "optical center" will be at the exact center of figure of the lens. Otherwise it will be found on the "principal axis" at distances from the two faces of the lens proportional to their radii. Thus, in Fig. 88, C being the "optical center," $\frac{R_1}{R_2} = \frac{CF}{CG}$. Any ray of light passing through this optical center will emerge parallel to its direction at incidence, and, as in general the

thickness of the lens is slight, the direction of such a ray after emergence may be said to be a continuation of its direction at incidence, or "rays which pass through the optical center undergo no deviation."

On the principal axis and at a distance, known as the "focal length," from the lens, is situated a point known as the "principal focus," *F*, Fig. 89. Rays of light impinging on the other side of the lens, and parallel to its principal axis, will, approximately, be converged to, and pass through, this "principal focus." There will, of course, be two principal foci, one on each side of the lens, and at the same distance from the optical center. Any straight line passing through the optical center of a lens is known as an "axis." The axes, other than the principal axis, are further known as "secondary axes." On each secondary axis, Fig. 90, will be two points corresponding to

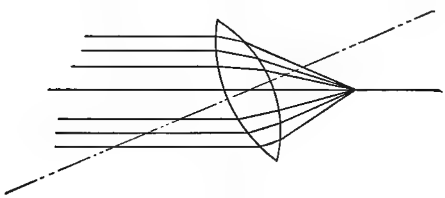


FIG. 90.

the principal foci on the principal axis. To these "secondary foci" are converged approximately all rays of light parallel to the secondary axis on which they are situated. The distance of these secondary foci, measured along their secondary axes from the optical center, except in the case where the angle between the principal and secondary axes is large, is practically equal to the focal length. Conversely, if a point of light is found at a principal or secondary focus, the rays radiating from it, and impinging on the lens, will emerge after refraction, parallel to the axis, principal or secondary, on which the focus in question is situated.

We may then deduce the following principle: "If a point of light is situated at a distance from a convex lens equal to the focal length of the lens, all the rays of light radiating from this point and impinging on the lens, will

emerge, after refraction, in a direction parallel to the straight line joining the point of light with the optical center of the lens, or, in other words, parallel with the axis on which the point of light is situated." Now, if the point of light is moved farther away from the lens, the radiating rays, after refraction, will be converged approximately to another point on the same axis, but on the other side of the lens, and farther from it than the focal length. These two points are known as "conjugate foci" and are both real, as the rays actually pass through each. (Figs. 91 and 92.) On the other hand, if the

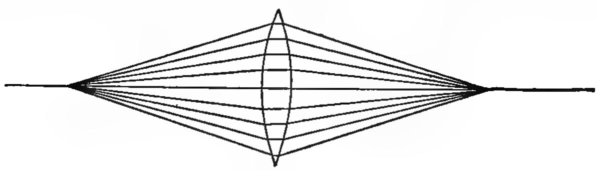


FIG. 91.

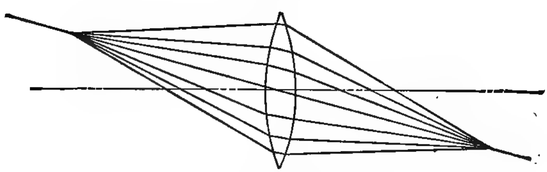


FIG. 92.

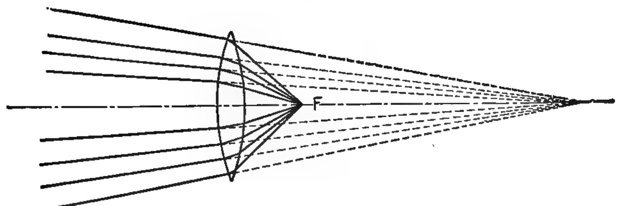
point of light is moved closer to the lens, the rays, after refraction, will emerge from the other side of the lens, *still divergent*, though less so than before. If produced backwards through the lens they will intersect approximately at a point on the axis on which is situated the point of light. The two points are still known as "conjugate foci." The one at which is found the point of light is "real." The other is "virtual," because the rays do not actually pass through it. (Figs. 93*a* and 93*b*.) Note in connection with conjugate foci that, while a point and its image are on opposite sides of the lens, and each farther from it than the focal length, as one approaches the lens the other recedes from it.

In the case of any two conjugate foci, let their distances from the optical center be denoted by f_1 and f_2. Then if f is

the focal length of the lens, in works on Optics, the following equation is shown to be true;

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (34)$$

FIG. 93*a*.



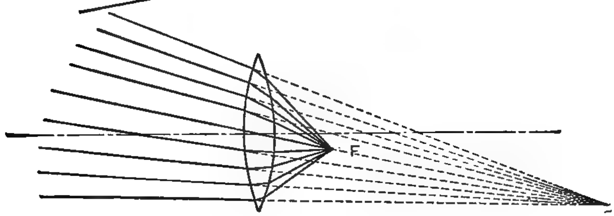


FIG. 93*b*.

56. Formation of Images.—An object may be described as a system of points. The formation of images by lenses depends on the principal of conjugate foci. In Fig. 94, the rays

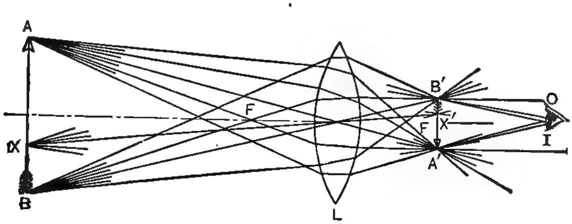


FIG. 94.

of light radiating in all directions from all points of the arrow AB impinge on the lens L. A cluster of rays radiating from a point is known as a "pencil." The pencil of rays proceeding from the point A and impinging on the lens is converged

to the conjugate focus A', while the pencil from B is converged to B'. The rays from points intermediate between A and B are converged, approximately, to points intermediate between A' and B'. These rays, passing through the points of convergence and entering an eye at OI, there appears to the observer at that point an image of the arrow AB, inverted, and situated at $A'B'$.

Note that the object viewed is situated at a distance from the lens *greater* than the focal length. Fig. 95 shows the case

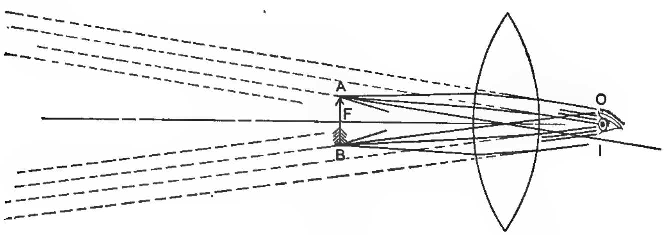


FIG. 95.

where the distance between the object and the lens is *equal* to the focal length. Here the rays from any point of the object become parallel before reaching the eye.

Supposing, now, that an object or an image of an object is placed between a lens and its principal focus. The resulting phenomena are shown in Fig. 96.

As in Fig. 94, AB represents the object or image viewed and OI the eye of the observer. A' is the virtual conjugate focus of A, and B' that of B. The rays from A entering the eye at O appear to be coming from A' and those from B appear to radiate from the point B', and consequently a magnified, erect, and virtual image of AB is seen at $A'B'$.

The foregoing constitutes a summary of some of the principles on which are based the theory of convex lenses. We will now proceed to the discussion of these as applied to the telescopes of surveying instruments.

57. Construction of the Telescope.—The telescope of a surveying instrument consists of four principal parts:

1. The "object-glass" or image-forming apparatus.

2. The "eye-piece" or image magnifying and viewing apparatus.

3. The cross hairs.

4. The tube holding in place all the above.

A longitudinal section of such a telescope in its elementary form is shown in Fig. 97.

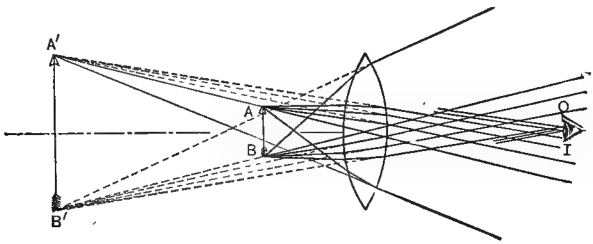


FIG. 96.

Rays from the object *AB*, passing through the object-glass *O*, form, just as in Fig. 94, an inverted image of *AB* in the plane *CH* of the cross hairs, and this inverted image is magnified by the eye-piece *E* just as in Fig. 96. For reasons ex-

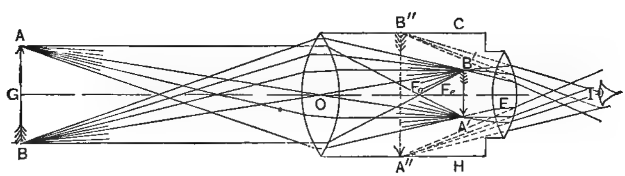


FIG. 97.

plained a little further on, it is necessary that the image formed by the object-glass should be situated in the plane *CH* of the cross hairs, and, as the distance from the image to the object-glass will change with the distance from the object to the object-glass, the plane of the cross hairs being fixed, the object-glass must be movable. A longitudinal section of the complete telescope is shown in Fig. 98. The object-glass is held in the cylindrical slide *S*.

The movement of the slide in and out of the main telescope tube *T* is controlled by a rack and pinion motion operated by the milled screw *R*, the slide being extended for viewing objects near the observer, and drawn in for viewing objects at a distance, so that the image will always lie in the plane *CH* of the cross hairs. The operation of bringing the image, by means of the movement of the object-glass, into the plane of the cross hairs, is known as "focussing."

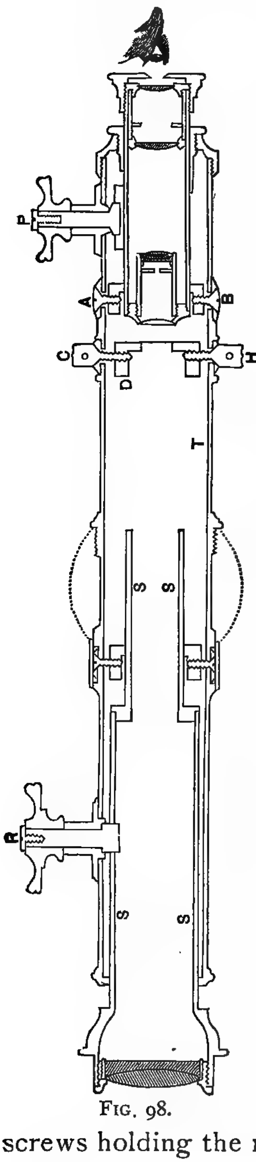


FIG. 98.

If the image is to be magnified by the eye-piece, it must lie between that lens (or combination of lenses) and its principal focus. In addition to the necessity for the formation of the image in the plane of the cross hairs and between the eye-piece and its principal focus, the image and the cross hairs must be separated from the eye-piece by the distance of most distinct vision. This will vary with different observers. On this account the eye-piece is movable in the same manner as the object-glass, the motion being controlled by the milled screw *P*, Fig. 98. In some telescopes, the rack and pinion motion for the eye-piece is omitted, and the eye-piece slide is moved in and out by hand simply. In the plain surveyor's telescope, the cross hairs are two in number, one vertical and the other horizontal, and are attached to a small flanged ring, which in turn is held in place, in the tube, by four capstan-headed screws. Two of these screws holding the ring *D* are shown at *C* and *H*, Fig. 98. A cross-section of a telescope near the ring is shown in Fig. 100,

while in Fig. 99 are given three views of the ring. In Fig. 100, *TT* represents the telescope tube, *RR* the ring and *FF* its flange, into which are screwed the capstan-headed screws *C*, *H*, *C′*, and *H′*. Between

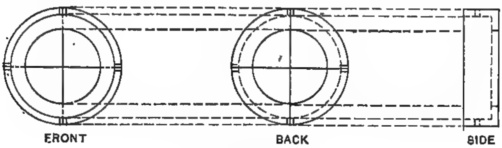


FIG. 99.

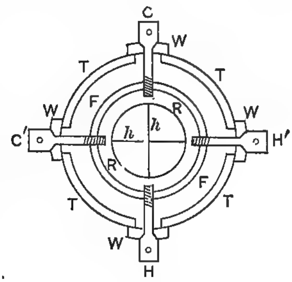


FIG. 100.

the heads of these screws and the telescope tube are placed the washers *WW*. The wires or hairs (platinum wires or spider webs) are glued to the back of the ring.

A " line of sight " of a telescope is any axis of the object-glass, that is, any straight line passing through the optical center of the object-glass. In connection with surveyors' telescopes, when the line of sight is mentioned, the particular axis referred to is the one connecting the intersection of the cross hairs with the optical center of the object-glass. The reason that lines passing through the optical center are called lines of sight is that only those rays of light passing through the optical center suffer no deviation. The use of the cross hairs and the necessity for the formation of the image in the plane of the cross hairs now become apparent. If the image of a point is formed in the plane of the cross hairs, and the observer sees through his telescope the intersection of the cross hairs coinciding with this point, he knows that, except for the effect of refraction in the atmosphere, the intersection of the cross hairs, the optical center, and the point are in the same straight line. If the image is not situated in the plane of the cross hairs, the intersection of the hairs will appear to coincide with the image of the point only when the eye of the observer is in line with these two, and the eye moved to one side or the other can see between them. From which it is evident that, if the image of the point is not in the plane of the cross hairs, when the image of the point and the intersection of the hairs

appear to coincide, the intersection of the hairs, the optical center of the object-glass, and the point may or may not be in the same straight line. The condition, resulting from the fact that the cross hairs and image are not situated in the same plane, is known as "parallax." In the elementary telescope shown in Fig. 97 it will be noted that both the image formed by the object-glass and the virtual image viewed by the eye-piece are inverted. In the complete telescope shown in Fig. 98 the eye-piece, instead of being one lens, is made of four lenses, the arrangement being such that the image is re-inverted and appears to the eye in an upright position. For the purpose of the present discussion it will not be necessary to go into the details of the construction of this eye-piece. For this consult works on Optics.

SECTION II.

Defects and Qualities of the Telescope.

58 **Defects.**—The elementary telescope shown in Fig. 97 possesses several serious defects. Among these we may notice in connection with the object-glass:

1. Spherical aberration. In the figures hitherto the rays of light proceeding from any point of an object have all been represented, after passage through the object-glass, as refracted to one identical conjugate focus on the other side. This, however, is not strictly true, especially where the object-glass is a single lens. The rays of light which impinge on a lens near its edge undergo more refraction than those which pass through the lens near its center. The effect of this is that for every point of the object viewed there will be shown many points in the image formed, and the image consequently will be hazy and blurred. The fault is corrected by making the object-glass of two lenses, as shown in Fig. 98. The two lenses are made of different kinds of glass, crown and flint in general, having different dispersive and refractive powers, and the effect of this combination, when properly adjusted, is to remove almost entirely the blurring due to spherical aberration.

2. The aberration of sphericity. Rays of light proceeding

from a flat object and passing through a simple convex lens are refracted in such a manner that the image formed does not lie in a plane, but is concave towards the lens. In the case of the object-glass the aberration of sphericity is inappreciable. This aberration of sphericity is entirely different from spherical aberration. It will introduce no error where the telescope is used simply for the measurement of angles or for leveling, as in these cases only one point is observed at a time. In the case of stadia measurements, to be discussed later, it becomes objectionable.

3. Chromatic aberration. This is due to the fact that white light, in passing through a prism or simple convex lens, is separated into the colors of the spectrum. The different kinds of colored light are refracted differently. Supposing, for instance, that rays of light from a white object impinge on a simple convex lens. In passing through the lens the white rays are separated into the colors of the spectrum, and the violet rays, being more refrangible, form a violet image of the object closer to the lens than the images formed by rays of other colors. The red rays form a larger image further away from the lens, and the rays of other colors images intermediate in size and position between those of the violet and red. If a circular white disk is viewed with a telescope whose chromatic aberration has not been corrected, it will appear white at the center and colored at the edges, the extreme edge being red. This is because the red image is the largest and extends beyond all the others. In the center all the images combine to give a white effect. Each one of these images is affected by spherical aberration and, in a very slight degree, by the aberration of sphericity. The resulting view is disfigured and indistinct. This error in the object-glass is corrected in the same way as the spherical aberration. The object-glass shown in Fig. 98 corrects both the chromatic and spherical aberration, and is known as an achromatic lens. (From the Greek *α*, without, and *χρομα*, color.) An eye-piece consisting of a simple convex lens is affected by the same errors as the simple object-glass, the aberration of sphericity in this case, however, being much more pronounced. In the eye-piece, different combinations of

lenses are used to correct the errors described above. A common arrangement is shown in Fig. 98. In this the two lenses nearest the object-glass are inserted for the purpose of re-inverting the image and presenting it erect to the eye. Every extra lens that is put in a telescope causes a certain loss of light, and, on this account, most authorities prefer the telescope which shows the object inverted. Such telescopes are found in the instruments of the highest class, but are rarely seen in ordinary American surveying instruments. Among foreign engineers they are much more common.

59. Qualities of Telescopes.—The defects in telescopes which have been mentioned so far, viz., spherical and chromatic aberration and the aberration of sphericity, are inherent defects and exist *originally* by no fault of the manufacturer, though, of course, in a well made telescope they will be corrected. We now come to the consideration of other qualities in the telescope which are not inherent, but depend entirely on the way in which the instrument is made. These qualities are four in number, viz.: (1) Definition, (2) Illumination, (3) Magnification, (4) Size of the field of view.

1. Definition. The "definition" of a telescope is the clearness, distinctness, and sharpness with which it presents the image to the view. It depends on the accuracy with which the lenses have been ground and polished and on the "centering." By the "centering" of the lenses is meant the placing in coincidence of the principal axes of those lenses which go to make up a compound lens, as the object-glass in Fig. 98. The definition depends also on the spherical aberration, but lack of clearness due to this is an inherent defect.

2. Illumination. By the "illumination" of a telescope is meant the relative amount of light conveyed by its lenses from a point to the eye, compared with the light sent from the point to the naked eye. An object viewed through a telescope will appear brighter in proportion to the amount of light passing from it to the eye. Or, in other words, each point of the object will appear brighter in proportion to the number of rays of light in the pencil of rays proceeding from that point and entering the eye. The larger the object-glass

the more rays in the pencil of light which enters it from any point. Consequently, the larger the object-glass, the higher the illuminating power of the telescope. The "aperture" of the object-glass is the diameter of that part of it which transmits light to the eye. It is difficult to grind the edges of lenses perfectly. It is, therefore, customary to place diaphragms in the tube to exclude rays passing through these edges. On this account, in general, the aperture of an object-glass will not be the same as its actual diameter.

A certain amount of light is diffused or reflected by the lenses. The best surveying telescopes transmit only 85% of the light impinging on the object-glass. In poor instruments this percentage falls to seventy. Loss of light from this cause is an *inherent* defect, but is mentioned in this connection because of its affecting the illumination.

The eye-piece, in its capacity as a magnifying glass, pulls apart, so to speak, the rays of light proceeding from any point of an object viewed and distributes them over a larger area, by which process the illumination is diminished.

An object viewed through a telescope, except in the case of stars, will never appear as bright as when seen by the naked eye. At dusk, a rod, which can just be seen by the naked eye, cannot be seen through a transit telescope. An object seen through a telescope will appear at its brightest when the arrangement of the lenses is such that the pencil of rays issuing from the eye-piece is as large or larger than the pupil of the observer's eye. In this case the brightness of the object is the same, except for the losses due to loss of light in the lenses, as when seen by the naked eye. When the pencil is larger than the pupil of the eye the magnification will be proportionally small. The best arrangement of the lenses from the standpoint of illumination and magnification then, will be when the emergent pencil is the same size as the pupil of the observer's eye.* If this condition is fulfilled, and the lenses, as in the best instruments, transmit 85% of the light, the brightness of the object seen through the telescope will be 85% of its brightness as

* Deschanel's Natural Philosophy ,Part IV, Art. 1038.

seen by the naked eye. If the emergent pencil is smaller than the pupil of the observer's eye, the brightness will be proportionally decreased. The illumination is, therefore, seen to depend on (1) the aperture of the object-glass, (2) the light-transmitting capacity of the lenses, and (3) the magnification. The brightness of any object presented to the vision depends on the size of the pupil of the observer's eye, but this is not a part of the telescope.

3. Magnification. The "magnification" or magnifying power of a telescope is the ratio of the angular size of an object as seen through the telescope to its angular size as seen by the unaided eye. In Fig. 97, I represents the position of the observer's eye. The magnification, which we will represent by M,

$$= \frac{\angle B''IA''}{\angle BIA'}.$$

It is to be observed that while in the figure the rays appearing to proceed from B'' towards the eye have greatly varying directions, as an actual fact, the pupil of the eye is so small that these rays are practically parallel. Therefore, the angle at the eye made by a ray from B'' and a ray from A'' remains the same, no matter what pair of rays may be selected. The same statement will apply to rays coming to the unaided eye from A and B. As the distance OI is inconsiderable in comparison with GI, angle AOB equals angle AIB. Therefore

$$M = \frac{\text{angle } B''IA''}{\text{angle } BOA},$$

and, as angle BOA equals angle $B'OA'$,

$$M = \frac{\text{angle } B''IA''}{\text{angle } B'OA'}.$$

Angle $B''IA''$ is practically equal to angle $B'EA'$. Therefore

$$M = \frac{\text{angle } B'EA'}{\text{angle } B'OA'}.$$

Let F equal the focal length of the object-glass and f that of the eye-piece. In the figure, F_O represents the principal focus of the object-glass, and F_E that of the eye-piece. In practice,

the distance of either of these points from the image is inconsiderable, and they may, therefore, be considered to lie in its plane. The thickness of the lenses is also inconsiderable. From which facts,

$$\text{angle } B'EA' = 2 \tan^{-1} \frac{A'B'}{2f}$$

$$\text{and angle } B'OA' = 2 \tan^{-1} \frac{A'B'}{2F}.$$

Angles $B'EA'$ and $B'OA'$ are small, and we may therefore substitute for them in the value of M the tangents of the half angles, whence

$$M = \frac{\dfrac{A'B'}{2f}}{\dfrac{A'B'}{2F}} = \frac{F}{f}.$$

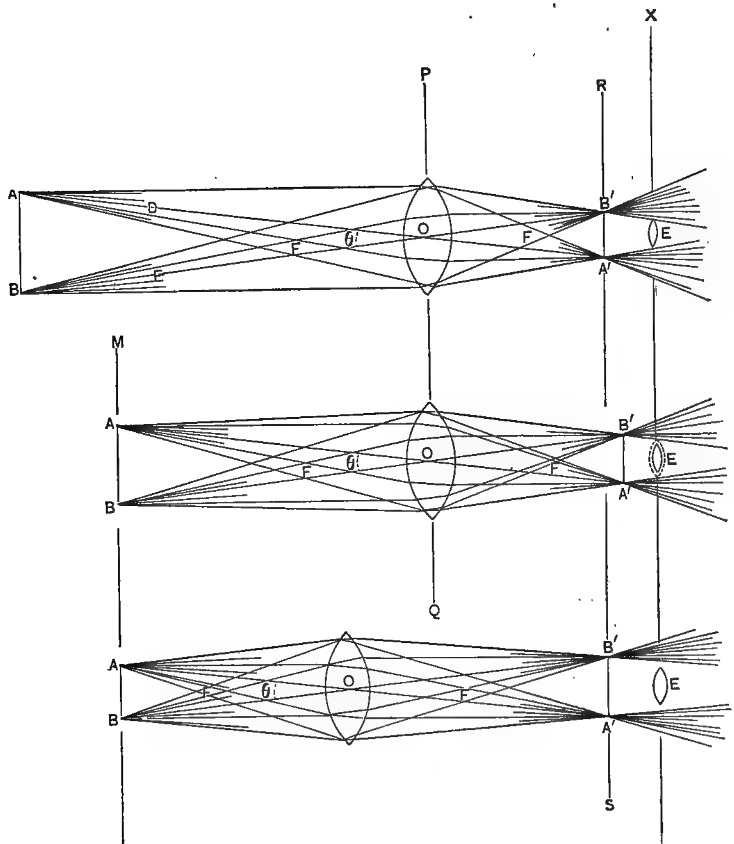
Consequently the linear magnification of the telescope equals the ratio of the focal lengths of the object-glass and the eye-piece; or,

$$M = \frac{F}{f}.$$

From what has been said, it will be understood that the illumination is in inverse proportion to the magnification. The magnification, however, is a factor of importance also. It must be great enough to enable the observer to point his telescope accurately. On account of these facts, in the manufacture of surveying telescopes, a proper proportion, based on experience, should exist between the magnification and the illumination. A well-known firm give their transit telescopes a magnification of 24 diameters. Another firm state that the brightness of an object seen through a telescope should never be allowed to fall below half that when seen by the naked eye. A third firm make their level telescopes with a magnification of 33 diameters.

4. Size of the field of view.—By the "field of view" of a telescope is meant the whole area seen by the observer when looking through the instrument. This area, of course, is a circle,

and by the "size" of the field of view is meant the angle at the optical center of the object-glass subtended by a diameter of this circle. Fig. 101 shows that the size of the field is inde-



FIGS. 101–103.

pendent of the diameter of the object-glass and that it may be increased theoretically at will by increasing the diameter of the eye-piece.

AB represents a line just longer than the diameter of the field of view. The pencil, or cone, of rays proceeding from *A*, passes through the object-glass, is refracted to the focus *A′*,

and passes on tangent to the lower edge of the eye-piece. In like manner, the cone of rays from B is tangent to the upper edge of the eye-piece. Evidently, by enlarging the eye-piece, rays from A and B could be intercepted, and this process of enlargement, theoretically, be carried on indefinitely.

In Fig. 102, the focus of the telescope remains the same as in Fig. 101; but the objects viewed are supposed to be nearer at hand. The image, of course, is formed nearer to the eye-piece than in Fig. 101. Now, if under these circumstances the size of the field was the same as in Fig. 101, two rays passing, from opposite extremities of a diameter, through the optical center of the object-glass would make the same angle, θ, as in the former figure. In Fig. 102, therefore, the rays AO and BO are shown making this angle at the optical center. In this case, however, the cones of rays are seen to miss the eye-piece entirely, the size of that lens being the same in both figures. From which we have the fact that, with the same focus of the telescope, the angular size of the field of view varies with the distance, growing larger as we depart from the instrument. Thus in Fig. 101, with the telescope focussed as there shown, and the eye-piece just large enough to intersect the cones proceeding from A and B, those points would be visible, while the points D and F, on the lines connecting A and B with the optical center, would not be seen.

In Fig. 103, the focus is changed, the object-glass being run out. This would correspond to focussing for objects nearer at hand than those focussed on in Fig. 101. The distance between the image and the eye-piece remains the same as in Fig. 101. While in Figs. 101 and 102 the eye-piece is shown the same size, in Fig. 103 it is the size of the dotted eye-piece in Fig. 102. The plane of the field of view is the same distance from the telescope as in Fig. 102. The rays OA and OB make the same angle θ, and the cones of rays proceeding from A and B are seen to miss the eye-piece entirely. Also, the size of the field decreases as the magnification increases. This may be seen as follows: In actual practice, the diameters of the cones of rays shown in Figs. 101, 102, and 103 are so small that the situation is practically as represented

in Fig. 104, where the rays AO and BO, passing from opposite extremities of a diameter of the field of view, are shown tangent to the edges of the eye-piece. As shown in the dis-

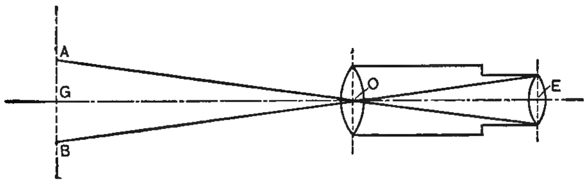


FIG. 104.

cussion of the magnification, F_O and F_E are practically coïncident, and OE practically equals F plus f. Also,

$$M = \frac{F}{f}.$$

Let the size of the field be represented by S and the diameter of the eye-piece by d. Then

$$S = 2 \tan^{-1} \frac{d}{2(F+f)} = 2 \tan^{-1} \frac{d}{2f\left(\frac{F}{f}+1\right)} = 2 \tan^{-1} \frac{d}{2f(M+1)}.$$

The conclusions thus reached may be summed up as follows: The size of the field of view (*a*) is independent of the size of the object-glass, (*b*) increases with the size of the eye-piece, (*c*) increases with the distance from the instrument, (*d*) increases as the object-glass is moved in towards the eye-piece, and (*e*) increases as the magnification decreases.

The question of the proportion between the size of the field and the magnification comes in here as in the case of the proportion between the illumination and the magnification. Practice will differ with different makers. A large size of field, will, of course, be found convenient. Generally between magnifications of 20 and 30 the size of the field will vary from 1° 30′ to 1° 00′. Between magnifications of 30 and 40, from 1° 00′ to 0° 45′ about.

SECTION III.

Optical Tests of the Telescope.

60. A good telescope for a surveying instrument, then, should possess the following qualities: It should be free (1) from spherical aberration, (2) from the aberration of sphericity, (3) and from chromatic aberration; (4) the definition should be good, (5) the illumination should be in suitable proportion to the other qualities of the telescope, and incidental to this (6) the aperture of the object-glass should be as large as practicable; (7) the magnification should be at its maximum compatible with a sufficiency of illumination, and (8) the field of view should embrace as large an area as possible. These qualities may be tested by the following methods:

1. Spherical aberration. Determine, by the method given below, the aperture of the object-glass. Cover this with black paper, leaving in the middle a circular hole whose diameter is half the object-glass aperture. Focus on some well-defined object, as clear print. Remove the black paper from the object-glass, and, instead, place thereon a circular disk of black paper, so as to cover the part left open before. If the spherical aberration has been thoroughly corrected, the print will appear as distinct as before. If it does not, note how much the object-glass must be moved to make the print so appear. The movement of the object-glass is a measure of the spherical aberration.

2. The aberration of sphericity. This test is also known as the "Test for Flatness of Field."

Draw with India ink on white paper a square whose side is about four inches. The lines should be heavy. Fasten the paper to some flat surface, as a drawing-board, and set the board on edge with its face perpendicular to the line of sight of the telescope, and at such a distance from the instrument that, when focussed on it, the square appears to fill nearly the whole of the field of view. If the paper appears to be a flat surface and the sides of the square straight, the aberration of sphericity has been corrected. As before stated, in ordinary work a fault of this kind will cause no error. It is only when

two objects are to be viewed at the same time, as in stadia measurements (see Chap. VIII), that it becomes of moment.

3. Chromatic aberration. The test for this is simply to focus on a white disk, and note whether it appears fringed with color. If the edges appear sharp and white, the chromatic aberration has been corrected.

4. Definition. The definition of a telescope is good when, with is aid, fine clear print can be read at a distance of from thirty to fifty feet, as well as by the naked eye at the distance of most distinct vision. Otherwise the lenses are at fault in their curvature, finish, or centering. However, if spherical aberration has before become apparent, indistinctness may be partly or wholly due to that fault. In this case it becomes difficult to decide whether or not the definition, in the strict sense of that term, is satisfactory.

5. Illumination. There being no absolute unit of bright ness, the illumination is merely relative, and can be determined only by a comparison between different instruments. The expression for the illumination is:

$$I = \frac{LA^2}{M^2},$$

where L is the proportion of light transmitted by the lenses, A the aperture of the object-glass, and M the linear magnification. The brightness of an object viewed through a telescope being in inverse proportion to the area of the pupil of the observer's eye, if this is considered, and the diameter of the pupil represented by e,

$$B = \frac{LA^2}{e^2M^2}.$$

To test the illumination, at about dusk place two telescopes side by side. As darkness comes on, observe the same object through both telescopes. The telescope which makes the object visible later has the better illumination of the two.

6. The aperture of the object-glass. Focus the telescope for distant objects and point toward the sky. There will be formed by the eye-piece, and just behind it, a real image of

the aperture of the object-glass. This image can be viewed by holding the eye some inches behind the telescope and looking through an ordinary magnifying glass. Hold the point of a pencil in contact with the outer face of the object-glass and move this in, if necessary, until it is just visible at the edge of the image formed by the eye-piece. To obtain the aperture of the object-glass, from its diameter subtract twice the distance of the point from the edge of the lens.

7. Magnification.—Place a divided rod from 100 to 200 feet from the telescope. Observe this rod at the same time through the telescope with one eye and with the other eye unaided. Suppose that D divisions seen on the rod by the naked eye appear to cover the same space as d divisions seen through the telescope. If M, as before, equals the magnification,

$$M = \frac{D}{d}.$$

8. Size of the field of view.—See Fig. 104. Focus for objects about 300 feet from the telescope and note two points, A and B, in opposite extremities of the field of view. Measure the distance AB between these points and the distance OG from the center of the objective to a point G midway between A and B. This will be the same practically as the distances OA and OB. Therefore, in practice, measure one of these instead of OG. If a rod is held, first on A and then on B, having focussed on A, B can be made the same distance from the telescope as A, by moving B to and from the telescope until it appears as distinct as A, the focus in the meantime remaining unchanged. If more precision is desired, measure the distances OA and OB, making them equal. The size of the field of view equals the angle AOB and

$$AOB = 2 \tan^{-1} \frac{AG}{GO}.$$

As before stated, the size of the field will depend somewhat on the distance from the telescope of the objects viewed. Focussing for 300 feet gives approximately an average size of the field.

SECTION IV.

Adjustment of the Line of Sight.

61. Method of Inserting Cross Hairs.—Fig. 100 is a general view of the cross hairs and the ring and screws by which they are attached to the telescope tube.

Fig. 99 shows three elevations of the ring and hairs.

Spider webs make the best material for cross hairs, though fine platinum wires are sometimes used. Select fibers as near opaque as possible. They may be obtained from spider nests. Having removed the cross hair ring carefully from the telescope tube, there will be found on it lines at right angles to each other, marking the position of the cross hairs. Clean the ring and place it, with the marks up, so that when the hairs are laid across it, the ends may hang down on the sides. Having selected a fiber, attach to each end of it, by means of shellac varnish, a weight of paper or of wood, as heavy as practicable, and place it in water for five or ten minutes. Having removed the fiber from the water, lay it across the ring and on the marks for one hair, moving it exactly into place with a pin. When in position, let a drop of shellac varnish fall gently on one end over the mark. When this has hardened, making sure that the web is stretched tight across the ring, fasten the other end in the same way. The other hair is attached in the same manner. If the work is carefully done, the hairs will not become loose even in the dampest weather.

62. Methods of Telescope Attachment.—Telescopes are attached to surveying instruments, generally, in one of three different ways:

(*a*) The telescope tube rests in two "wyes," one near each end. Cross-sections of these wyes are supposed to present equal angles perpendicular to the geometric axis of the telescope. The wyes themselves are rigidly attached to the body of the instrument. The telescope tube admits of revolution in them, around its own axis of figure. Such a method of attachment is used in the wye level (Fig. 116).

(*b*) The telescope tube is rigidly attached to a "horizontal axis," the axes of figure of the "horizontal axis" and of the telescope tube being supposed to intersect at right angles.

The horizontal axis, of course, does not extend through the telescope tube, but is attached to it at either side, and has its ends resting in bearings rigidly attached to the body of the instrument. This method of attachment is used in the engineers' transit (Fig. 155).

(*c*) The telescope itself may be rigidly attached to the body of the instrument. This is the case in the dumpy level (Fig. 119).

Other methods of attachment are modifications of the three given above. Note in regard to the movement of the telescope tube with reference to its immediate supports, that the first method of attachment admits of only a rotation around the telescope tube's axis of figure; that the second admits of only a rotation around a line perpendicular to the telescope tube's axis of figure, and that the third permits of no movement at all. It may be stated further, however, that field-instruments, generally, admit of a rotation of the telescope *with its immediate supports* around a *vertical* axis.

63. Remarks Preliminary to Discussion of Adjustments.—The tests of the telescope heretofore described have been those pertaining to its optical properties only. To insure correct work certain conditions must be fulfilled concerning the line of sight. These vary somewhat with the method of attachment of the telescope. In distinction to the optical tests heretofore mentioned, those about to be described may be known as "geometrical tests." The method of adjustment is given in each case. It should here be stated that many writers style the "line of sight" (Art. 57) the "line of collimation." Others give the same definition of the line of sight as found in Art. 57, and state that the line of collimation is "the mathematical line at right angles to a certain axis," also that the adjustment of the line of sight consists in its being brought into coincidence with the line of collimation.

For our present purpose it will be sufficient to state that the line of collimation practically coincides with the geometric

axis of the telescope tube. On account of the ambiguity above mentioned the expression will not be further used.

64. Centering the Eye-piece.—Before making any of the following tests and adjustments the telescope should be pointed to the sky and the eye-piece moved in or out until perfectly clear vision of the cross hairs is had.

The fact that the intersection of the cross hairs does not appear in the center of the field of view is no indication that there is anything radically wrong with the telescope.

The instrument may be in perfect working order, with the hair intersection appearing far out of center. The apparent situation of the hairs depends on the position of the eye-piece only. It is convenient, however, to have the cross hairs appear in the center of the field of view. Also during the following adjustments the actual position of the hairs is changed.

Consequently, wait until all the tests and adjustments are finished; and then, by moving the ring in which the inner end of the eye-piece slide works, bring the hairs into the center of the field of view. This movement of the ring is controlled by the screws shown at A and B, Fig. 98.

In the tests described in the remainder of this chapter the vertical axis is supposed always to be truly vertical.

65. The Telescope Attached by the First Method.—With this method of attachment it is necessary for the line of sight to be parallel with the geometric axis. The test of this parallelism depends on the rings, i.e., the part of the tube which rests in the wyes being truly cylindrical. They may be tested with a pair of calipers. If found defective the further test cannot be made.

Let us examine the relative positions in which the line of sight and the geometric axis may be found.

(*a*) The optical center and the intersection of the hairs, both on the geometric axis.

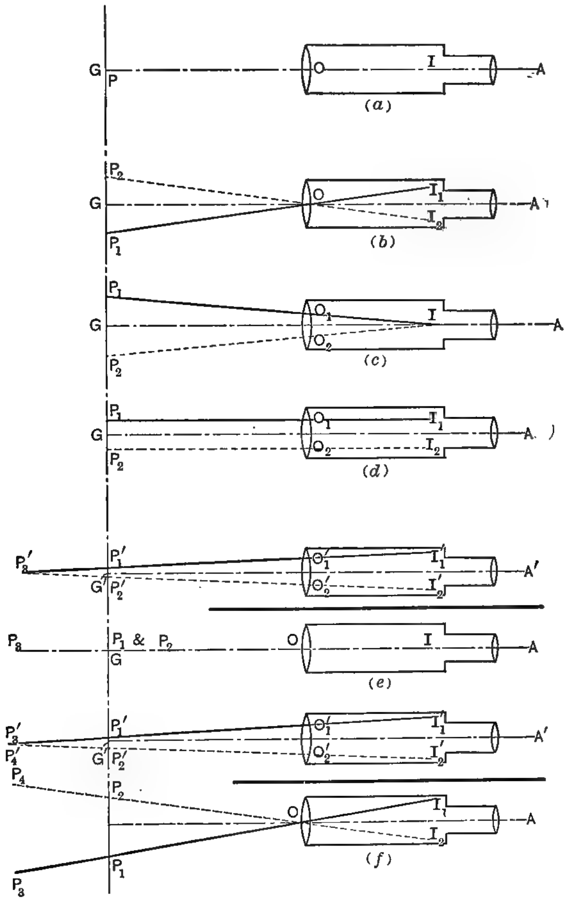
(*b*) The optical center on, and the hair intersection *not* on, the geometric axis.

(*c*) The intersection of the hairs on, and the optical center not on, the geometric axis.

(*d*) Neither the optical center nor the intersection of the

hairs on the geometric axis, but the line of sight and the geometric axis parallel.

(*e*) Neither the optical center nor the intersection of the



FIGS. 105-110.

hairs on the geometric axis, the line of sight and the axis intersecting at some angle.

(*f*) The line of sight and the geometric axis not lying in the same plane.

The six conditions are illustrated in Figs. 105 to 110, in

which I represents the intersection of the hairs, O the optical center, and GA the geometric axis.

The vertical projections in Figs. 109 and 110 are the same. The horizontal projections, however, show that in Fig. 109, IP and GA lie in the same plane, while in Fig. 110 they do not. Fig. 110 shows the horizontal projections I_1P_1 and I_2P_2 crossing at O. This, of course, is not necessarily true. They might just as well have crossed at any other point on the horizontal projection of GA. If the line of sight is directed to some point P, Figs. 105–110, and the telescope revolved 180° around its axis in the wyes, the intersection of the hairs I may or may not continue to cover the point P. Let us examine the movement of the line of sight during such a revolution.

Condition (*a*), Fig. 105. IP and GA coincide. I will continue to cover P.

Condition (*b*), Fig. 106. During the revolution, IP will describe the surface of a cone, and after 180° will be in the position I_2P_2 directed to P_2.

Condition (*c*), Fig. 107. IP will describe a cone pointing to P_2 at the end of 180°.

Condition (*d*), Fig. 108. IP describes a right cylinder. After the revolution it is found at I_2P_2 pointing to P_2.

Condition (*e*), Fig. 109. IP describes a cone. After the revolution it points to P_2, unless P has been taken at P_3, the intersection of IP and GA. In the figure, O is shown closer than I to GA. The reverse might have been true, in which case P_3 would not exist.

Condition (*f*), Fig. 110. In this case IP generates a hyperboloid. After the revolution it points to P_2. Note that this is the only case in which IP and GA do not lie in the same plane.

The test for the parallelism of the line of sight and the geometric axis is seen to be a test for discovering whether or not the line of sight and the geometric axis are in one of the relative positions shown in Figs. 105 and 108.

Test. To make the test, then, proceed as follows:

Focus on a point so distant as to require the object-glass to be run entirely in. Next revolve the telescope around its

axis in the wyes 180°, and note whether the hair intersection still covers the point. If so, either condition (*a*) or condition (*e*) (point P_3, Fig. 109) has been shown to exist. Assuming for the time that (*a*) prevails, focus on a near point, thus requiring the object-glass slide to be run out to its full length, and test again. If the test is again fulfilled, we may reasonably suppose that condition (*a*) exists once more, i.e., that the optical center has continued on the geometric axis during the whole operation. It is possible, but extremely improbable, that, owing to a special position of the hair intersection and the peculiar ill-fitting motion of the object-glass slide, the telescope has been directed to two cone vertices similar to P_3 (Fig. 109). This last supposition is so highly improbable, however, that in practical work it need not be considered.

Adjustment. If, however, with the object-glass slide run in, as in the first case, the test is not fulfilled, one of conditions (*b*), (*c*), (*d*), (*e*) (points P_1 and P_2, Fig. 109), or (*f*) exists, and an adjustment, except in the case of condition (*d*), becomes necessary. It should here be remarked that in good instruments, with the object-glass slide run entirely in, the optical center will lie on the geometric axis. At least, its departure from that line will be too small to be observed by the engineer. Consequently, with the slide run in, all the conditions except (*a*) and (*b*) are of very rare occurrence. In fact, they are generally not considered. The test, not being fulfilled, we, therefore, assume that (*b*) exists and adjust by moving the ring containing the hairs until the line of sight is directed to a point G midway between P_1 and P_2. Evidently, Fig. 106, this will bring IP and GA into approximate coincidence. Repeat the test and adjustment until the test is fulfilled, next focus on the near point and make the test on that. If the test is now not fulfilled, the adjustment has been destroyed by the optical center's leaving the geometric axis as the slide has been run out.

Correct, as before, by altering the direction of the line of sight until it points midway between P_1 and P_2, but do this now by moving the inner end of the object-glass slide instead of the cross hairs. Test on the distant point once more, adjusting if necessary, and observing first the distant and then the near

point until the test is fulfilled in both positions of the slide. Fig. 111 shows the case in which condition (*a*) has been changed to condition (*c*) by the running out of the slide. The slide is not shown, but, as it has been run out, the optical center is supposed to have moved from O to O_1, when the line of sight IO_1 is directed to P_1. After the revolution, O_1 moves to O_2 and the line of sight is found in the position IO_2P_2. Moving

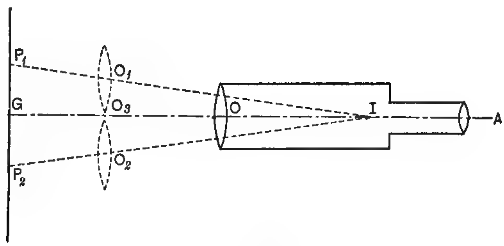


FIG. 111.

the inner end of the slide until the optical center assumes the position O_3 and the line of sight is directed to G, a point midway between P_1 and P_2, is seen to effect the desired adjustment.

If after the revolution the line of sight appears to be out of adjustment (*appears*, because condition (*d*) is really a condition in adjustment) and a condition other than (*b*) exists, the observer, in his efforts to make the adjustment, will induce condition (*e*) with the line of sight directed to a cone vertex P_3. It is possible that he might then adjust the intersection and the slide going from one to the other that the instrument would finally be in *apparent* adjustment.

This, as before stated, is an improbable case. The observer would probably find it impossible to get the telescope into apparent adjustment with the slide both in and out, and after a number of trials would be justified in concluding that, with the slide run in, the optical center was off the geometric axis. The beginner should bear in mind, however, that he will probably experience some difficulty in adjusting the instrument in any case, and so not be too hasty in deciding that the instrument is ill made. However, with instruments from a reputa-

ble maker, the optical center, with the slide run in, is pretty sure to lie on the geometric axis.

66. Movement of the Diaphragm and the Slide.—As shown in Figs. 98 and 100, the ring, or diaphragm to which the cross-hairs are attached, is held in its place in the tube by four capstan headed screws, two vertical and two horizontal. Screwing in one of these pulls the diaphragm towards it. Do not pull on one screw without loosening its opposite, as also one of the screws in the other pair. The method of adjusting instruments by means of capstan headed screws and adjusting pins is not very delicate. However, it appears to be the only one practicable.

Note that if, after the revolution, the line of sight appears too low, to adjust, the cross-hairs must be moved still lower. This is on account of the fact that the line of sight revolves around the optical center. The tendency of the beginner is to move the hairs in the other way.

In the case of the object-glass slide, however, if the line of sight appears too low the optical center must be raised. This is done by screwing the inner end of the slide down as shown in Fig. 111. The screws holding the ring in which the inner end of the slide moves are generally counter-sunk and covered with a ring (Fig. 98).

In some instruments the slide is not adjustable. In such a case, with the slide run out, the line of sight can be tested only. If found wrong it cannot be corrected by the engineer.

In adjusting either the hairs or the slide, correct the line of sight first for one hair, and then for the other.

In connection with the foregoing test the conditions consequent on the optical center's being off the geometric axis have been shown. The further discussion will assume that, with the slide run in, the optical center lies on the geometric axis.

67. Adjustment of the Telescope Attached by the Second Method.—With this method of attachment it is necessary that the horizontal axis, at its point of intersection with the vertical axis, should be intersected at right angles by the line of

sight. Bringing the line of sight into this position involves two tests and the corresponding adjustments. These may be known, respectively, as the adjustment of the line of sight to intersect the horizontal and vertical axes and its adjustment to be perpendicular to the horizontal axis. They are often known as the "adjustments of the vertical and the horizontal hairs."

1. The adjustment to make the line of sight intersect the horizontal and vertical axes.

Test. Set up the instrument opposite a vertical wall, and so far from it that focussing for a point on it will require the slide to be run entirely in. With the telescope tube at about right angles to the face of the wall, mark the point to which the line of sight is directed. The front end of the tube should rest in a notch cut in a board driven firmly in the ground, the whole being arranged to admit of the instrument's rotation around its vertical axis. Revolve the telescope 180° around first its vertical axis and then its horizontal axis, allowing the front end of the tube to rest in the notch once more.

This is technically known as "reversing in azimuth and altitude." Reversing in altitude is further known as "plunging" or "transiting." The telescope will now point in the direction of the wall, but the tube will be upside down.

Note whether the line of sight now covers the same point as before. If so, it intersects the horizontal and vertical axes. Next, make the test, using instead of the wall a board set up near the instrument, thus requiring the slide to be run out to its full extent.

If the test is again fulfilled, we may conclude that it would be fulfilled for any intermediate position of the slide, though this is not *necessarily* true. If desired, the test can be made with several intermediate positions of the slide.

Adjustment. If the test is not fulfilled with the slide run in, adjust by moving the hair intersection up or down, and right or left, as may be necessary, until the line of sight cuts a point midway between the first and second marks on the wall. Having done this, test on the near point. This time, if an adjustment is found necessary, make it by moving the inner end of the slide as described in the preceding article.

Continue observing, and adjust until the test is fulfilled for both distances.

Explanation. When the telescope is reversed in altitude and azimuth and made to rest in the notch again, the position of the line of sight is the same, supposing the geometric axis to intersect the vertical and horizontal axes, as if the tube had been attached by the first method and revolved around its geometric axis in the wyes, just as in the test and adjustment of Art. 65. Indeed, to make this adjustment, some engineers prefer to take the telescope from its supports and place

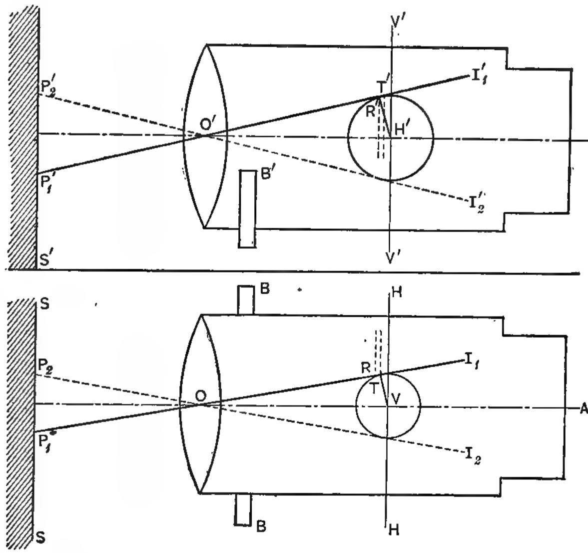


FIG. 112.

it in a pair of specially made wooden wyes. Note, however, that the method of revolving around the geometric axis brings the line of sight into coincidence with that axis, while the method of this article makes it intersect the vertical and horizontal axes. Practically this amounts to the same thing, as the manufacturer makes the telescope with its geometric, horizontal, and vertical axes intersecting in a point. Fig. 112 may serve to make this explanation somewhat more plain.

The telescope tube is shown in vertical and horizontal pro-

jection. In the first position of the tube resting in the notch in the board *B*, the line of sight I_1O covers the point P_1 on the vertical wall or board at *S*. As the telescope is revolved in azimuth and altitude around the vertical and horizontal axes, I_1O swings to the position I_2O.

Note that the optical center is now found in the same position as at first. To accomplish this has been the purpose of the notched board. Instead of using the board some writers recommend determining, by means of certain divided circles to be described later (Chap. VI), when the optical center has come back to its first position. This makes the whole adjustment depend upon the accuracy with which the circles have been divided and the vernier read.

Using the notched board eliminates both these sources of error.

With the line of sight in the position I_2O, the point P_2 is covered on the wall. An examination of both projections shows that the line of sight will intersect both the horizontal axis *H* and the vertical axis *V*, when by moving the hair intersection it is made to cover the point *G* midway between P_1 and P_2. Note that, although the manufacturer makes the telescope with the vertical and horizontal axes intersecting, this adjustment does not depend on that fact. In Fig. 112 the slide is supposed to be run in.

A similar figure can be drawn to illustrate the adjustment with the slide run out.

2. The adjustment to make the line of sight perpendicular to the horizontal axis.

Test. Set up the instrument in a fairly level place, having the slide run in, and sight on the bottom of a pin placed in the ground far enough away to avoid parallax. Plunge the telescope and set another pin in the same way and at the same distance from the instrument. Revolve the instrument in azimuth and sight again on the first pin.

Plunge, and note whether the line of sight, with no further revolution in azimuth, may be once more directed towards the second pin. If so, the adjustment is perfect.

Adjustment. If not, set a third pin in the line of sight in

its last position and the same distance from the instrument as the second pin. By motion of the hair intersection only, direct the line of sight to a point between the second and third pins and one fourth the distance from the third to the second. Test, and, if necessary, adjust again until the test is fulfilled. Next, with the slide run out and the pins brought correspondingly near, repeat the whole operation, correcting this time by the movement of the object-glass slide, and so on, observing first the distant and then the near points until the test is fulfilled with both positions of the slide.

Explanation. It should here be stated that, if the instrument has been carefully made, the first adjustment of this article effects the second also. For, in the correct telescope with the slide run in, the optical center lies on the geometric axis, which, in turn, meets at right angles the horizontal axis at its intersection with the vertical axis. However, with the second method of attachment, it is of particular importance that the line of sight should be perpendicular to the horizontal axis, and, as the present operation tests this directly, it should always be performed.

Fig. 113 shows the movement of the line of sight in plan and elevation. H_1A_1 and I_1O_1 are the first positions of the horizontal axis and the line of sight, directed to the first pin at P_1. Note that the points P are shown in the same horizontal plane with the point V. The proof depends on this, but in practice, if the instrument is set up in a fairly level space, the condition may be approximated sufficiently by setting the pins in the ground. The line of sight is shown passing through the vertical and horizontal axes at V and making with the horizontal axis the angle whose horizontal projection $O_1VA_1 = \theta$. As the telescope is plunged, the line of sight describes the surface of a cone whose vertex is at V and whose axis is H_1A_1, and is finally directed to P_2. Then, when the revolution in azimuth takes place, the horizontal axis describes a cone whose vertex is at V and whose axis is the vertical axis of the instrument; the line of sight describes a horizontal plane; the optical center moves to O_2, coinciding with O_1, and the line of sight is again directed to P_1. While the revolution in azimuth takes place,

the horizontal axis moves on ahead of the line of sight by the horizontal angle θ, and, when O_2 reaches O_1, H_1A_1 is in the position H_2A_2.

Now, when the telescope is plunged, the line of sight again describes a cone and indicates finally the point P_3. Evidently

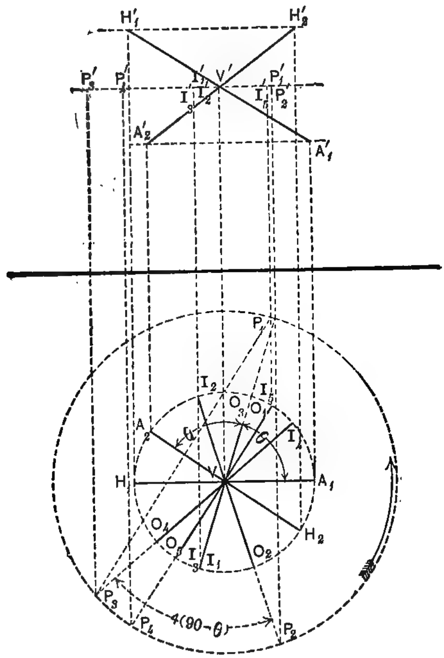


FIG. 113.

P_2VI_1 and I_1VP_3 each equals $2(90-\theta)$, and, therefore, their sum P_2VP_3 equals $4(90-\theta)$.

Consequently, the point P_3 must be moved to P_4, one fourth the distance to P_2. When this is done, and the line of sight, by means of the movement of the cross hairs, directed to P_4, the line of sight and the horizontal axis will make a true right angle, as may be proven by the ordinary methods of descriptive geometry.* Note that this discussion assumes the line of sight, as it is adjusted, to revolve about V, whereas it

* Church's Descriptive Geometry, Art. 36.

actually revolves about O. In practice the two motions will not differ appreciably. Having adjusted the instrument with the slide run in, if, with the slide run out, the test is not fulfilled, the error is due to the motion of the slide, and the correction must be made, as in the preceding similar cases, by the movement of that part of the telescope. As before remarked, the first adjustment of this article should prove sufficient. If, however, the first has been performed and the test of the second is found to be unfulfilled, make the second. In such a case this will throw the line of sight out of intersection with the vertical axis, but it is more important that it should be perpendicular to the horizontal.

However, the discrepancy should be very small if accurate work is desired. Further, the explanation of Fig. 113 depends on the line of sight's intersecting the vertical axis. Assuming that the adjustment for axis intersection has been made and the test for perpendicularity then found unfulfilled, the first attempt at adjustment will destroy the intersection with the vertical axis. During the further manipulation of the instrument, until the adjustment is effected the movement of the line of sight will not be *exactly* as shown in Fig. 113. However, it will be practically the same, as the line of sight will miss the vertical axis by a very small distance, and a few approximations should serve to complete the adjustment. Be careful to sight always on the bottom of the pins. In some instruments the slide is non-adjustable. In this case, with the slide run out, the engineer can only *test* the adjustment.

68. The Telescope Attached by the Third Method.—With this method of attachment, the slide is non-adjustable and the geometric axis is always rigidly fixed in a position at right angles to the vertical axis. It is necessary for the line of sight and the geometric axis to be parallel, or at least to lie in planes perpendicular to the vertical axis.

Test. By the method of Art. 84, 2, second general method, place the optical center and a given point in the same horizontal plane.

Then, the line of sight being directed to the vertical line

in which the point is situated, if it covers the point the test is satisfied.

Adjustment. If not, by moving the hair intersection up or down, make it cover the point.

Explanation. A full discussion of this test and adjustment cannot well be had until some of the principles of leveling are understood. Consequently, it will be taken up in the chapter under that head.

CHAPTER V.

LEVELING.

SECTION I.

Definitions and Divisions of the Subject.

69. The science of leveling comprises two distinct operations. First, the determination of the difference in elevation between any two points, and second, the elevations, with respect to some given datum, of any number of points, a point's elevation being plus or minus as the point is above or below the given datum. The datum is always some surface parallel with the surface which the earth would assume were it entirely liquid; in other words, some level surface (Art. 28). It is customary, however, to speak of it as a plane. In the case of small areas this is approximately true.

The second operation depends on the first.

Having found the difference in elevation between two points, we can then determine the difference in elevation between the second point and a third, and so on, from the data thus obtained, referring all the elevations to that of one of the points. The whole process of leveling, then, depends on the determination of the difference in elevation between two points. Such a determination may be effected by the methods of (1) spirit leveling, (2) barometric leveling, or (3) trigonometric leveling.

SECTION II.

Spirit Leveling.

70. In ordinary engineering operations the methods of spirit leveling are by far those most often employed.

We will proceed to a discussion of these methods and of the instruments used in connection with them.

71. Water and Pendulum Levels.—Before taking up the subject of spirit leveling proper, two devices of some practical utility will be described. These are the water level, Fig. 114, and the pendulum level, Fig. 115.

The water level consists simply of a float, as a short piece of plank, with two pin-hole sights of the same height attached,

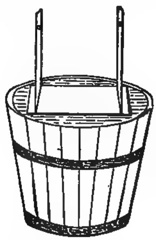


FIG. 114.

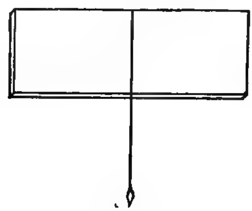


FIG. 115.

one at each end. If the float is placed in a pail of water, the line of sight determined by the two pin-holes will be a level line. The writer has used this device quite successfully in leveling off a small area.

The pendulum level consists of a smooth board with a line drawn on it at right angles to one edge, a brad driven in this line, and a plumb-line attached to the brad.

The board being held so that the plumb-line, swinging free, covers the line drawn at right angles to the edge, the edge itself determines a level, or, rather, horizontal line. The manner of using both the water level and the pendulum level will become evident from the directions given for the use of the spirit-level (Art. 76).

72. Principle of the Engineer's Spirit Level.—In Art. 52 (1) it was explained that the axes of figure of the level bubble tubes found on engineering instruments are arcs of circles and that by the expression "tangent of the tube" is meant a straight line lying in the same plane with the tube's axis of figure and tangent to it at the middle point. When the bubble appears at the middle of the tube the tangent evidently becomes a horizontal line (Art. 28). Now, if by any means we

attach rigidly to the bubble tube a line of sight, and make that line of sight parallel with the tangent, then, when the bubble appears in the center, the line of sight will become a horizontal line. This is the fundamental principle of all spirit leveling. The essential elements of a spirit level are seen to be

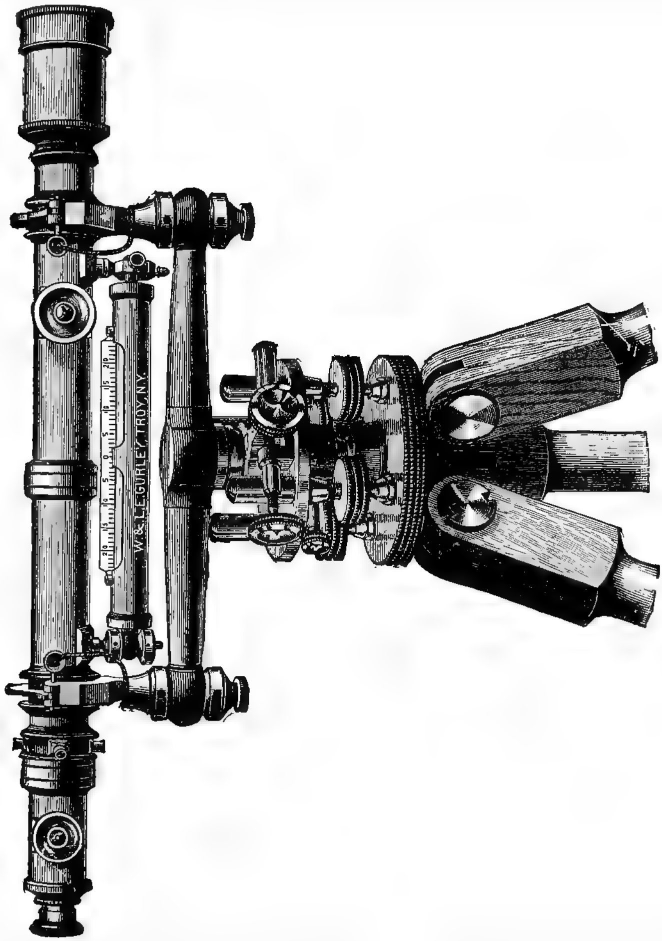


FIG. 116.

a line of sight and a bubble tube. In engineering instruments the line of sight is telescopic and capable of adjustment into parallelism with the tangent of the bubble tube. The tangent permits of adjustment into perpendicularity to the vertical

axis, around which it and the line of sight revolve. There are two types of levels in common use by engineers, the wye level and the dumpy level. The principal difference between the two types is in the method of attachment of the telescope.

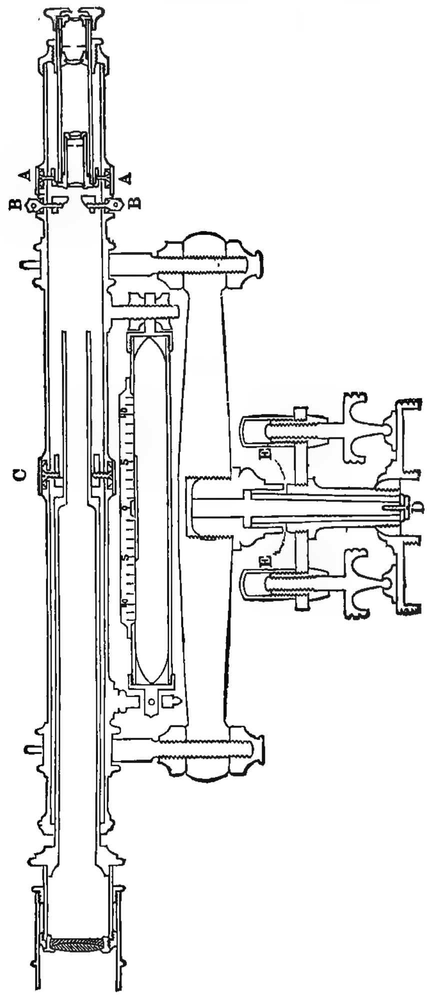


FIG. 117.

73. The Wye Level.—A cut of this instrument is shown in Fig. 116, and in Fig. 117 is given a longitudinal vertical section through the geometric axis of the telescope tube. The tele-

scope is seen to be attached by the first method of Art. 62, the tube resting in the wyes (*YY*, Fig. 116), in which it is fastened by the clips (*CC*) and their pins (*PP*). To admit of rotation around the geometric axis the clips can be turned back on hinges. The height of the wyes may be adjusted by the nuts (*NN*). The bubble tube is held in a metal case fastened to the telescope. One end of the case permits of being adjusted horizontally and the other vertically. This adjustment is effected by means of the nuts and capstan-headed screw shown in the figure. The bar to which the wyes are attached is known as the level bar. Into this bar is screwed a steel spindle (*D*, Fig. 117), whose axis of figure is the vertical axis of the instrument. This spindle turns in a hollow cylinder of bell metal revolving in the socket (*EE*) of the "leveling head." This last contrivance consists of two plates and four leveling screws, arranged in pairs perpendicular to each other. The plates and two of the screws are shown in Fig. 117. The screws fit into nuts in the upper plate and have their bottoms resting in caps on the lower.

When the instrument is in use, the lower plate is screwed on a tripod, and, by the motion of the leveling screws, the upper plate, level bar, bubble tube, etc., are brought into any desired position. (Fig. 117.) As any one of the leveling screws is unscrewed, it presses against the lower plate and consequently forces upward the side of the upper plate into which its thread fits. The other side of the plate is forced down and, to admit of this motion, the screw on that side must be screwed up. Thus the screws work together; while one is screwed up the other must be screwed down. To level the instrument described in this article, having set up the tripod, turn the bubble tube parallel to one pair of leveling screws, and by their use bring the bubble to the center. Then turn it over the other pair and repeat the operation. If the instrument is in adjustment (Art. 84) it will now be level. In "leveling up," the operator uses both hands at the same time, turning one screw up and the other down. Note that the thumbs move in opposite directions and that the bubble follows the left-hand thumb. Be careful not to work the screws

too tight. If they become so, loosen all four and start again.

The level of precision (Fig. 118) is simply a wye level,

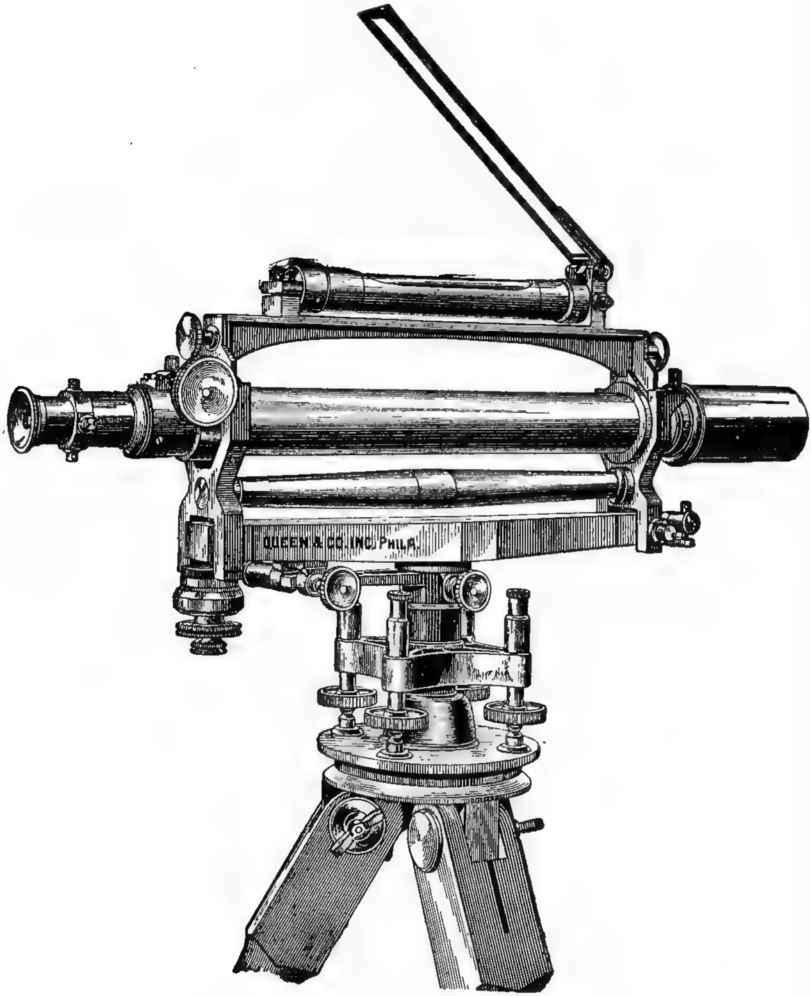


FIG. 118.

made with the utmost care, and having some modifications designed to increase to the highest degree possible the accuracy of its work. For a description of an instrument of this

class, see the U. S. Coast and Geodetic Survey Report for 1879, Appendix 15, p. 202.

74. The Dumpy Level.—In the dumpy level, Fig. 119, the telescope is attached by the third method and the bubble tube is fastened to the level-bar instead of to the telescope tube. Otherwise, it is practically the same instrument as the wye level. It receives its name from the shortness of its tube, due to the

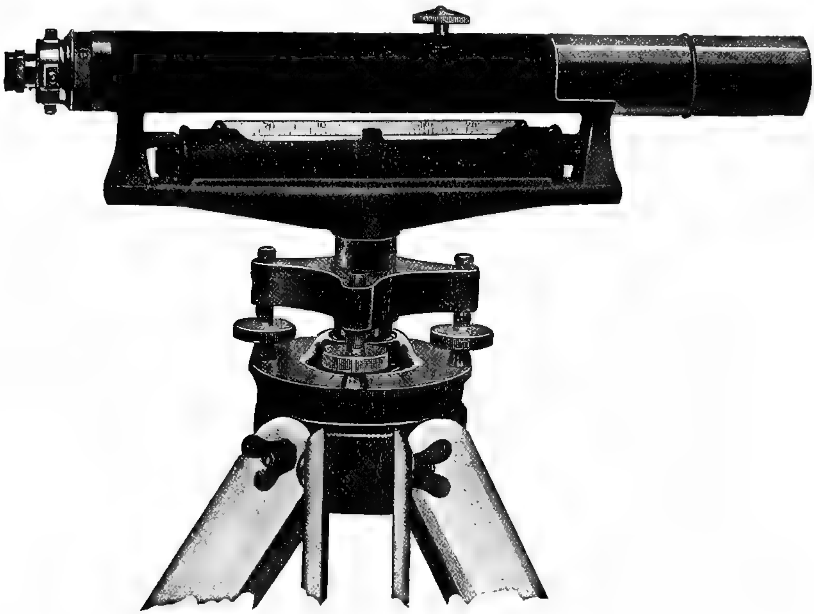


FIG. 119.

fact that the telescope is generally inverting. The wye level is the instrument in general use by American engineers, while the dumpy level is more common abroad. In some ways, the latter is the better instrument. While more difficult of adjustment, it retains its adjustments longer and is cheaper. An inverting telescope is always an advantage.

75. Leveling Rods.—The use of these in connection with spirit levels is explained in the next article.

There are two kinds, target rods, and self-reading rods. In

both, the rod is simply a straight wooden bar, of rectangular cross-section, graduated in some unit of length. In the first, however, a "target" slides on the bar. The level being turned towards the rod, the rodman shifts the target, until a line on it at right angles to the bar is intersected by the line of sight. He can then "read" the rod, that is, determine the position of the target on it. With the self-reading rod no target is used, the graduations being such that the observer at the level can read them through the telescope. With target rods, to increase the length, the bar is always made in two or more pieces sliding on each other and held in any desired position by a clamp. The self-reading variety may be made in the same way. There are a number of patterns of each kind. In target rods the most common are the Philadelphia, New York, and Boston rods. The Philadelphia rod, Fig. 120, can be used either as a target or a self-reading rod. It is made of two strips, each about three quarters by one and one half inches in cross-section and seven feet long. Both faces of the back strip and the front face of the front strip are recessed, painted white, and graduated to feet and tenths, or feet, tenths, and hundredths. The feet and tenths are numbered. When the two strips are slid together the front face reads from zero up to seven feet. By extending the rear strip the front face of the rod gives a continuous reading from zero up to thirteen feet. If used as a self-reading rod, or "speaking" rod, the reading is taken to the nearest hundredth of a foot. When the target is used some makers add to it a short scale reading to half hundredths.

Others attach a vernier. In Fig. 120, the scale or vernier is shown at the side of the opening in the target. For a target-reading over seven feet, the target is clamped at the seven-foot graduation and the rod extended. The figures on the back read downwards from seven to thirteen feet, with the intermediate divisions. The height to which the target has been raised by the extension of the rod is read off by a scale or vernier on the side of the front piece at its upper end. The New York rod, Fig, 121, is similar to the Philadelphia rod, except that the graduations are made on the wood in its natural

color. This makes it difficult to use as a speaking-rod. Fig 122 shows the Boston rod.

FIG. 120.

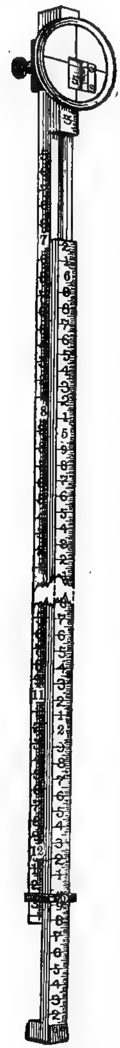


FIG. 121.

FIG. 122.

With this instrument the graduations are entirely on the side of the rod. The target is fixed permanently with its

center line three tenths of a foot from the end of one of the strips. This strip is graduated to hundredths from zero up to six feet on one side, and from six feet down to twelve on the other. The other strip carries a vernier at each end. When a reading less than six feet is to be taken, the rod is held with the target end down, and one vernier is read. For readings over six feet, the rod is turned upside down and the other vernier used.

FIG. 123.

Speaking-rods are made in numerous patterns. A plain rod, graduated like the Philadelphia, is very serviceable. Others are designed so that the intermediate tenths and hundredths can be read off at a glance. Fig. 123 shows a rod of this kind.

76. **Use of the Spirit Level.**—The vertical axis being truly vertical, the line of sight assumed perpendicular to it and revolving around it, generates a horizontal plane. If the level rod is held vertically on any point, the line of sight directed to it reads on the rod the elevation of the line of sight above the point. If now the elevation of the line of sight is known, the rod reading subtracted from it will give the elevation of the point. In practice, it is customary to start from some point of known or assumed elevation. The level being set up at any convenient place, the reading of the rod held on the point, added to the known or assumed elevation, will give the elevation of the line of sight, or, as it is commonly termed, the "height of instrument."

Having the height of instrument, subtracting from it the rod-reading on any point gives the elevation of that point.

A sight taken on a point to determine the height of instrument is known as a plus sight. This is because, to determine the height of instrument, we *add* the rod reading to the elevation of the point. For a similar reason, a sight taken on a point to determine the elevation of the point is a minus sight. Plus sights are sometimes known as "backsights" and minus sights as "foresights," but the use of these terms is not to be recom-

mended. So proceeding, we may find the elevations of any number of points. In the course of this operation it may be necessary to take one or more "turning points." In Fig. 124, for instance, supposing the elevation of the point A to be either known or assumed, it is desired to determine the elevation of the point D. The level being set up at 1, the rod is too short for a reading on D. However, by reading the rod on B, the

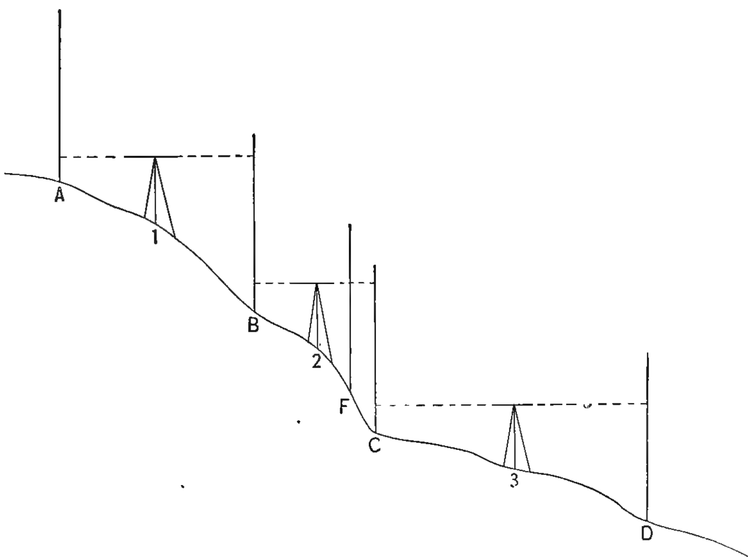


FIG. 124.

elevation of that point can be determined. Having the elevation of B, by setting up at 2 we can pass from B to C, and so on. The points B and C are known as "turning points."

A "bench mark" is a point of permanent absolute elevation, such as the highest point on a stone monument, a nail driven in a tree, etc., whose elevation with reference to some datum has been determined by the instrument.

In leveling over any country it is customary to leave bench marks at convenient points, as the tops of hills, the bottoms of valleys, the banks of streams, etc. The point from which the leveling starts, as A, Fig. 124, is generally a bench mark.

Turning-points are taken with the rod resting on a point whose elevation will not change during the two settings of the instrument. In Fig. 124 the rodman at *B* would be careful to hold the rod on a prominent point of a stone or other unyielding surface. The idea is to have the absolute elevation of *B* exactly the same while the instrument is at 1 and at 2. It will be found convenient for the rodman to carry a triangular piece of iron with the corners bent down and a rivet set in the middle, as shown in Fig. 125. With the corners driven firmly in the ground, the rivet makes a very good turning-point. The use of this little appliance saves a great deal of time and trouble.

FIG. 125.

On bench marks and turning points, the rod is read to thousandths of a foot. At other points, as *F*, Fig. 124, the nearest tenth suffices. Evidently, it is necessary for the rod to be held vertically. The rod level, Fig. 126, enables this to

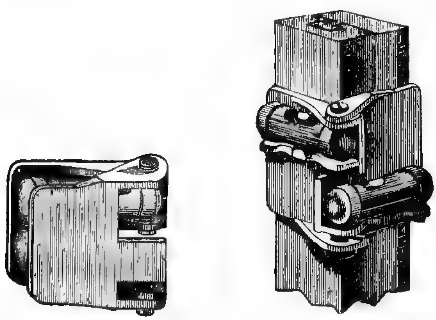


FIG. 126.

be done by the rodman. The level man is also able, by means of the horizontal hair, to judge the verticality of the rod. The clip of one of the wyes has a little pin projecting from it. This pin fits into a recess in the telescope tube and prevents its rotation in the wyes. While the line of sight is being adjusted, it is customary to keep the hairs truly vertical and horizontal by comparing them with a vertical line, as the edge of a building, a plumb line, or a horizontal line, as the line between two courses of stone in a wall. As the hairs are set

at right angles, it is necessary to compare only one of them, in the case of the level, preferably the horizontal hair.

The usual pattern of target on a level rod is that shown in Fig. 127

A target in which the horizontal line is shown by two points, as in Figs. 128 and 129, can be read more accurately.

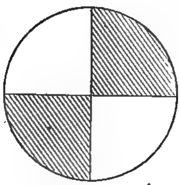


FIG. 127.

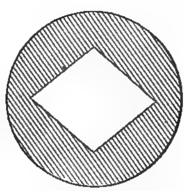


FIG. 128.

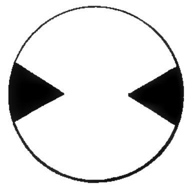


FIG. 129.

With the target of Fig. 127, the setting may be in error by the width of the horizontal hair as magnified by the eye-piece of the telescope. However, each of these targets presents a horizontal line in the direction of the instrument. The coincidence of the horizontal hair with this will show whether the rod is inclined from the vertical sideways. It will not, however, indicate whether or not the rod is inclined from the vertical directly towards or from the instrument. Thompson's leveling target, shown in Fig. 130, accomplishes this purpose.

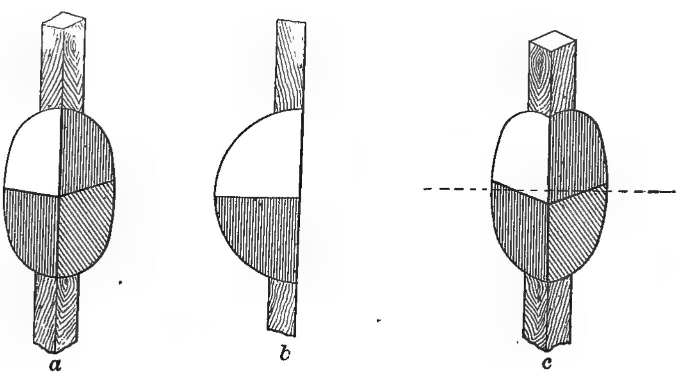


FIG. 130.

In this target, to quote from the maker's catalogue, "the horizontal dividing line is carried over two surfaces placed at right

angles to each other, thus showing a continuous and unbroken line only when the rod is held vertical." The rod is held with the corner of the target towards the level. If held vertical, the horizontal hair coincides with the horizontal line on the target.

If inclined to or from the instrument, the horizontal hair and the parts of the horizontal line on the two faces of the target form a triangle, as in Fig. 130*c*.

Another very convenient feature in this target is its bearing against the rod on rollers. By means of these it can easily

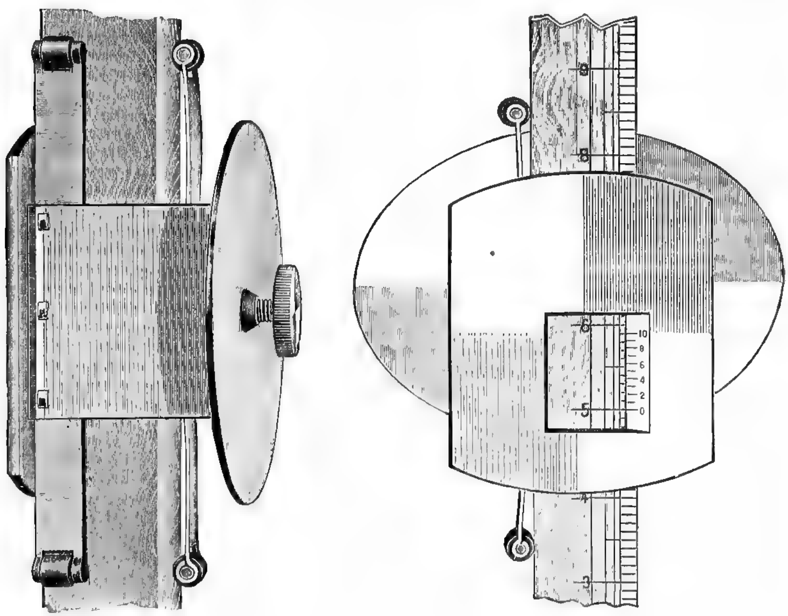


FIG. 131.

be moved along the rod, a great improvement on the stiff and jerky motion of the old targets. The Patent American Target, Fig. 131, consists of two plates, one on each side of the rod. With the rod vertical, the horizontal lines on the two plates appear to coincide. With the rod inclined to or from the instrument, they separate. This target also has a roller bear-

ing. It would be improved by having a design, similar to that of Fig. 128, painted on the front plate.

The level man should exercise care that, while a rod reading is being taken, the level bubble appears in the center of the tube. The line of sight, being parallel to the tangent of the bubble tube (Art. 52), will not be horizontal unless the bubble appears in the center. The level of precision, Fig. 118, has a mirror attached on top, so that the observer may, from the same position, read the rod and note the position of the bubble. In passing from one turning point to another, or between a turning-point and a bench mark, make the lengths of the minus sight and the plus sight equal. For if, by any combination of fixed conditions, in the instrument or elsewhere, one of the points appears too low or too high, the other elevation will be wrong by the same amount and in the same direction. Consequently, the relative elevation of the points will be correctly determined. In determining the elevation of intermediate points, such as (*F*) Fig. 124, this caution need not be observed. In going up or down a hill it is tedious to follow the above rule. If preferred, the sights can be taken of unequal lengths, and, as soon as practicable, an error of equal amount made in the opposite direction. An exasperating experience, in leveling over steep grades, is to set the instrument up and then discover that the line of sight, when leveled, cuts below the point on which the rod is held, or above the rod when extended to its full length. To avoid this, before setting up, with the tripod legs held together and the telescope turned in the direction of the rod, incline the whole instrument until the bubble comes to the center. Then, by sighting through the telescope or along its side, it can be seen whether or not the point is suitable for setting up.

For short sights the error of setting the target is greater. In long sights errors in adjustment will have greater effect. For bench marks and turning-points, about 300 feet seems to be the best length of sight. For intermediate points, read to tenths, the length of sight does not matter.

77. Curvature and Refraction.—In Fig. 132 the circle of

radius OA represents the earth. The level is supposed to be set up at any point A on the earth's surface with the rod held at any other point B having the same elevation as A. Evidently, owing to the curvature of the earth, the instruments will indi-

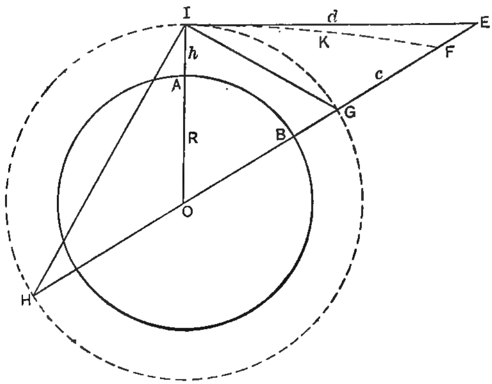


FIG. 132.

cate that B is lower than A by the amount EG, or c. From geometry

$$IHG = \frac{1}{2} IOG; \qquad EIG = \frac{1}{2} IOG.$$

Therefore, in the triangles IEG and IEH, the angle IHG equals IEG, the angles at E are identical, and the triangles are similar. Hence

$$IE : EG :: EH : IE. \qquad (35)$$

From which, letting

$$\begin{aligned} OB &= OA = R \\ GE &= \quad c \\ IE &= \quad d \\ IA &= BG = h \end{aligned}$$

$$d^2 = [2(R + h) + c]c = 2Rc + 2hc + c^2 \qquad (36)$$

Since the expressions $2hc$ and c^2 are extremely small in comparison with $2Rc$, we may write $d^2 = 2Rc$ and

$$c = \frac{d^2}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Unless the error is eliminated by some special method, curvature makes all elevations obtained by the spirit level too low. Practically, the length of sight d equals the distance, measured along the surface between the points where the level and the rod are set up. Knowing this and the mean radius of the earth, we can determine the correction for curvature in any particular case.

For a distance d equal to 1 mile, and using for R its approximate value, 3958 miles,

$$c = \frac{(5280 \times 12)^2}{2 \times 3958 \times 5280 \times 12} = 8 \text{ inches about.} \quad . \quad (38)$$

Eq. (3) shows that c varies with the square of the distance.

This makes it easy, having the result of (38), to calculate the correction for any given distance.

Refraction in the atmosphere, on the other hand, makes elevations obtained by the spirit level too high. In the preceding discussion it has been assumed that the line of sight, tangent at I (Fig. 132) to the circle IGH, intersected the target at E. This would be true if there were no refraction, so the *error due curvature* has been correctly computed. As a matter of fact, the target would be in some position as F. The rays of light proceeding from it are bent into the path FK and enter the telescope at I.

Generally speaking, the error due to refraction is about one seventh that due to curvature. Hence, to determine the total error, take six sevenths of the error due to curvature.

In passing from one turning point to another, or between a turning point and a bench mark, observance of the caution of

Art. 76, to make the minus sight and plus sight of equal length, will eliminate all errors due to curvature or refraction.

78. Reciprocal Leveling.—In some cases, as at the crossing of a river, it is impossible to have the plus and minus sights of equal length. In such a case as this, establish two bench-marks A and B, one on each side of the river. With the level at C, Fig. 133, near one of these, as A, determine their differ-

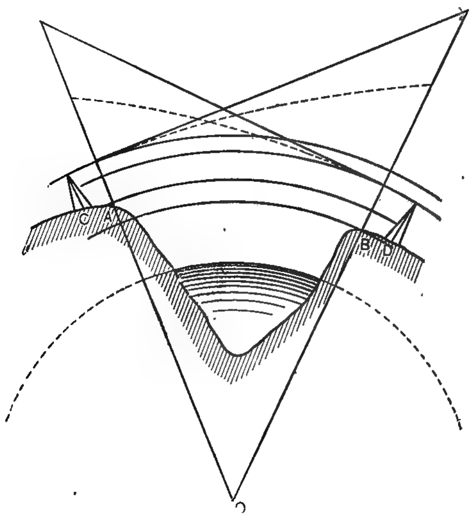


FIG. 133.

ence of elevation. Then repeat the operation with the instrument at D, a point on the other side of the river, such that $BD = AC$ and, therefore, $AD = BC$. The mean of the two differences is the correct difference of elevation. If E_1 and E_2 are the absolute elevations of A and B, their true difference of elevation is

$$X = E_1 - E_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Letting r_1 represent the correction for curvature and refraction for the distance $AC = BD$, and r_2 the corresponding correction for $CB = DA$, the difference determined with the instrument at C is

$$X' = (E_1 - r_1) - (E_2 - r_2). \quad . \quad . \quad . \quad . \quad . \quad (39)$$

With the level at D, the difference found is

$$X'' = (E_1 - r_2) - (E_2 - r_1). \quad . \quad . \quad . \quad . \quad (40)$$

Adding (39) and (40) and dividing by 2

$$\frac{X' + X''}{2} = E_1 - E_2. \quad . \quad . \quad . \quad . \quad . \quad (41)$$

79. **Profiles.**—Lines of levels may be run for various purposes. One of the commonest uses to which the information thus obtained is put is the making of a "profile." Suppose that a traverse has been run on the ground, and the elevations of its angles and any number of intermediate points on its lines determined. Further, that a horizontal line on a sheet of paper is taken to represent the datum, and that the points plotted to

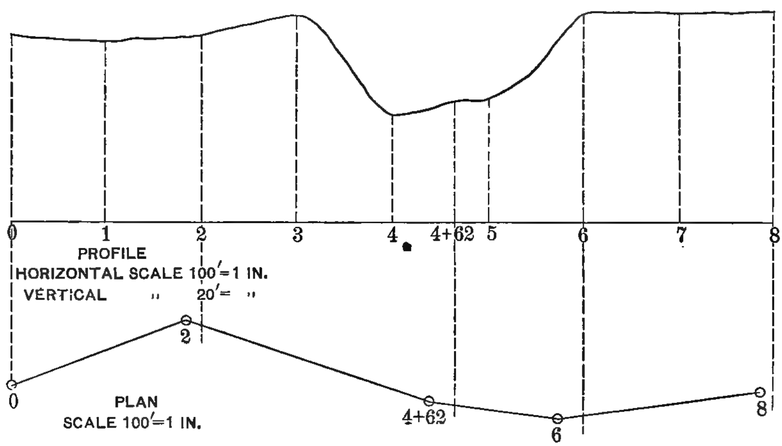


FIG. 134.

scale, according to their respective elevations and horizontal distances apart, are joined by a continuous line. This line on the paper, or what it represents, is known as the "profile" of the traverse or "line" run on the ground. It is seen to be a kind of vertical projection of the traverse, with all its lines swung into one plane parallel with the vertical projection plane.

Fig. 134 shows a traverse both in profile and in plan.

The horizontal scale being the same, note that the points on

the plan are not directly under the corresponding points on the profile. This is on account of the swinging of the lines alluded to above. In traverses generally, it is customary to divide the distance run over into stations of 100 feet each. Thus, station 2 would be at the end of the first 200 feet, station 2 + 35, 35 feet farther along, etc. This numbering is kept up continuously without regard to angles, either horizontal or vertical, in the line. In leveling for profile, it is necessary to take an elevation at every change of slope. It is customary, further, to take the elevation of all even stations and all horizontal angles in the traverse. In the plotted profile, owing to the fact that the vertical distances are so small in comparison with the horizontal, the vertical scale is taken much larger than the horizontal. The line joining the points is generally drawn in free-hand, as better representing the surface, though it may be ruled between adjacent points.

The paper used, known as profile paper, Fig. 135, is

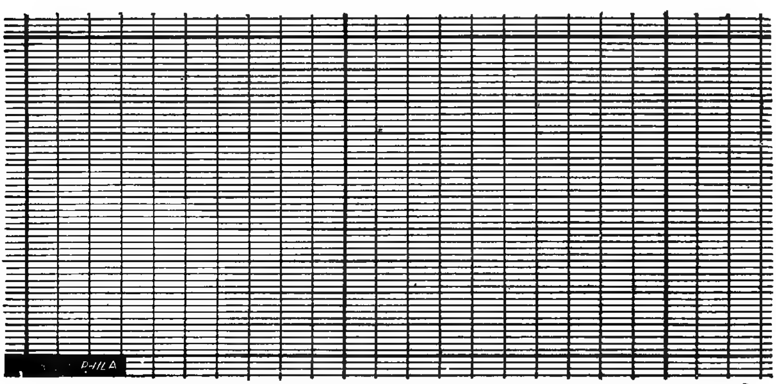


FIG. 135.

specially prepared for the purpose. It is ruled in squares or rectangles, divided, in either case, into smaller rectangles, having their longer dimensions horizontal. There is an even number of the larger divisions to the inch each way, vertically and horizontally, thus making it easy to plot to any desired scales.

80. Notes.—Fig. 136 shows a specimen double page of an ordinary level note-book. From these notes the profile of Fig. 134 was plotted. Note that each bench mark (B. M.) is described and that each height of instrument (H. I.) has a horizontal line to itself. This last is to indicate that the instrument stands between the turning point on which a plus sight has just been taken, and the turning point next below.

35	LEVELING SOUTH B'DY CAMPUS, MAY 10, 1901. INSTRUMENT, ROE. ROD, DOE.						36
STA.	+	H. I.	—	ROD	EL.		
B.M. STA. 0	4.632				200.000	CENTER STONE MON. S.W.	COR. IRVING & CROTON
		204.632					
1				6.3	198.3		
2				5.0	199.6		
3				1.2	203.4		
T.P.	2.104		12.819		191.813		
		193.917					
4				8.5	185.4		
4+62				8.0	185.9		
5				7.6	186.3		
T.P.	13.812		2.406		191.511		
	20.548	205.323					
6				1.2	204.1		
7				1.0	204.3		
8				0.5	204.8		
B.M (T.P.)	10.322		0.631		204.692	CROSS ON LARGE STONE	10′ SOUTH OF STA. 8
			15.856 20.548		200.000		
			4.692		4.692		

FIG. 136.

Generally, the H. I. is entered on the same horizontal line with the turning point next above. This saves space in the note-book, but it is believed that the method of Fig. 136 will serve to make clearer to the beginner the general plan of level note keeping. As described in Art. 76, the elevation of the first bench mark is known or assumed. In Fig. 136 it is assumed as 200 feet. The idea is to assume a beginning point of such elevation, that no point among those to be leveled over will have an elevation less than zero, or minus.

Assuming, then, the elevation of the first B. M. as 200.000, the plus sight on it from the first position of the instrument is

4.632. This added to 200.000 gives 204.632 as the first H. I. Subtracting from this the rod readings (minus sights) taken to the nearest tenth at stations 1, 2, and 3, gives the elevation of these points to the nearest tenth.

Next the elevation of turning-point No. 1 is found to be 191.813 by subtracting the minus sight on that point, 12.819, from the H. I. above, 204.632. The instrument having been moved to a new position, the plus sight on turning point (T. P.) No. 1, of 2.104 added to its elevation 191.813, gives the H. I., 193.917, and so on. Adding up all the quantities in the — and + columns between any two points, the difference between the two sums should equal the difference in elevation of the two points. This gives a check on the arithmetical work as far as turning points are concerned. Note that the minus sight on the first point and the plus sight on the second are not included in the two sums. This check should be applied to every page of the note-book. For convenience in adding, the last plus sight on each page can be written at the top of the + column over the heading on the next page.

Except where B. M.'s are used as T. P.'s, this proceeding does not check the arithmetical work on B. M.'s and intermediate points. This work should be gone over a second time and checked by the leveler, before accepting any of his figures as correct. Such checking can be done after the day's work is finished, though sometimes it is important to have the work kept up in the field. At all events, the *first* calculation of elevations, etc., should be made as the leveler goes along. It is bad practice to let this work accumulate. The check on turning points should be applied immediately at the end of every page of notes.

81. **Precision.**—It is customary, at the end of each day's work, to run back and check on the starting point. Without a check of this nature, no leveling can be relied on. The fewer the number of rod observations, the more closely will the original and check elevations agree. Experience has shown that the accuracy of such work is proportional, approximately, to the square root of the number of observations. In ordinary work with a wye level, after a day's work, the original and

check elevations are required to agree within one tenth of a foot.

82. Tests of the Wye Level.—The reasons for the first two of these will be given in Art. 84. The third has been discussed in Art. 65; the fourth is explained in the present article.

(1) *The Test of the Rings.* The rings of the telescope tube should be equal cylinders. One reason for this has been given already in Art. 65. As there explained, they may be tested by calipers.

(2) *The Test of the Wyes.* Both wyes should have the same cross-section perpendicular to the telescope tube. They may be compared by tracing on stiff paper the cross-section of each.

(3) *The Test of the Object-glass Slide.* In instruments with a non-adjustable object-glass slide, the motion of the slide should be parallel with the geometric axis. The test can be made as explained in Art. 65.

(4) *The Test of the Magnification in Comparison with the Sensitiveness of the Bubble Tube.* The proportion between the magnifying power of the telescope and the sensitiveness of the bubble tube should be such that the smallest appreciable motion of the line of sight on the rod will cause a noticeable movement of the bubble, and *vice versa*. The sensitiveness of the bubble is proportional to the radius of its tube. If the sensitiveness is too great in proportion to the magnifying power, the observer will waste time bringing the bubble exactly to the middle of its run, while for some time probably the line of sight has been directed to the same division on the rod that it will cover when the instrument is finally level. On the other hand, if the magnification is too great, illumination is lost and to no purpose.

To make this test, focus carefully on the rod, the bubble being at the center, and note the division cut by the line of sight.

By means of the leveling screws, throw the instrument slightly out of level. Then level up again and note whether the line of sight is directed towards the same division.

If so, the proportion is correct.

83. General Adjustment of the Wye Level.—The line of sight must be horizontal. In general, the only means of judging of its horizontality is by the bubble. Therefore, the line of sight and the tangent of the tube must be parallel. If this condition is fulfilled, the telescope can be turned in any direction, the bubble brought to the center by means of the leveling screws, and the work carried on. It would be tedious, however, to level up each time a new sight is taken, consequently it is convenient to have the line of sight and the tangent of the bubble tube perpendicular to the vertical axis, so that, once level, they will remain so as the instrument turns in azimuth (Art. 52, 1).

There are two general methods of adjusting the wye level. In one of these, first, the line of sight is brought into coincidence with the geometric axis of the telescope tube.

Next, assuming that the rings and wyes have been correctly made, and that, therefore, the geometric axis is parallel with the axis of the wyes, the tangent of the bubble, by making it parallel with the axis of the wyes, is made parallel with the geometric axis of the telescope. Lastly, for the sake of convenience, the line of sight and the tangent of the bubble tube are together made perpendicular to the vertical axis.

The other method differs from the one just given, only in the second step. After bringing the line of sight into the geometric axis, the whole telescope tube is moved until the line of sight becomes horizontal. The bubble tube is then moved until the bubble appears in the middle, and the third part of the whole adjustment is done as in the former method.

The operations above mentioned will next be taken up and discussed separately.

84. Adjustments of the Wye Level.—(1) The adjustment of the line of sight to coincide with the geometric axis of the telescope tube. This adjustment is the same as the adjustment of the telescope attached by the first method (Art. 65).

(2) By the first general method. The adjustment to make the tangent of the bubble tube parallel with the axis of the wyes, and hence parallel with the line of sight.

Test. The clips being open, set up the instrument and level

it over both pairs of screws. Then level it very carefully over one pair and clamp the spindle in this position. Note particularly that the bubble must be in the center after the spindle is clamped and the hands taken off the instrument. Next take the telescope tube from the wyes, being careful not to jar the rest of the instrument, turn it end for end, and replace it in the wyes with the bubble below as before.

If the bubble remains in the center of the tube, the first part of the test is satisfied. Next rotate the telescope tube around its geometric axis in the wyes through an angle of 25 or 30 degrees, and note whether the bubble remains in the center. If so, the second part of the test is satisfied also.

Adjustment. If the first part of the test fails, by means of the capstan nuts working vertically at one end of the bubble tube case, bring the bubble half way back to the center. Test again, adjusting if necessary, and so proceed until the first part of the test is satisfied.

If, then, the second part of the test is unfulfilled, by means of the capstan-headed screws working horizontally at the other end of the case, bring the bubble entirely back to its original position at the center. Before leaving the adjustment, be sure that both parts of the test are satisfied.

Explanation. In Fig. 137, *VV* represents the vertical axis,

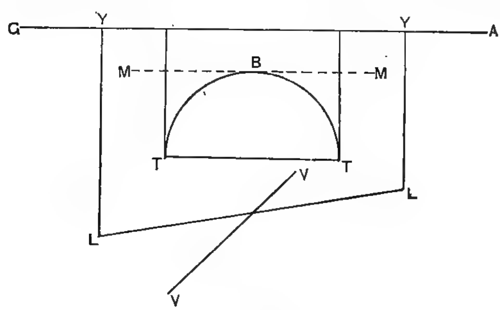


FIG. 137.

YY the wyes, *GA* their axis, and also the geometrical axis of the telescope tube, *TBT* the bubble tube, *MM* its tangent, and *B* the position of the bubble. Note that the points *GA*, *MM*,

TT, and *B* are rigidly fastened together, as are also *YY*, *LL*, and *VV*.

MM is parallel to *GA*, so that when the bubble appears at the center, *B*, of its run, *GA* is horizontal. Evidently, on lifting *GA* out of the wyes, turning it end for end, and replacing it, the bubble will again be found at the center *B*. Note that the wyes are of unequal height, the level bar is perpendicular neither to them nor to the vertical axis, which last is not truly vertical. All these are drawn this way to indicate the fact that they have nothing to do with the adjustment. Fig. 138 shows

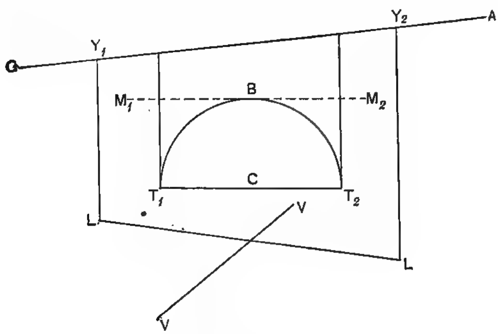


FIG. 138.

the case in which *MM* is not parallel to *GA*. *C* is the center of the bubble tube's axis of figure. On *GA*'s being lifted out of the wyes, turned end for end, and replaced, by a method similar to that of Art. 52, 1, it can be shown that the bubble moves to B_2, Fig. 139, and that the angle B_1CB_2 equals twice the angle between *MM* and *GA*. Consequently, if the bubble is made to move through half the angle B_2CB_1, *MM* and *GA* will become parallel. For an explanation of the second part of the adjustment, observe Fig. 140. A side elevation, end view and plan of the instrument are shown.

The first part of the adjustment is supposed to have been completed, and the instrument leveled. If the points T_1 and T_2 and the geometric axis of the telescope tube lie in the same plane, as the telescope is revolved 25° or 30° in the wyes, the bubble will remain in the center.

If, however, the above condition does not exist, as the

telescope is revolved in the wyes the bubble will run to one end or the other. This is the case shown in Fig. 140.

The side elevation and plan, or the side elevation and end

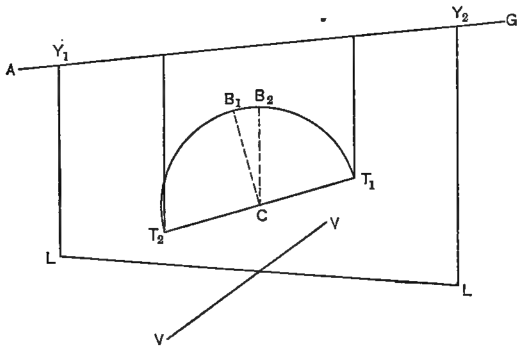


FIG. 139.

view, taken together, show that T_1, T_2, and the line GA do not lie in the same plane.

If, now, the telescope is revolved in the wyes, say in the same direction as the hands of a watch, the end view shows

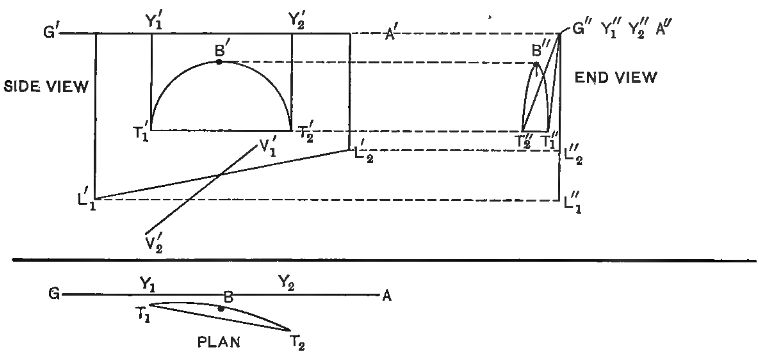


FIG. 140.

that the end T_2 of the bubble tube will rise higher than T_1, and consequently the bubble will move towards T_2. Evidently, in this case the whole error must be corrected. In modern levels the second part of this adjustment is not of much importance. If the first part of the test is satisfied, it shows

that the axis of the wyes, while it may not be parallel to the *tangent* of the bubble tube, is parallel to the horizontal *plane* in which the tangent, for the time being, lies. Now, if the telescope is kept in the same position as far as regards rotation in the wyes, when the bubble is brought to the center, the line of sight will be horizontal. As described in Art. 76, the best levels have a pin on one of the clips which prevents the rotation of the telescope. In old levels this pin was omitted.

(2) By the second general method. The adjustment to make the line of sight parallel with the tangent of the bubble tube.

Test. Measure off, on comparatively level ground, a line four or five hundred feet in length, and drive a stake at each end With the level set up on the line and just half way between the stakes, determine their difference of elevation. The difference thus found will be correct. Moving the levei to a point on the line continued beyond one of the stakes and distant from it one tenth of the distance between the two, find again their difference of elevation. If the two differences agree, the line of sight is parallel with the horizontal plane in which the tangent lies.

Next, as in the corresponding step in the first general method, rotate the telescope a few degrees in the wyes and note whether the bubble remains in the center. If so, the line of sight is parallel to the tangent.

Adjustment. If the test is not satisfied, having the level set up at the point on the line continued, and one pair of leveling screws directly under the telescope and in line with the stakes, proceed as follows: With the bubble in any position, conveniently at the center, note the reading of the rod on the near stake. Denoting by d the true difference of elevation, found with the level half way between the stakes, and by r_1, the rod-reading on the near stake, the reading, r_2, on the far stake, supposing the line of sight to be horizontal, should be

$$r_2 = r_1 \pm d + c,$$

where c is the combined correction for curvature and refraction. In a distance of four or five hundred feet, with the ordinary wye level, it may be neglected.

Therefore, set the target at the reading $r_1 \pm d$, and have the rod held on the far stake. The line of sight, being out of adjustment, will not intersect the horizontal line of the target, but will cut below or above it on the rod. Denoting by x the distance up or down by which the line of sight misses the target, by means of the two leveling screws in line with the stakes, depress or elevate the telescope until the line of sight moves over a distance on the rod equal to eleven tenths of x. Then, by means of the nuts working vertically at one end of the bubble tube case, bring the bubble to the center. The second part of the adjustment is made in the same way as under the first general method.

Explanation. With the telescope half way between the stakes, as stated in Art. 77, the combined effect of curvature, refraction, and any errors due to the line of sight's deviation from the horizontal, will be to make the elevations of both stakes too low or too high by the same amount. Consequently their difference of elevation will be correctly determined. Be sure, however, that the bubble is at the center of its run when the reading is taken on each stake. It may depart from the center as the telescope revolves in azimuth. Before reading on the second rod, by means of the leveling screws bring the bubble back to center. The operation of setting the instrument half way between the stakes is seen to be a scheme merely for determining their true difference of elevation.

When the telescope is moved to its second position the situation is as shown in Fig. 141. The line of sight is supposed to be pointing up. From the similarity of the triangles GFI and DFE, the distance FI through which the line of sight must be moved to bring it into the horizontal GI is $\frac{11}{10}x$.

(3) The adjustment to make the tangent of the bubble tube perpendicular to the vertical axis of the instrument. As before stated, this adjustment is not absolutely necessary. If it is not made, however, the instrument will have to be lev-

eled for each separate sight. Further, this might cause slight difference in the height of instrument, though probably too small to be of any moment.

The adjustment is entirely similar to the first adjustment of the compass, Art. 52, 1, with the exception that when the ends of the bubble tube are to be raised or lowered it is done

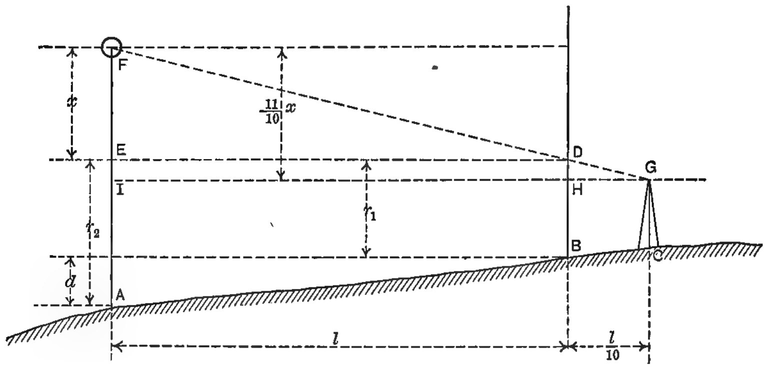


FIG. 141.

by raising or lowering the wyes, and not the screws and nuts which attach the bubble tube directly to the telescope.

The instrument is first leveled carefully over one pair of leveling screws, and then turned 180 degrees on the vertical axis until it comes again over the same pair of screws. If now the bubble has left the center, it must be brought half way back by means of the movement of the wyes as described above.

85. Adjustments of the Dumpy Level.—In this level the object-glass slide is non-adjustable, and the geometric axis of the telescope is permanently fixed at right angles to the vertical axis of the instrument.

The bubble tube is attached to the level bar and not to the telescope. It is considered best in this case to describe the adjustments before the tests. The conditions to be fulfilled are essentially the same as with the wye level.

The adjustments are two in number:

(1) The adjustments to make the tangent of the bubble tube perpendicular to the vertical axis of the instrument.

This adjustment is the same as the first adjustment of the compass, Art. 52, 1.

(2) The adjustment to make the line of sight parallel to the tangent of the bubble tube. This adjustment is very similar to the second adjustment of the wye level under the second general method. The only difference is that when the instrument is moved to the point beyond one of the stakes the bubble *must* be brought to the center; the reading is then had on the two stakes and the line of sight made to pass over the distance $\frac{11}{10}x$, not by using the leveling screws, but by moving the hair intersection.

86. Tests of the Dumpy Level.—There are three of these: (1) The test of the movement of the object-glass slide. This movement should be in a plane perpendicular to the vertical axis of the instrument; (2) the test of the perpendicularity of the geometric axis of the telescope and the vertical axis of the instrument; and (3) the test of the sensitiveness of the bubble tube in comparison with the magnifying power of the telescope.

(1) and (2) If these two conditions do not obtain, it will be indicated by the difficulty which the observer will experience in effecting the second adjustment. In that adjustment, with the telescope set up at the point on the line continued and focussed on the far rod, the slide will be run nearly in. It is with the slide in this position that the hair intersection is moved over the distance $\frac{11}{10}x$.

Now, if either the geometric axis is not parallel with the bubble tube tangent or the optical center does not move in a plane perpendicular to the vertical axis of the instrument, with the slide run farther in or out and the instrument leveled, the line of sight will not be horizontal. Consequently a further determination of the difference in elevation of the two stakes, with the instrument near one of them, will not agree with the true difference *d* already found. If, therefore, after several careful trials, the observer finds it impossible to effect satisfactorily the second adjustment, he may assume (*a*) that the geometric axis is not perpendicular to the vertical axis, or (*b*) that the optical center does not move as indicated above.

These defects are beyond remedy by the engineer. A further test concerning the perpendicularity of the geometric and vertical axes may be made as follows: By the method of the 5th test of the compass, Art. 51, determine whether the level bar is perpendicular to the vertical axis.

Then, by direct measurement, determine whether the telescope tube is parallel to the level bar. In all discussions of the object-glass slide heretofore it has been assumed that, if found correct in two positions, the slide will be correct in all other directions. This is probably, though not necessarily, true. If the engineer is anxious on this point, his only remedy is to make the tests with the slide in a number of other positions.

The third test is made as with the wye level.

87. General Remarks on Adjustments.—In the case of the wye level, the second general method is more accurate than the first. This is because the first general method assumes that the rings or collars are of the same size, and while these are supposed to have been tested with a pair of calipers, the test may not have been accurately performed. However, the adjustment is made more easily by means of the first general method, and on that account it is usually adopted. If the first method is used, when the adjustments are completed make the test of the second adjustment under the second general method. This is known as taking a "test level." In leveling work with either the wye or the dumpy level, be careful to have the instrument set just half way between the last T. P. and the B. M., where the work is to be taken up and carried on. Then, before starting again, set up near the B. M. and determine again the difference in elevation of the two points. This amounts to taking a test level, and will serve as a check on the adjustment of the instrument.

88. Determination of the Sensitiveness of the Bubble Tube.—In Art. 52, 1, it was shown that, if the tangent of the bubble tube is moved from one position to another in the plane of the bubble tube's axis of figure, the bubble will move through an angle equal to the angle between the two positions of the tangent. Consequently, if the tube and its tangent are moved in the way described, the longer the radius of the

bubble tube, the longer the linear distance through which the bubble will move. In other words, the sensitiveness of a bubble tube is directly proportional to the length of its radius, that is, to the length of the radius of the path which the bubble follows. This will be a trifle longer than the radius of the axis of figure of the tube.

It is customary to express the sensitiveness either by the length of the radius or, what amounts to the same thing, the angular divisions corresponding to the linear distance moved through by the bubble. The bubble tube levels have scales attached to them, either on a strip of metal fastened just above the tube, or graduated on the tube itself. The latter method is preferable. These scales are divided in different ways, sometimes to tenths of inches. Thus the length of the bubble's run can be read off, and the sensitiveness of the tube expressed by the number of seconds of arc that correspond to one scale division.

It becomes necessary to have an instrument by which the angular motion of the bubble can be measured. Such an instrument is known as a level trier or tester. In connection with the trier is used what is known as a micrometer (small measuring) screw. This is a plain, but carefully made, screw of known pitch, having attached to its head a graduated disc, by means of which can be read the number of revolutions and fractions thereof through which the screw is turned.

The level trier, Fig. 142, consists of a bar supported at one end on a knife edge, and at the other by a micrometer screw. In the figure, B represents the bar, K the knife edge, S the micrometer screw, D its graduated disc, and P a pointer and scale by which the disc is read. Attached to the bar are two wyes, Y Y, in which the level tube is placed. B' is a board on which the tester is placed. By means of the leveling screws R the apparatus can be made approximately level without the aid of the micrometer screw S.

As the screw is turned, the distance KK' being very small, the bar moves practically as if pivoted at K'.

Knowing the distance $KS = K'S'$ and the height through which the turning of the screw has elevated the point S', the

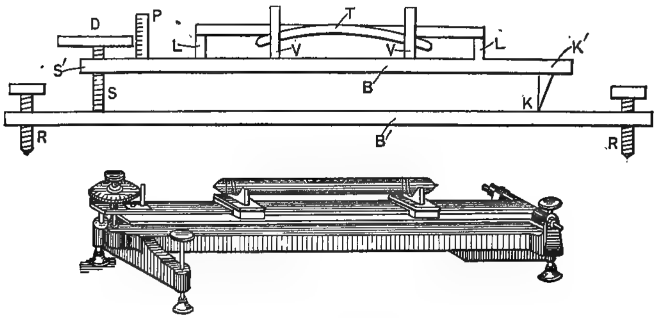


FIG. 142.

angular motion of the bar can be computed. Denoting this by θ, and the height through which S' has been raised by h, then

$$\theta'' = h \div \frac{2\pi \times KS}{360 \times 60 \times 60} = \frac{360 \times 60 \times 60 \times h}{2\pi \times KS}$$

$$= 206265 \times \frac{h}{KS}. \quad . \quad . \quad (42)$$

If now, while S' has moved the distance h, the bubble has been noted to run past n divisions on the scale, the value of one division in seconds of arc is

$$d'' = \frac{\theta''}{n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Further, if it is desired to obtain the radius of the bubble's path, denoting by d the absolute length of a division on the bubble scale, $R : nd : : KS : h$ and

$$R = \frac{nd \times KS}{h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Also, the radius being 206265 times as long as the arc of one second,

$$R = 206265 \frac{nd}{\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

It is desirable to note the movement of both ends of the bubble. This is because the ether in the tube expands as it grows warm, thus contracting the bubble. The movement of an end of a bubble is seen to be a combination of the motion due to contraction and that due to the level trier.

If it could be done accurately, the center of the bubble might be read. The best results will be obtained by reading both ends and taking the average. In high class levels the bubble tube is "chambered," that is, made so that the amount of liquid in the tube may be regulated. The arrangement is

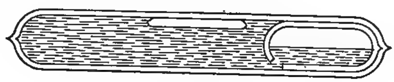


FIG. 143.

shown in Fig. 143. A long bubble is more sensitive than a short one, the tube in each case having the same radius, so that with a chambered tube the bubble may be arranged somewhat in proportion to the magnifying power.

In a common form of trier the screw has a pitch of one sixtieth of an inch, the disk is divided into one hundred parts, and the distance KS equals eighteen inches. The movement past the pointer P of one division of the disk thus corresponds to a value of h equal to one six-thousandth of an inch. As a check on the number of total disk revolutions the scale divisions on P are one sixtieth of an inch. In a modified form proposed by the writer the pitch of the screw, the divisions on the disk, and the length KS are such that the angular movement θ of the bubble can be read off at once. Thus, keeping the pitch one sixtieth as before, let the distance KS be made 17.188. Then an entire revolution of the screw would correspond to 200 seconds of arc. With the disk divided to hundredths, each division would represent two seconds, and, with the proper numbering of the graduations on the disk and vertical scale P, the angular movement could be read at once without the use of equation (42). With the ordinary form it is necessary to use equations (42), (43), and (44), or (42), (43), and (45). With the modified form only equations (43) and (45) are needed.

The bubble scale is sometimes graduated on the tube. Where the tube itself is not graduated an auxiliary scale can be used. Such a scale, held in position by two vertical rods attached to the bar, is shown at *LL*, Fig. 142. The divisions are plus or minus as they lie to the right or to the left of zero. The method of procedure is as follows: By turning the screws *R* and *S*, bring the left end of the bubble to the first plus division which will allow the right end to appear under the graduation. Note their readings, as also the readings of the vertical scale and the micrometer. Then, by moving the screw, bring the left end of the bubble to the next division, note all the readings, and so on to the minus end of the scale. Then come back in the same way. Take the entire run of both ends of the bubble, forward and back, average the four values thus found, and compare the result with the distance *h* determined from the micrometer readings. Average values of d'' and R can thus be found. If desired to test the uniformity of the tube curvature and bubble scale divisions, values can be determined by readings from one division to the next. As above, each value will be the average of four taken from the notes. The following form of notes will be found convenient for use:

		Bubble-scale Reading.			
		Forward.		Back.	
Vert. Scale.	Micrometer.	Left End.	Right End.	Left End.	Right End.

The leveling instrument itself may be used as a level trier. To do this, set up the instrument, having the telescope directly over one pair of leveling screws, the rod being held at any convenient distance from the instrument, say 100 feet. This distance corresponds to *KS* with the level trier. The leveling screws under the telescope take the place of the micrometer screw in changing the inclination of the bubble tube tangent,

and the quantity h is determined from the movement of the line of sight along the rod.

89. Hand Levels.—Rough leveling is sometimes done with these. They are used mostly in connection with topographical surveying, and their discussion will be deferred until that subject is taken up.

SECTION III.

Barometric Leveling.

90. General Explanation and Kinds of Instruments.—The word barometer is derived from the Greek and means pressure measure. The pressure of the atmosphere decreasing with the elevation, the barometer is used to measure this pressure directly, and hence indirectly the elevation of the point at which the instrument is held. There are two kinds of barometers, mercurial and aneroid, in use by engineers.

91. The Mercurial Barometer.—Fig. 144 shows a section of the elementary mercurial barometer. It consists of a vertical glass tube T closed at one end, and having the other inserted in a reservoir of mercury R. The tube being exhausted of air and the reservoir open, the height of the column of mercury in the tube above the surface of the mercury in the reservoir will be a measure of the pressure of the atmosphere. The height of the column is measured by the scale S and vernier V. The zero of the scale is the extremity of the pointer P. The tube and scale being fixed, it is necessary for the reservoir to be movable vertically, so that when a reading is to be taken the surface of the mercury in the reservoir may be brought just in contact with the end of the pointer.

Figs. 145–148 show the mecurial barometer (Green's Standard) as practically constructed. The following description is taken from the tenth annual report of the Smithsonian Institution:

"The barometer consists of a brass tube (Fig. 145), terminating at the top in a ring A, for suspension, and at bottom in a flange B, to which the several parts forming the cistern are attached.

The upper part of this tube is cut through so as to expose the glass tube and mercurial column within, seen in Fig. 148. Attached at one side of this opening is a scale, graduated in

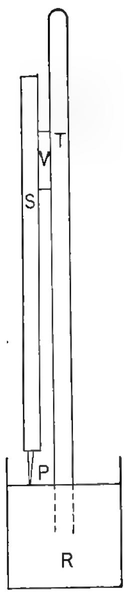


FIG. 144.

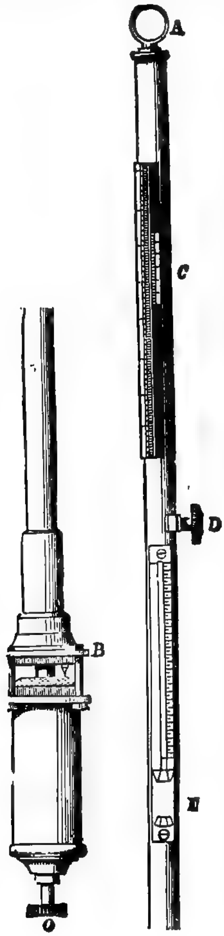


FIG. 145.

inches and parts; and inside this slides a short tube C, connected to a rack-work arrangement, moved by a milled head D: this sliding tube carries a vernier in contact with the scale, which reads off to $\frac{1}{500}$ (.002) of an inch.

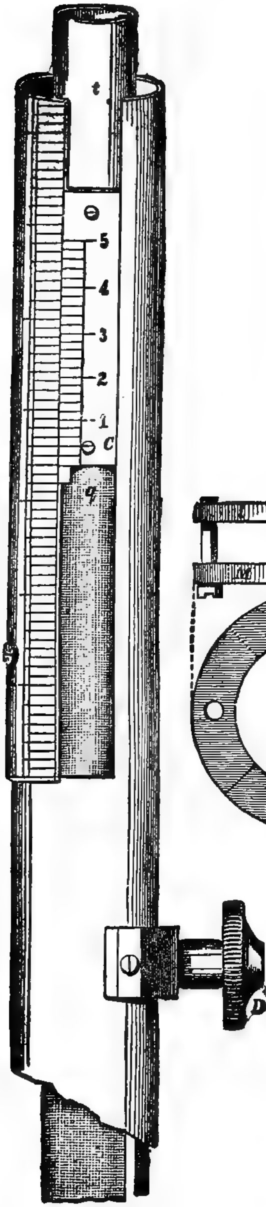


FIG. 148.

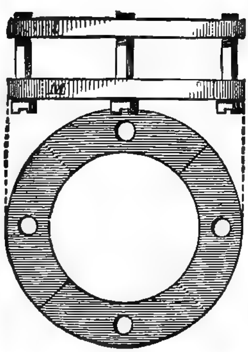


FIG. 147.

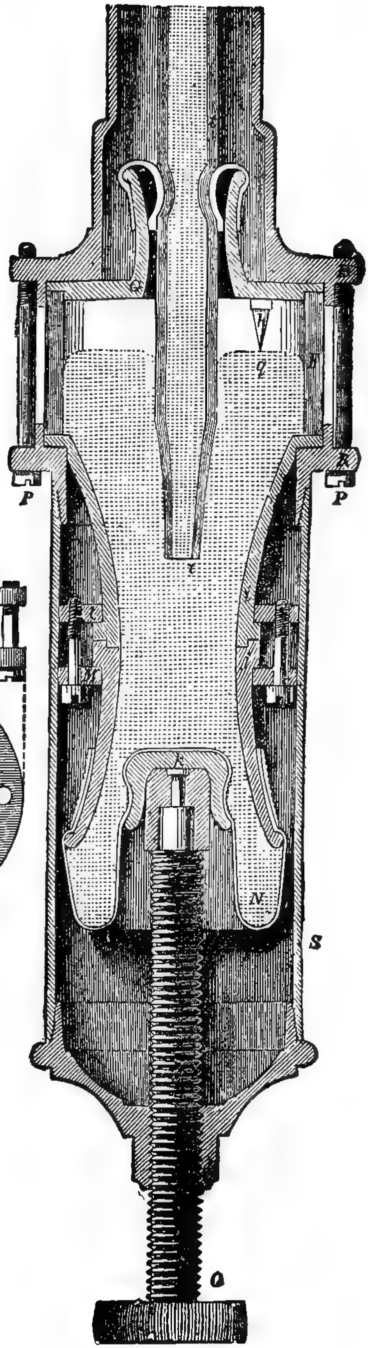


FIG. 146.

In the middle of the brass tube is fixed the thermometer *E*, the bulb of which being externally covered, but inwardly open, and nearly in contact with the glass tube, indicates the temperature of the mercury in the barometer tube, not that of the external air. This central position of the thermometer is selected that the mean temperature of the whole column may be obtained; a matter of importance, as the temperature of the barometric column must be taken into account in every scientific application of its observed height.

The cistern (Fig. 146) is made up of a glass cylinder *F*, which allows the surface of the mercury *q* to be seen, a top-plate *G*, through the neck of which the barometer tube *t* passes and to which it is fastened by a piece of kid leather, making a strong but flexible joint. To this plate also is attached a small ivory point *h*, the extremity of which marks the commencement, or zero, of the scale above. The lower part, containing the mercury, in which the end of the barometer tube *t* is plunged, is formed of two parts *i* and *j*, held together by four screws and two divided rings *l* and *m*, in the manner shown in the figure. To the lower piece, *j*, is fastened the flexible bag *n*, made of kid leather, furnished in the middle with a socket *k*, which rests on the end of the adjusting screw *O*. These parts, with the glass cylinder, *F*, are clamped to the flange, *B*, by means of four long screws, *P*, and the ring, *R*; on the ring, *R*, screws the cap, *S*, which covers the lower parts of the cistern and supports at the end the adjusting screw, *O*. *G*, *I*, *J*, and *k*, are of boxwood; the other parts of brass or German silver. The screw *O* serves to adjust the mercury to the ivory point, and also, by raising the bag, so as to completely fill the cistern and tube with mercury, to put the instrument in condition for transportation."

92. **Aneroids.**—The aneroid barometer consists of a metallic box, exhausted of air and connected by a system of mechanism with a pointer reading on a graduated scale or dial. The scale is graduated arbitrarily by comparison with a standard mercurial barometer. The pressure of the atmosphere acting on the box is transmitted to the pointer, and read off on the scale.

There are two iorms in use, the Vidi or Naudet aneroid, Fig. 151, and the Goldschmid aneroid, Fig. 153.

The Naudet Aneroid.—The elementary form of this instrument is shown in elevation in Fig. 149. The lower side of the

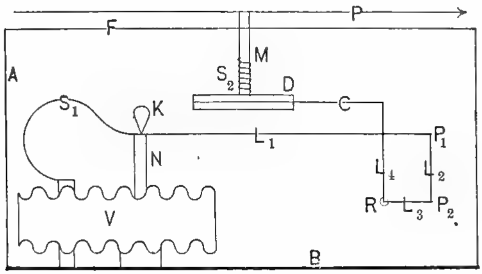


FIG. 149.

vacuum chamber V is fastened to the bottom plate B of the outside case. In order to increase its sensitiveness to a change in the atmospheric pressure, V is made of corrugated metal.

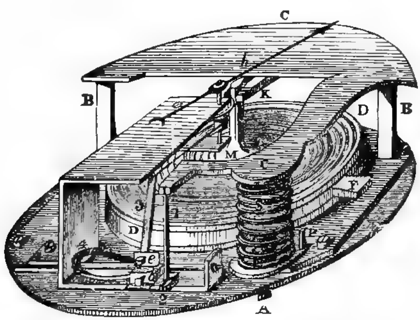


FIG. 150.

Attached to its top and passing through the spring S_1, against which it bears by the knife-edge K, is a rod N. The spring S_1 is also attached to the lower plate B. The levers L_1, L_2, and L_3 are connected by the pins P_1 and P_2. L_1 is attached to S_1, and L_3 to the shaft R. L_4 is a vertical lever also attached to R. The chain C connects L_4 with a drum D whose axis M carries

the pointer P. A spiral spring S_2 coiled on the central shaft M pulls against the chain. The motion of the top of the vacuum box is thus communicated to the pointer P.

Figs. 150 and 151 show the Naudet aneroid as practically

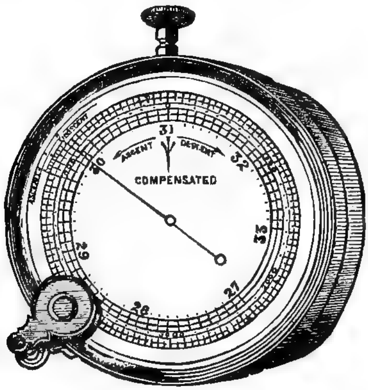


FIG. 151.

constructed, Fig. 150 being a view with the outer case removed.

The Goldschmid Aneroid.—Fig. 152 shows the essential parts

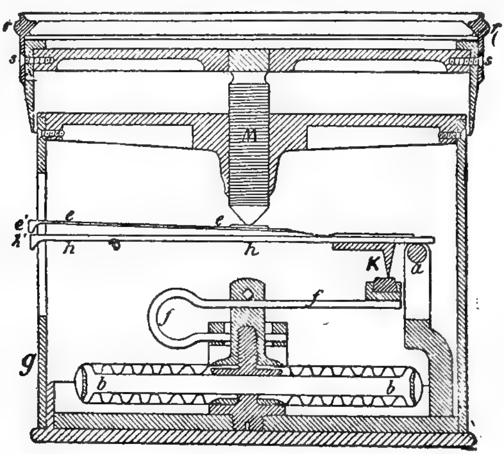


FIG. 152.

of this barometer. As in the Naudet form, the vacuum chamber is rigidly fastened to the bottom of the outside case. From its

top projects the bent arm f terminating under the knife edge K. K is attached to the lower side of the lever hh hinged at a. Its outer end, h', works in a slit in the side of the case. At the side of this slit is found a vertical scale, as shown in Fig. 153. This scale is graduated downward* to agree with the even inches of the mercurial barometer, and is read by a horizontal line on the lever hh. To read the tenths and hundredths a spring ee is used in connection with the micrometer screw M. M is moved by turning the top of the outside case, its graduations being on the side of this top, and the zero on the side of the box just below. The micrometer is divided into one hundred equal parts. The spring ee is soldered to the side of the

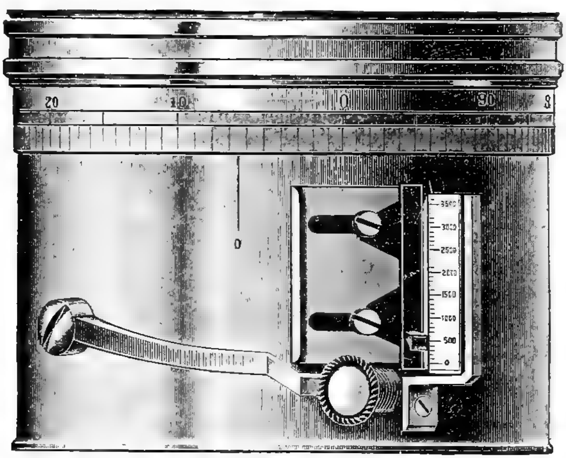


FIG. 153.

lever hh near the end a. On its other end is found a horizontal line similar to the one on the lever. With the micrometer reading zero, the spring presses upward against the point of the screw. As h' rises, its horizontal line reads less on the scale. A complete revolution of the micrometer corresponds to one division of the vertical scale. To read the instrument the micrometer is turned down until the horizontal line on e' coincides with that on h'. When the line on e' coincides with the division just above h', the micrometer reads zero.

* In Fig. 153, *upward*, giving a scale of altitudes.

Consequently its reading, when the two lines are made to coincide, will give the tenths and hundredths between h' and the scale division next above, the combined scale and micrometer readings giving the total reading of the instrument.

Fig. 153 shows the Goldschmid aneroid as practically made.

93. **Barometric Formulæ.**—The atmospheric pressure per unit area at any point is equal to the weight of the column of the atmosphere of unit cross section whose height is the vertical distance from its upper limit down to the point in question. The analytical representation of this fact is

$$P = \int a\,dz, \quad \text{or} \quad -dP = a\,dz, \quad . \quad . \quad . \quad . \quad (46)$$

in which $-dP$ is the decrease in pressure due to an increase in elevation dz, and a is the weight of a unit volume of the atmosphere. If the values of P and a are known at any elevation, say P_0 and a_0, at any other elevation, from Marriotte's law,

$$\frac{P}{P_0} = \frac{a}{a_0} \quad \text{or} \quad a = a_0 \frac{P}{P_0}. \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Substituting (47) in (46), we have

$$-dP = \frac{a_0}{P_0} P\,dz, \quad dz = -\frac{P_0}{a_0}\frac{dP}{P},$$

and

$$z = -\frac{P_0}{a_0}\int_{P_2}^{P_1}\frac{dP}{P} = -\frac{P_0}{a_0}(\log P_1 - \log P_2),$$

in which z represents the difference in elevation between two points, at the upper and lower of which the pressures are P_1 and P_2 respectively. The two points may or may not be in the same vertical line.

Writing the expression $(\log P_1 - \log P_2)$, $-(\log P_2 - \log P_1)$, the last equation becomes

$$z = \frac{P_0}{a_0} \log \frac{P_2}{P_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Equation (48) is the fundamental equation of barometric hypsometry. It assumes that the atmosphere is in static equilibrium. To determine the value of P_0/a_0 observe that at any given point where the height of the mercury column is h_0 and the weight of the mercury per unit volume is m_0, $P_0 = m_0 h_0$, or putting $P_0 = a_0 H$,

$$\frac{P_0}{a_0} = H = \frac{m_0 h_0}{a_0}. \quad \ldots \ldots \quad (49)$$

Evidently H is the height of an atmospheric column of uniform density a_0, which weighs the same as the mercurial column whose unit weight is m_0 and whose height is h_0.

Substituting H for P_0/a_0, (48) may be written

$$z = H \log \frac{P_2}{P_1}. \quad \ldots \ldots \quad (50)$$

In latitude 45° at the temperature 32° F. at the sea level, the following values have been experimentally determined: *

$$h_0 = 29.92''; \; a_0 = .00129303; \; m_0 = 13.59593.$$

Substituting these values in (49), we find

$$\frac{P_0}{a_0} = H = 26220 \text{ ft.} \quad \ldots \ldots \quad (51)$$

The coefficient of expansion of air is .002034. Hence, at any other temperature t,

$$H = 26220[1 + .002034(t - 32)°]. \quad \ldots \quad (52)$$

With these values, and, to pass from the Napierian to the common system, substituting for $\left(\log \frac{P_2}{P_1}\right)$, $2.30258\left(\log_{10} \frac{P_2}{P_1}\right)$, (48) or (50) becomes

$$z = 26220[1 + 0.002034(t - 32)]\, 2.30258 \log_{10} \frac{P_2}{P_1}. \quad . \quad (53)$$

The expression $\frac{H}{M} = 26220 \times 2.30258$ is known as the "principal barometric constant" or the "barometric coefficient."

* U. S. C. & G. S. Report for 1881, Appendix 10, p. 234.

The pressure can be determined only by the height of the mercury column. However, supposing the mercurial heights h_2 and h_1 to correspond to the pressures P_2 and P_1, the equation $\frac{P_2}{P_1} = \frac{h_2}{h_1}$ will not be exactly true. The absolute pressure at any point is equal to the weight of the column of mercury which it will support. If m_1 is the weight of a unit volume of mercury at the point where the barometer height is h_1, the pressure at the point will be m_1h_1. In like manner

$$P_2 = m_2h_2, \quad \text{and} \quad \frac{P_2}{P_1} = \frac{m_2h_2}{m_1h_1}. \quad . \quad . \quad . \quad . \quad . \quad (54)$$

The attraction of gravity varies inversely as the square of the distance. Consequently if h_2 corresponds to a point r ft. from the center of the earth, $\frac{m_1}{m_2} = \frac{r^2}{(r+z)^2}$, from which

$$m_1 = m_2 \frac{r^2}{(r+z)^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Substituting (55) in (54), we have

$$\frac{P_2}{P_1} = \frac{m_2h_2}{m_2 \frac{r^2}{(r+z)^2} h_1} = \frac{h_2}{h_1} \frac{(r+z)^2}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad (56)$$

The expression $\frac{(r+z)^2}{r^2}$ may be written

$$\frac{r^2\left(1+\frac{z}{r}\right)^2}{r^2} = \left(1+\frac{z}{r}\right)^2 = 1 + \frac{2z}{r} + \frac{z^2}{r^2}.$$

Dropping the last term as being very small in comparison with the second, and substituting the resulting value in (55) we have:

$$\frac{P_2}{P_1} = \frac{h_2}{h_1}\left(1 + \frac{2z}{r}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

$\frac{2z}{r}$ being a very small fraction, the hyperbolic logarithm of $1 + \frac{2z}{r}$ is very nearly equal to $\frac{2z}{r}$, and its common logarithm to $\frac{2Mz}{r}$, where M equals 0.4342945, the modulus of the common system.*

With this value, passing to logarithms in (57),

$$\log_{10} \frac{P_2}{P_1} = \log_{10} \frac{h_2}{h_1} + \frac{2Mz}{r}. \quad . \quad . \quad . \quad . \quad . \quad (58)$$

Again, $\frac{2Mz}{r}$ is very small. Consequently we may substitute in it for z the approximate value, $z = \frac{H}{M}\left(\log_{10}\frac{P_2}{P_1}\right)$, deduced from (50) by passing from the Napierian to the common system.

We have, therefore, $\log_{10} \frac{P_2}{P_1} = \log_{10} \frac{h_2}{h_1} + \frac{2M}{r}\left(\frac{H}{M} \log_{10} \frac{P_2}{P_1}\right)$, from which

$$\log_{10}\frac{P_2}{P_1} = \frac{\log_{10}\frac{h_2}{h_1}}{1 - \frac{2H}{r}} = \log_{10}\frac{h_2}{h_1}\left(1 + \frac{2H}{r}\right) \quad . \quad . \quad (59)$$

practically, the expression $\left(1 + \frac{2H}{r}\right)$ being obtained from $\frac{1}{1 - \frac{2H}{r}}$ by dividing out and dropping all except the first two terms. Substituting (59) in (53),

$$z = 26220 \times 2.30258 \times \log_{10}\frac{h_2}{h_1}\left(1 + \frac{2H}{r}\right)[1 + 0.002034\,(t - 32)]. \quad (60)$$

From (51), $H = 26{,}220$ ft. r is the distance in feet from the center of the earth to the lower point at which the barometer is held. The fraction $\frac{2H}{r}$ will be sufficiently correct if we use

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* Laplace, Mécanique Celeste, Bk. 10, Chap. IV, p. 8680.

in it always for r the radius of the earth. This, as given by Col. Clark,* is 20,890,548 ft. Substituting these values in (60) and multiplying out, we have

$$z = 60,521.5[1 + .002034(t-32)] \log_{10} \frac{h_2}{h_1}. \quad . \quad . \quad . \quad (61)$$

If the barometric height at the lower station is $h_2 = 30''$, which corresponds to the approximate sea level, and if the temperature t may be assumed as the mean of the temperatures t_1 and t_2 at the upper and lower stations respectively (61) becomes:

$$z = 60,521.5[1 + .001017(t_1 + t_2 - 64)] \log_{10} \frac{30}{h_1}, \quad . \quad . \quad (62)$$

from which, assuming further $t_1 + t_2 = 100°$, we have

$$z = 60,521.5(1 + .001017 \times 36) \log_{10} \frac{30}{h_1}. \quad . \quad . \quad . \quad (63)$$

The temperature conditions being as noted above, (63) gives the elevation of any point at which the barometer height is h_1 above another point at which the height is $30''$. To obtain the difference in elevation between the points by means of a barometer and the formula, obtain the elevation of each above a point where the pressure is $30''$ and take the difference of the results. Table VII † is computed from (63). It contains values of z for different values of h [$= h_1$ of (63)] varying by one tenth of an inch from eleven to thirty-one inches. To use the table, it is entered twice with the heights of the barometer at the two points at which it has been held, as arguments. The difference between the corresponding tabular values of z will give the difference of elevation of the two points. Interpolation is necessary for the values of h not in even tenths of an inch. If the sum $(t_1 + t_2)$ differs from 100°, Table VIII must be used in connection with Table VII.

To illustrate the use of Table VIII suppose that with the barometer heights h_1 and h_2 the two values taken out of Table VII are z_1 and z_2. Then

$$z = z_1 - z_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

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* Clark's Geodesy.

† Taken from U. S. C. & G. S. Report for 1881, Appendix 10, p. 265.

supposing the value of $(t_1 + t_2)$ to be 100°. If, however, the sum $(t_1 + t_2)$ is more or less than 100°, Table VIII must be entered with the argument $(t_1 + t_2)$, the corresponding value of C taken out and multiplied into the result z obtained from the first table and the product added algebraically to that result. so that finally

$$z = (z_1 - z_2)(1 + C). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (65)$$

Table VIII has been compiled from Tables I and IV, Appendix 10, U. S. C. & G. S. Report for 1881. Its use corrects for a supposed average condition of humidity in the atmosphere, the value of a_0 used in deducing eq. (51) being for dry air.

Strictly speaking, the value of a (eq. 46) should be corrected for latitude and also for elevation, but these corrections are very small, and in ordinary engineering operations may generally be omitted.

The barometer heights should be reduced to the same temperature. It is customary to use the temperature of freezing.

Let h = observed height of barometer in inches.

t = temperature of attached thermometer (Fahrenheit).

m = coeff. of expansion of mercury = 0.0001001.

1043

b = coeff. of expansion of brass = 0.0000104344, the brass scale of the barometer being standard at 62°.

c = the correction to the observed height to reduce it to absolute units at the freezing point.

h' = the absolute height of the mercury column at temp. t.

h_0 = absolute height of mercury column at temperature 32°.

Then

$$h' = h_0[1 + m(t - 32)], \quad . \quad . \quad . \quad . \quad . \quad . \qquad (66)$$

$$h' = h[1 + b(t - 62)]. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (67)$$

From (66) and (67),

$$h_0 = \frac{h[1 + b(t - 62)]}{1 + m(t - 32)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

$$c = h - h_0 = h\left(\frac{m(t - 32) - b(t - 62)}{1 + m(t - 32)}\right). \quad . \quad . \quad (69)$$

Extensive tables have been calculated from eq. (69). See Williamson's "On the Barometer," Table A; "Smithsonian Meteorological Tables," 1893, Table 10.

94. **General Remarks on the Barometer.** — Barometric Hypsometry is a science in itself, and has been the subject of much study and investigation. It furnishes a means of determining quickly the difference in elevation of widely separated points, but its results can be regarded only as approximations. These become most useful in the exploration, or, technically, "reconnaissance," of a new country.

Special methods of observation eliminate in part the errors to which barometric results are liable. (See Baker's "Engineers' Surveying Instruments," Arts. 371–381.) From the standpoint of precision the mercurial barometer is a better hypsometrical instrument than the aneroid. The aneroid, on the other hand, is more easily portable, and therefore more serviceable. It is also thought to be more sensitive. "Even in observatories, where mercurial standards are in use, the aneroid is most valuable in its capacity of giving earlier indications than can be obtained from the more sluggish mercurial column."

SECTION IV.

Trigonometric Leveling.

95. **Definition.**—Trigonometric leveling means, literally, leveling by means of three angles. Generally only one angle is used, but the work is based on the theory of the right-angled plane triangle. In Fig. 154 an observer at A, knowing the distance AB and the angle BAC, can determine the height BC by means of the formula $BC = AB \tan BAC$. This method of

leveling is especially convenient in the case of a triangulation (Art. 106), where the tract surveyed is divided into a number of triangles, an instrument capable of measuring both horizontal and vertical angles being set up at each vertex. In such a case the horizontal angles and distances have to be determined,

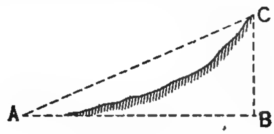

FIG. 154.

and the additional labor involved in measuring the vertical angles is comparatively slight.

If work of the highest possible precision is desired, the computations must be corrected for curvature of the earth and for refraction.* These corrections, however, especially that for the curvature of the earth, lie somewhat without the field of plane surveying. In mine surveying the methods of trigonometric leveling become very useful. See the discussion of that subject in Chapter X.

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* See Van Nostrand's Science Series, No. 91.

CHAPTER VI.

TRANSIT SURVEYING.

96. The Engineer's Transit.—The essential elements of this instrument are a line of sight, a vertical and a horizontal axis, and the graduated circles by means of which may be measured the revolution of the line of sight around the two axes. In the transit instrument the primary object is the determination of angles, and with the construction above described both horizontal and vertical angles can be measured. As in the level, the line of sight is telescopic.

The complete instrument is shown in Fig. 155. The telescope is seen to be attached by the second method of Art. 62, and has its horizontal or "transverse" axis resting on two standards shaped so as to give the maximum strength with the minimum weight. The standards are attached to what is known as the "alidade" or "upper plate." One of them has a vertical adjustment at its upper end by which the end of the horizontal axis resting there can be raised or lowered (Fig. 185). The adjustment is effected by means of a capstan-headed screw. Attached to the other standard is found a vernier by which is read the vertical circle of the horizontal axis. The upper plate carries another vernier, and together with the "lower plate," which carries the horizontal graduated circle, revolves around the "vertical axis." The arrangement is such that the lower plate can be clamped to the vertical axis and the upper plate together with the standards, etc., revolved around it, the angle of revolution being read by the vernier and horizontal divided circle. Or the plates can be clamped together and revolved around the vertical

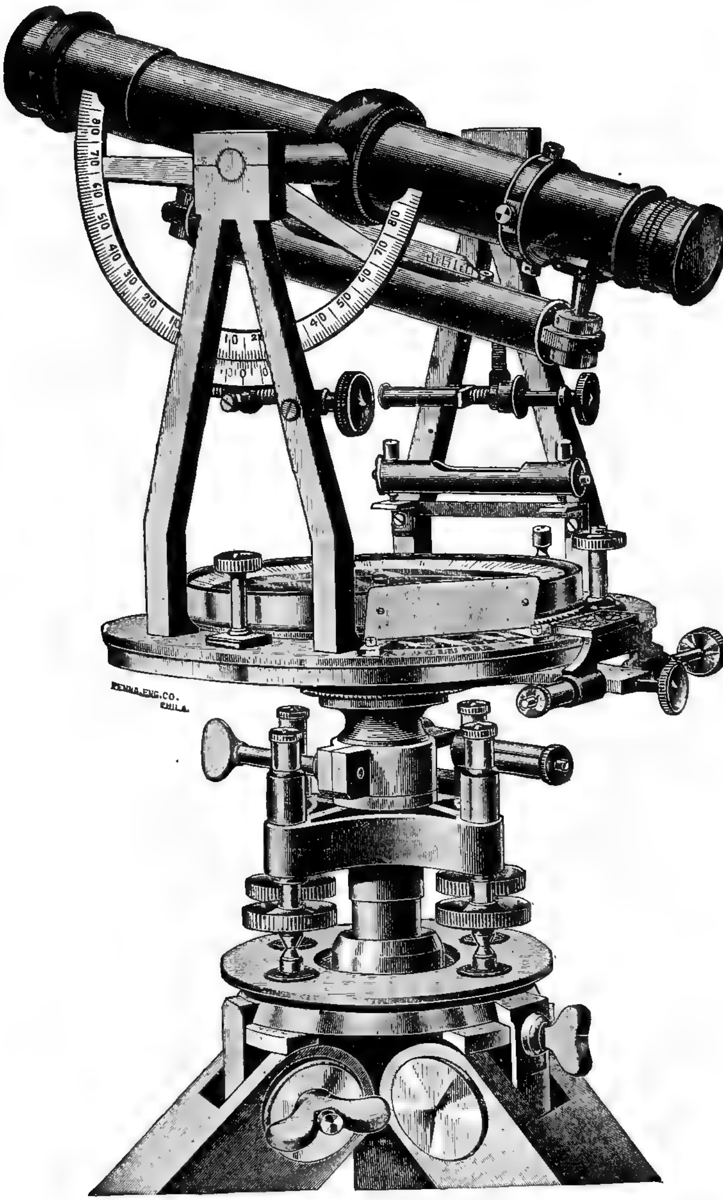


FIG. 155.

axis. The mechanism by which this is effected is shown in Fig. 156. There are seen to be two axes or "centers," V_1 and V_2, the inner attached to the upper plate, P_1, and fitting inside the outer, which, in turn, is attached to the lower plate, P_2, and works in a socket M in the leveling head (Art. 73). These axes are generally known as "centers." In cheap instruments they are made short. The movements described above evidently require two clamps, the upper to hold the plates together, the lower to hold the outer center in any position in its socket. In connection with each clamp a tangent screw is

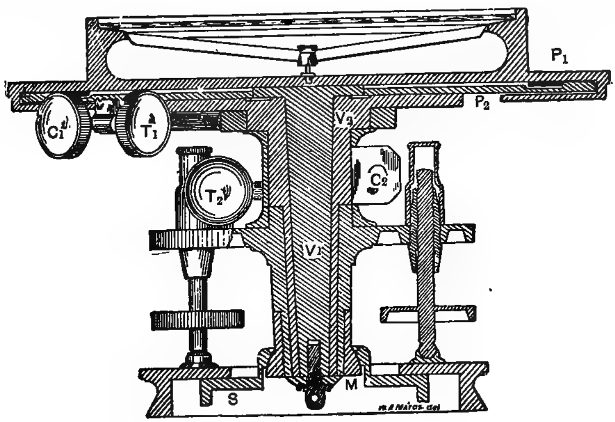


FIG. 156.

used. By means of these when the clamps have been fastened, they, together with the parts which they hold, can be turned around the vertical axis. The upper and lower clamps and tangent screws are shown at C_1, C_2, T_1, and T_2, Fig. 156.

Clamps and tangent screws are made in various forms. Their purpose is to enable the telescopic line of sight and graduated circles to be set accurately and held firmly in any desired positions. In addition to those already described, there is a clamp and screw (T_3 and C_3, Fig. 159) attachment used in connection with the horizontal axis and vertical circle. In distinction from the "upper" and "lower" clamps and tangent screws, we will speak of this last arrangement as the "vertical" clamp and screw. Figs. 157–159 show the con-

struction of some of the common forms. Fig. 157 gives side, front, and back views of the usual lower clamp and tangent screw. The clamp R_2 is in the form of a collar fitting loosely around the outer axis, V_2. By tightening the clamp screw C_2, shown in the side and back views, R_2 binds around V_2. On the side opposite to C_2 a flange F_2 (front view) extends out from R_2 between the spiral spring S_2 and the tangent screw T_2. T_2 and S_2 are both attached to the top plate H of the leveling head by the standards N_2 N_2. With R_2 clamped to V_2, by turning T_2, V_2, and with it the lower plate P_2, is revolved in the socket M of the leveling head.

The construction of the upper clamp and tangent screw is similar. Fig. 158 shows a front and side view of a good form. The bent piece N_1 is attached to the upper plate, P_1. By screwing in the clamp screw C_1, the collar R_1, by means of the movable piece A, binds around the outer axis V_2. C_1 passes through an opening in N_1 (see Fig. 155) and is pressed firmly against the end of the tangent screw T_1 by the spiral spring S_1. S_1 and T_1 are both attached to N_1. C_1 being clamped to V_2, by turning T_1, the inner axis, V_1, and with it the upper plate P_1, revolves inside of V_2.

Fig. 159 shows the vertical clamp C_3 and tangent screw T_3. The screw in the figure is also a "gradienter screw," and will be described more particularly later.

The upper plate carries two plate levels as in the compass. In the common forms of instrument, a complete compass is placed between the standards. The method of attaching the leveling head to the tripod, as well as many other details connected with surveying instruments, varies with the different makers. Their catalogues generally give full descriptions of the different parts. The device known as a "shifting center" deserves especial mention. It consists of the ring S, Fig. 156, working in the opening shown in the bottom plate of the leveling head. After the tripod has been set up, with the leveling screws rather loose, the whole upper part of the instrument can be shifted on the bottom plate of the leveling head and the plumb-bob brought exactly over the point occupied. Fig. 156 shows also the eye to which the plumb-bob is at-

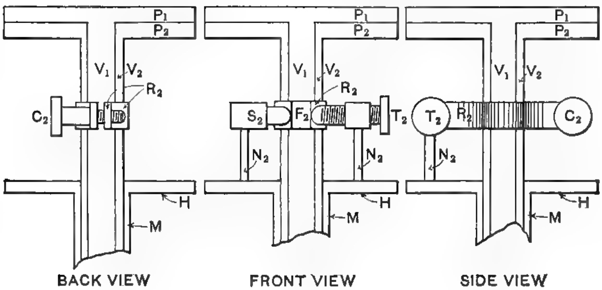


FIG. 157.

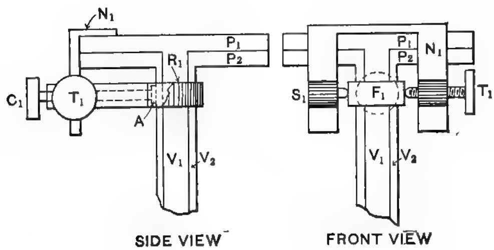


FIG. 158.

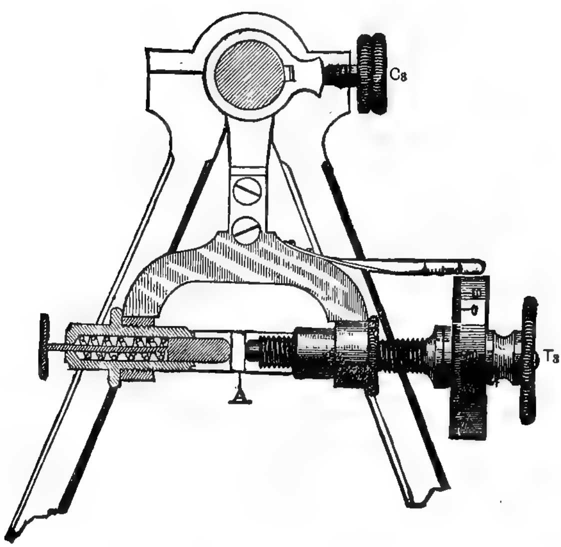


FIG. 159.

tached. An accessory to many transits is the level under the telescope shown in Fig. 155. By means of this the instrument can be used as a level.

Stadia wires are horizontal wires inserted in the telescope and used in connection with the determination of distances. Their use will be described in detail in the chapter on Topographical Surveying. The gradienter screw mentioned above is simply the vertical tangent screw, made into a micrometer screw by the addition of a graduated head. It is customary to make the pitch such that a complete revolution of the screw will elevate or depress the line of sight by an angle whose tangent is .01 or .005. This attachment may be used in a manner similar to the stadia wires (Art. 118), for the determination of distances, but its principal use is in the setting out of grades. Thus, suppose one revolution of the tangent screw to correspond to a change in elevation of one foot in a hundred. If it is desired to set out a grade rising one foot in a horizontal distance of one hundred feet, with the instrument on line, first level the telescope and then elevate the line of sight by turning the screw through one complete revolution. The line of sight will then be parallel to the desired grade line, and the level rod will read the same when held at any point on it. Grades are technically known as one per cent, one and three quarters per cent, and so on, as they rise or fall one foot, a foot and three quarters, etc., in a horizontal distance of 100 feet. In the preceding example, if it had been desired to set out a one and one half per cent grade, the screw would have been turned one and one half revolutions. The complete revolutions are read on the scale shown in Fig. 159, while the fractional revolutions are taken from the graduation, usually to 100 parts, on the head. It may seem to some, at first, that the theory of the gradienter is not correct; that, owing to the fact that the tangents do not vary as the angles, the gradienter will set out exactly correctly only the grade for which it has been graduated. A consideration of the fact that the motion of the tangent screw is always horizontal will prevent this mistake. In other words, if moving the screw through the distance d causes the line of sight to move through the angle θ whose tangent is

t, moving the screw through the distance nd will cause the line of sight to move not through the angle $n\theta$, but through the angle whose tangent is nt.

A convenient addition to a transit, in some cases, is the "quick leveling attachment" inserted between the instrument

FIG. 160.

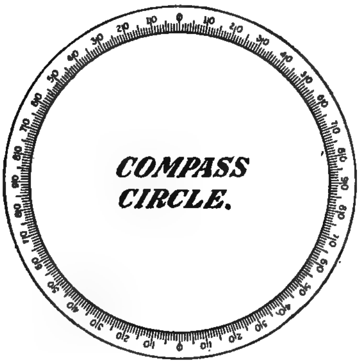
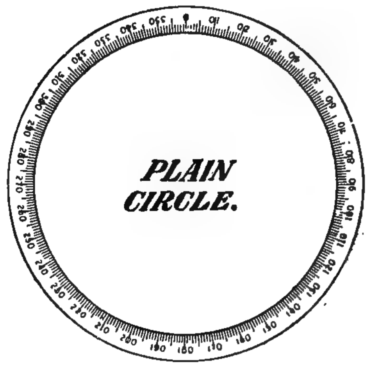


FIG. 161.



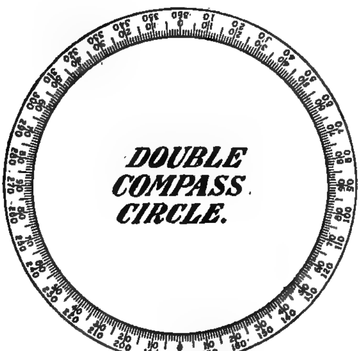


FIG. 162.

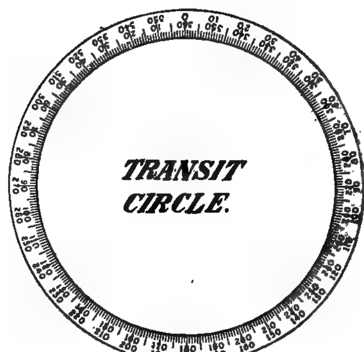


FIG. 163.

and the tripod head. For a description of one style of this device, see the "Manual" published by W. & L. E. Gurley, Troy, N. Y. Further accessories are the arrangement for offsetting at right angles to the line on which the instrument is set, the diagonal prism for vertical sighting (Fig. 255), and the reflector for illuminating the cross hairs in night work (Fig. 254). The offsetting arrangement consists of two pin-hole

sights attached to the standards in a line perpendicular to the line of sight of the instrument, a small hole pierced longitudinally through the transverse axis, or an arrangement of two prisms by which the observer can offset without leaving the eye end of the telescope. The two last devices are described in the catalogue of C. L. Berger & Sons, Boston, Mass.

The horizontal circle is numbered in one of the ways shown in Figs. 160–163. In Fig. 160 the numbering is the same as on the compass circle, while in Fig. 161 it extends all around the circle from 0 to 360. In Figs. 162 and 163 there are seen to be double rows of figures. Fig. 163 is probably the best style for ordinary use. Ordinarily the verniers read to one minute, though in the better grade of instruments the reading is sometimes as close as ten seconds. The verniers are like those described in Art. 49. There are generally two, placed so as to read 180° apart on the circle. The reason for this is discussed in Art. 110. Every transitman should carry a hand magnifying glass with which to read the vernier. For this purpose, some instruments have reading microscopes attached.

Fig. 164 shows a "transit theodolite" for use in work where a high degree of precision is required. Such an instrument would be unsuitable for the operations of Plane Surveying. For a description of the recent "cyclotomic" transit, see Appendix.

97. **Use of the Transit.**—The remarks of Art. 29 in regard to "setting up" are particularly applicable in the case of the transit. With all instruments it is important that the tripod should be set up firmly. With the transit, generally, it is further of importance that the plumb-bob should be accurately centered over the point occupied.

In the first place, be careful that the tripod legs are fairly well spread out and inserted in the surface so that they will not slip. If the instrument has to be set up on a floor, put the points in the cracks. In setting up on a railroad track do not let the tripod legs rest on the ties. If they do, stepping on the tie may throw the instrument out of level. If, as is almost always the case, the instrument is to be centered over a point,

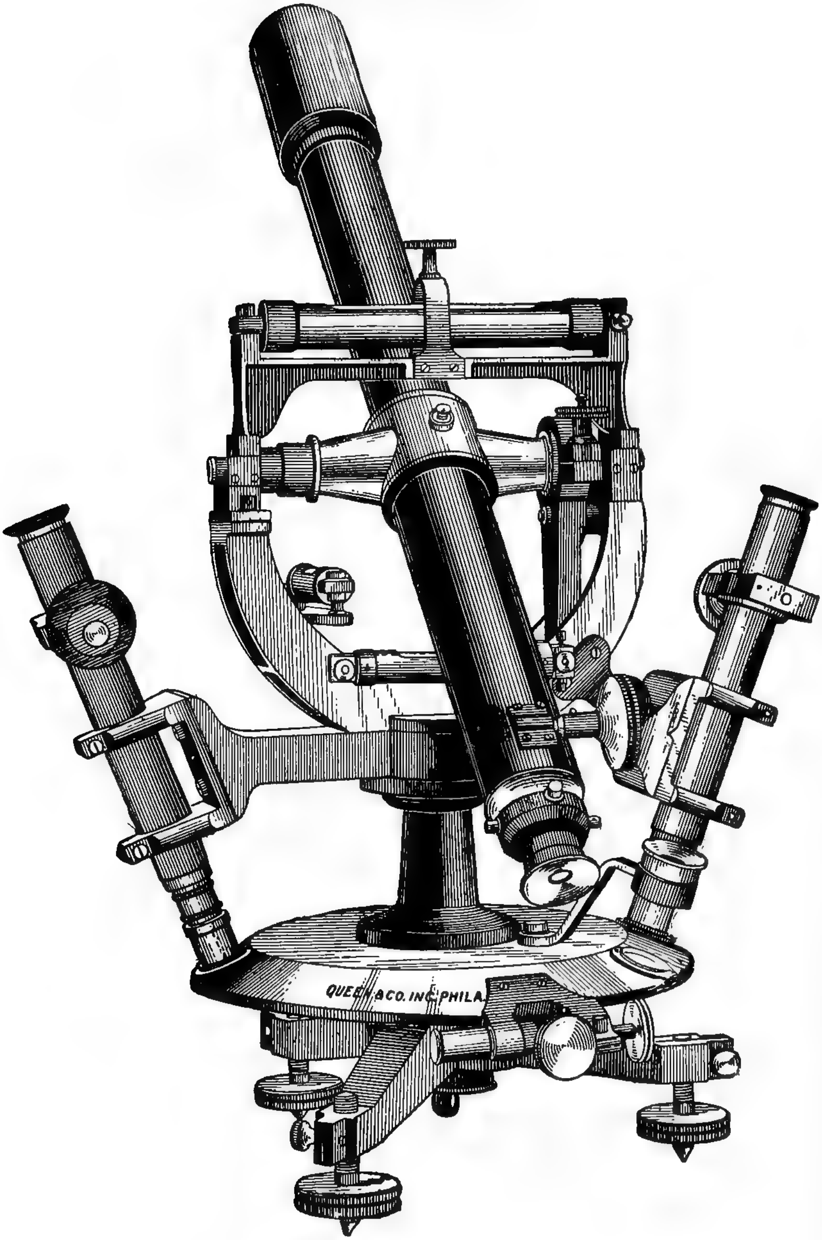


FIG. 164.

being careful to keep the head of the tripod approximately level, press one of the legs into the ground until the plumb-line swings in the plane of another leg and the point to be occupied. Then by moving the third leg bring the bob nearly over the point and complete the centering by means of the shifting center.* In turning the instrument in azimuth, with the clamps loose, take hold of the plates, never the standards or telescope. The clamps and leveling screws should be turned to a firm bearing only. The beginner is apt to wrench the instrument and strip the screws.

To level the instrument turn it on the vertical axis until the plate levels are each parallel with one pair of leveling screws. Then by manipulating the screws bring first one bubble and then the other to the center until both appear at the center at the same time. If the instrument has previously been centered over a point, note whether the leveling has disturbed this arrangement. If so, center and level again until both conditions are satisfied. The operation of "setting up" is generally understood to include both centering and leveling.

98. Simple Measurement of Horizontal Angles.—When not in use the transit should be kept with both clamps loose. To measure a horizontal angle first set up over the vertex of the angle, direct the line of sight nearly to one of the points between which the angle is to be measured, clamp both upper and lower motions, and by means of the lower tangent screw and the vertical motion, direct the line of sight exactly to the point.

A note should now be made of the vernier reading on the horizontal circle. Next, loosen the upper clamp, turn the line of sight nearly on the second point, clamp the upper motion once more, and finish the setting by the upper tangent screw and vertical motion. The difference between the vernier reading now found and that noted at first will give the angle required. If desired, the vernier may be set at zero at first. In this case the vernier reading at the last will give the angle at once.

---

* Baker's Engineers' Surveying Instruments, Art. 24.

It is often the case that a number of angles have their vertices at the same point, as shown in Fig. 165. The procedure in such a case is to start with the line of sight on one point, as *A*, and then direct it successively to the points *B*, *C*, *D*, and *A* again. This is known as "closing the horizon." The agreement of the final reading with the first reading will insure that

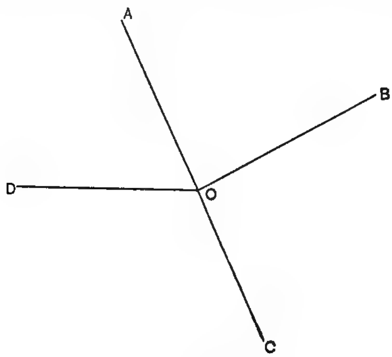


FIG. 165.

the instrument has not slipped or moved during the operation. With a transit reading to single minutes, the initial and final readings should not differ by more than one minute.

The notes are kept as shown in Fig. 166. The vernier is supposed to read to single minutes and only one vernier to be read. The two verniers on the horizontal circle are generally distinguished as verniers A and B. If only one vernier is read, be careful that it is the same one each time. In the example shown in Fig. 166, vernier A is supposed to have been read. The use of the column headed "Vertical Angle" will be explained in Art. 100.

FIG. 166.

X Y Z Survey.
May 10, 1900.
Instrument on Sta. O. Observer, Park.

Station observed.	Vernier A.	Horizontal Angle.	Vertical Angle.
A	0° 00′	0° 00′	3° 10′
B	85° 06′	85° 06′	− 10° 16′
C	178° 43′	93° 37′	− 4° 08′
D	294° 26′	115° 43′	6° 14′
A	359° 59′	65° 33′	7° 01′
		359° 59′	

99. Horizontal Measurement by Repetition.—If it is desired to measure the angle more accurately than can be done proceeding as just described, the method of this article may be adopted. The procedure is the same as in the simple measurement described in Art. 98, except that when the line of sight has been turned on the second point by the upper tangent screw, the lower clamp is loosened and the telescope again directed, by means of the lower motion, to the first point. The angle is then again turned off by the upper motion, the line of sight directed once more, by the lower motion, to the first point, and so on for as many repetitions as may be desired. At the conclusion of this operation both verniers are noted, and the mean of their readings divided by the number of repetitions will give the value required. If still further accuracy is desired, the measurement is repeated in the same way, except that the telescope is plunged and the *second* point sighted on first, thus causing the angle to be measured in the opposite direction. The final value of the angle is the mean of those obtained with the telescope normal and plunged. The reasons for plunging the telescope, reading both verniers, etc., will be better understood after a discussion of the tests and adjustments of the transit (Arts. 110 and 111). For the present it will be sufficient to state that making the observations with the telescope normal and plunged eliminates all errors due to lack of adjustment, reading both verniers eliminates errors due to eccentricity of the circle, and reading in both directions eliminates errors due to clamping, pointing, and twisting the tripod.

The following list of steps in taking the observations may make the method somewhat clearer. With the telescope at O, Fig. 165, suppose the angle AOB is to be measured, and that, with both upper and lower motions clamped, the line of sight is directed to A and both verniers read.

Telescope Normal.

1.	Unclamp	above	and	set	on	B
2.	"	below	"	"	"	A
3.	"	above	"	"	"	B
4.	"	below	"	"	"	A
5.	"	above	"	"	"	B
6.	"	below	"	"	"	A
7.	"	above	"	"	"	B

and so on until the whole circle has been covered, which will be, if the vernier has at first been set at zero, when the final reading is about 360 degrees. The final reading divided by the number of repetitions gives the value of the angle. The readings, so far, have been from left to right. Next, with conditions the same as at first, except that we have the line of sight directed to *B* and the

Telescope Plunged.

1. Unclamp above and direct to *A*
2. " below " " " *B*
3. " above " " " *A*

just as before. The mean of the values of the angle with the telescope normal and reversed will be the value finally adopted. The manipulation of the instrument eliminates all errors of adjustment and observation. Of course with this method the horizon can be closed, if so desired. This will require, however, that each angle about the point be repeated the same number of times, whereas in the method just described, in general, each angle is repeated until, if n be the number of repetitions and v the value of the angle, nv is about equal to 360 degrees.*

STA. OBSERVED	VERNIER A	VERNIER B	AVERAGE READING	TOTAL ANGLE	MEAN ANGLE
4	0°00′	01′	0°00′30″	0°00′10″	0°00′10″
6	148 03	04	148 03 30	49 21 10	49 21 00
7	222 02	03	222 02 30	74 00 50	24 39 40
9	311 32	33	311 32 30	103 50 50	29 50 00
10	496 42	42	496 42 00	165 34 00	61 43 10
4	1080 00	1079°59′	1079 59 30	359 59 50	194 25 50
					359°59′50″

FIG. 167.

With ordinary work and the usual engineer's transit a repetition all around the circle seems a useless refinement. For the purpose of a check, however, it is desirable to close the horizon. In such a case, therefore, a method will suffice which closes the horizon with the same number of repetitions for each angle, say three. From what has preceded the list of steps in taking the observations in this last case will be readily written out. The notes will stand as shown in Fig. 167.

* For another method, see Baker's "Engineers' Surveying Instruments," Chap. VII, § 131.

In this set of notes no vertical angle column is shown. If desired, another column can be added for this purpose, or the average reading can be omitted, the 5th and 6th columns each moved to the left and the 6th column used for vertical angles. The usual field-book is ruled in only six vertical columns. Additional columns would often be found convenient. They may be added on the right-hand page, though it is desirable to keep this page entirely for sketches and remarks.

100. Vertical Angles.—The same importance does not attach to these, generally, as to horizontal angles. The measurements are made in a similar manner, but ordinarily one observation will suffice. If the vertical arc is a complete circle, errors of adjustment can be eliminated by reading the angle with the telescope both normal and reversed, and taking the mean. Further, if there are two verniers attached, the mean reading of these will be free from any error due to eccentricity. Generally, the angle desired is simply the angle of elevation or depression from the horizontal, and in any position of the telescope this is read directly on the circle and entered in the vertical angle column, as in Fig. 166, + indicating elevation and − depression.

101. Tangents.—In surveying, especially road and railroad surveying, a straight line, or, more strictly, a line lying in one vertical plane, is known as a "tangent." The method of running a tangent is simple. Supposing the transit set up and in *perfect* adjustment, the line of sight as it revolves around the horizontal axis will describe a vertical plane. The general method, therefore, will be as follows: Set up the transit at one point, and direct the line of sight to another, both points being on the tangent. Then, by revolving the telescope around its horizontal axis, set as many more points as may be desired on either side of the instrument, and at any distance from it within the range of the telescope. By moving the instrument over any one of these a sight may taken to any other, and the tangent prolonged indefinitely. This assumes, however, that the adjustment is perfect. In practice, where any degree of accuracy is desired, always set a point with the telescope both normal and reversed, and take for its true position a

point half way between the two thus found. This operation is known as "double centering." It eliminates all errors due to lack of adjustment.

Evidently a sight to get the instrument "on line" should be taken as long as possible consistent with clear vision through the telescope.

102. Circular Curves.—These are discussed at length in treatises on railroad surveying. It has been found convenient to classify them according to the angles at the center subtended by a chord of 100 feet on the curve. Thus a circular curve is known as a one, two, or two-and-a-half degree curve as its 100-foot chord subtends at the center an angle of one, two, or two and a half degrees.

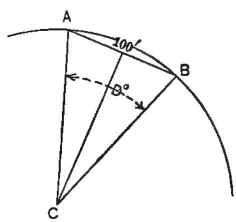


FIG. 168.

In Fig. 168, letting D represent the degree of the curve and R its radius CB or CA, we have evidently

$$R \sin \tfrac{1}{2}D = 50, \quad . \quad . \quad . \quad (70)$$

which is the equation connecting the degree of a curve with its radius.

The method of running out circular curves with a transit depends on the two following geometrical theorems:

(*a*) An angle formed by a tangent and a chord equals half of the central angle standing on the intercepted arc.

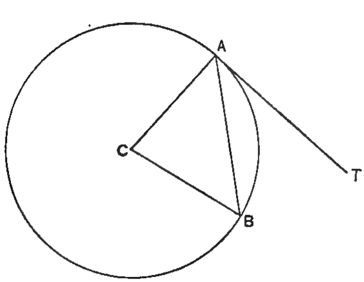


FIG. 169.

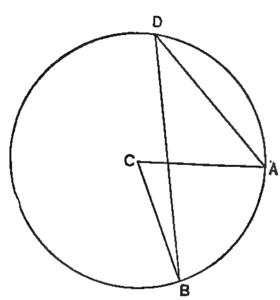


FIG. 170.

(*b*) An inscribed angle equals half the central angle standing on the same arc. Thus in Fig. 169 angle $TAB = \tfrac{1}{2}ACB$,

and in Fig. 170 angle $ADB = \frac{1}{2}ACB$. In Fig. 171 suppose it is desired to connect the intersecting tangents XE and EY by the circular curve AB of degree D and radius $AC = R$? The angle FEY is known as the "angle of intersection." It is evidently equal to the "central angle" ACB. Both are denoted by $\triangle$. The distance $AE = BE$ is known as the "tangent distance," and is generally represented by T. Evidently

$$T = R \tan \tfrac{1}{2}\triangle. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

The curve will start from the point A, known as the "point of curvature" or P. C., and end at the "point of tangency" or P. T., B. The first step, then, will be to locate these points.

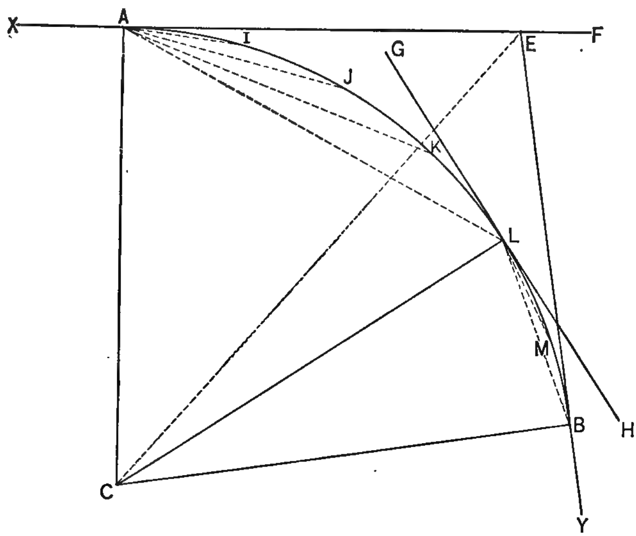


FIG. 171.

This is easily done by computing, by eq. (71), the length $AE = BE = T$, and measuring this distance back along each tangent from the point E. Having found these points, with the transit at A and the line of sight directed along AE, deflect to the right an angle EAI equal to $\frac{1}{2}D$. With one end of the 100-foot tape held at A, and the other swung until it intersects the line of sight, the point I on the curve will be located. Similarly, by deflecting the further angle $IAJ = \frac{1}{2}D$, and

having the tape swung from I, the point J will be found. So proceed around the curve. If it happens that L is the last even station that can be seen from A, having located L, move the transit to that point. Backsighting to A and deflecting the angle $GLA = \frac{1}{2}ACL$, the line of sight will again be in tangent, and further points can be located as at first. Usually, the curve will not correspond to an even number of 100-foot chords. This can be tested by noting whether D is contained in Δ an even number of times. Supposing that in the case represented in Fig. 171 it is found that D is contained in Δ five and three quarter times. The transit having been moved to L, and the point M located in the usual manner by deflecting from the tangent LH the angle $HLM = \frac{1}{2}D$, and measuring from L the distance $LM = 100$ feet, it is now desired to locate the point B and see whether it checks closely enough with its location as determined by measuring from E down the tangent EY. The angle MLB will be made $\frac{3}{4}$ $(\frac{1}{2}D) = \frac{3}{8}$ D, but the distance MB will have to be determined by the equation $MB = 2R \sin \frac{3}{8}D$. Evidently it will be a little longer than $\frac{3}{4} \times 100$ feet.

The foregoing constitute a brief summary of the most important principles in the theory of circular curves. A full discussion cannot be given here, but may be found in any good work on railroad surveying.

103. X Y Z Surveying.—It will be found good practice to calculate the coordinates of a number of points referred to some fixed point and line as the origin and axis of X. The general method is as follows: Assume a base line, AC, Fig. 172, and any number of other stations, B, D, etc. The base line should lie on high level ground and be as long as practicable up to about a quarter of a mile. The other stations may be any prominent fixed points, as the tops of church-steeples, etc. Set up at A and C and measure the horizontal angles made with the base line by the lines running from each of those points to the various outside stations, noting the angle of elevation or of depression in each case.

In Fig. 172, B represents a typical outside station. AC is the base, with A higher than C by the distance CC''. The line

AC'' is therefore horizontal and equal to the reduced length of base. $HI_a = AA'$ is the height of instrument at A, while HI_c is the corresponding height at C. A', C', and B' are in the

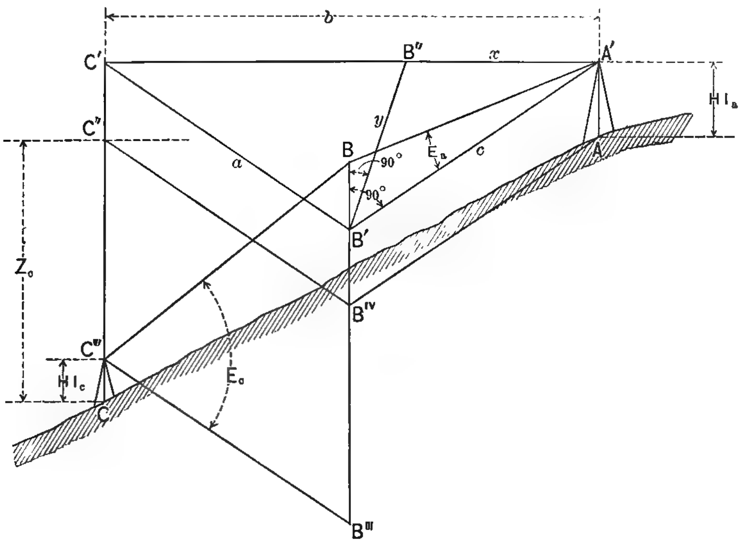


FIG. 172.

same horizontal plane, and the horizontal angles measured at A and C are $C'A'B'$ and $A'C'B'$. Having these angles and the length $b = AC'' = A'C'$, calculate a and c by the law of sines. Having a and c, calculate $B^{iv}B = z$ by the formulæ:

$$z = a \tan B'''C'''B - (C''C - HI_c)$$
$$= a \tan E_c - Z_c + HI_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

and

$$z = c \tan B'A'B + HI_a$$
$$= c \tan E_a + HI_a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

The agreement of these results will check the work both, instrumental and numerical, as $B'''C'''B$ and $B'A'B$ are independent angles. There will evidently be two formulæ for calculating each of the distances $B'B'' = y$ and $A'B'' = x$, namely

$$y = c \sin C'A'B' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$
$$y = a \sin A'C'B' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

and

$$x = c \cos C'A'B' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

$$x = b - a \cos A'C'B'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

The agreement of the results in these last cases will check only the numerical work. The angles may be measured by the method of Art. 98 or of Art. 99, and the field notes kept as therein described.

Forms D and E will be found convenient for the reduction of the notes. Form D is used in finding the lengths a and c, and form E for the later work of finding x, y, and z. A, B, and C denote the horizontal angles at those points. E_a and E_c the angles of elevation or depression of the station from those points. Z_c is the elevation of C with respect to A.

104. Transit Surveying.—The transit can be used in a manner similar to the compass for surveying any traverse, whether closed or not. The general method will be to set up the instrument at each vertex of the traverse and measure the angle, or, more conveniently, the *deflection angle*, at that point. In the case of a closed traverse, for instance, the deflection angles will be the supplements of the interior angles. Then having assumed, or found, the bearing of any one of the lines, from the known angles the other bearings may be computed and the rest of the work carried on as with the compass.

105. Traversing.—A somewhat simpler method is by "traversing." A transit instrument is said to be "oriented" when it is set up with the horizontal circle in such a position that if the vernier is made to read zero, the line of sight will be parallel to the meridian, true or assumed. In Fig. 173, suppose that the traverse $ABCDE$ is to be run out and that NS is the meridian through the initial point A. Then, with the instrument at A, if the vernier be set at zero, the upper motion clamped, and the line of sight, by means of the lower motion, brought into the line NS, the instrument will be oriented at the point A.

The term "azimuth" has been used before (Arts. 50 and 67) in connection with the determination of the meridian and the adjustment of the telescope. We may now define the azimuth

of a point, at any station, as the angle which the line from the station to the point makes with the meridian, true or assumed, through the station. The azimuth of the line joining the station and the point is the same as the azimuth of the point. In traversing, azimuth is seen to be very much the same as

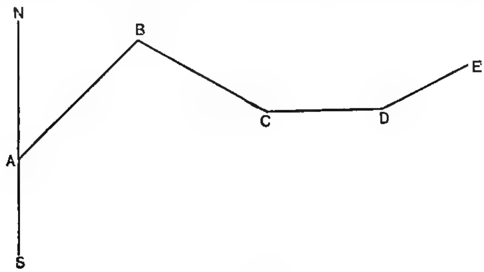


FIG. 173.

bearing. The difference lies in the fact that bearings are reckoned 90 degrees each way from the north and the south points, while azimuth is reckoned 360 degrees from any assumed point. The general practice is to start from the south point and continue around by the west, north, and east. Thus, in Fig. 173, if the bearing of the line *AB* is N 45 degrees E, its azimuth is 225 degrees, so that by giving the azimuth of a line, its direction may be known without using any of the terms north, south, east or west. The expression "azimuth of Polaris" (Art. 50) was not used in quite this general sense.

With the above explanation, it will be understood that if the transit is oriented at *A*, Fig. 173, the line of sight being directed to the south point *S* and then turned by means of the upper motion through the angle *SNB* until it points to *B*, the horizontal circle will read the azimuth of the line, *AB*. Note particularly, however, that the telescope is not plunged during this operation, nor for that matter during the whole work of traversing. Having obtained the azimuth of *AB*, the instrument is set up at *B*. By making the circle read 180 degrees plus the azimuth of *AB*, and then, by means of the lower motion, directing the line of sight to *A*, the transit becomes oriented at *B*, the azimuth of *BC* may be found, the instrument moved to *C*, and so on. The general rule is, on

setting up at a new station, to make the circle read 180 degrees plus the azimuth of the last line, and then, by means of the lower motion, to direct the line of sight to the last station. The instrument will then be oriented. The telescope is never reversed and the same vernier always read.

Traversing is seen to be entirely similar to compass surveying. The notes are kept in the same way, except that the azimuth of the line is written on it instead of its bearing. However, for the purpose of a check, the bearing should be written also. A compass circle, graduated all the way around from 0° to 360°, is convenient here. Or a circle may be graduated on paper and pasted on the compass box glass. If the needle bearings are to be used as checks, the meridian assumed for the traversing must coincide with that indicated by the compass. In calculating the latitudes and longitudes the same formulæ hold as in compass surveying, substituting azimuth for bearing. It must be remembered, however, that

azimuth	90° to 270°	latitude	+
"	0° to 90° and 270° to 360°	latitude	−
"	0° to 180°	longitude	−
"	180° to 360°	"	+

This is assuming the south point as zero azimuth. If the *north* point is assumed as zero azimuth, the algebraic signs of the functions of the azimuth angles will determine the algebraic signs of the corresponding latitudes and longitudes. For this reason the latter practice is somewhat more convenient than the former.

106. Triangulation.—The operation of triangulation consists, first, in dividing the tract to be surveyed into a number of adjoining triangles and, second, in measuring one or more sides and all the angles, so that all the remaining sides may be computed. The work may be hastened by measuring only two of the angles of each triangle, but this is not advisable. The triangles must be so arranged that, starting with the measured side or sides, it will be possible to compute sides in other triangles, passing from these to others, and so on throughout the whole chain. Thus it is possible, knowing the length

of one comparatively short line, to survey a large tract of country. Fig. 174 shows a case of a triangulation system applied to the survey of a river. It will be noticed that wherever possible, the vertices are taken on the bank. Any convenient side may be taken as a base line. In the case shown it would be best, probably, to assume the line 1—3 as the base. Having measured this, as well as all the angles of the various triangles, we can start with the triangle 1—2—3 and, knowing all its angles and the side 1—3, compute the sides 1—2 and 2—3. Next in the triangle 2—3—4, knowing all the angles and the side 2—3, the sides 2—4 and 3—4 can be computed, and so on throughout the chain. 1—3 will have been selected in the first place on ground suitable for measurement and its length determined by the methods of Art. 107. For a check, measure

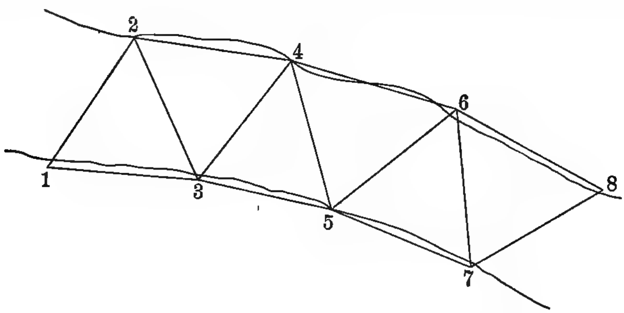


FIG. 174.

another line, as 6—8, at the other extremity of the survey. The agreement of the measured and computed lengths of 6—8 will check the work. Before any of the computations are undertaken, however, the angles must be balanced so that the condition is satisfied that the sum of the angles of each triangle equals 180°. The angles taken from the field notes will not fulfill this condition. In any triangle, if all the angles have been measured with equal care, one third of the total correction should be applied to each.

In general, the angles will be measured by the last method of Art. 99, and the notes kept as shown in Fig. 167. With this method of measurement and the ordinary engineer's transit, the total error in each triangle should not exceed one minute.

Form F will be found convenient for use in reducing the observations. Note that, starting with the base line, it will not be necessary to look up any of the anti-logs until we reach the second or check base line at 6—8, Fig. 174. More generally we may say that it will not be necessary to look up any of the anti-logs except for the purpose of a check. For, having balanced all the triangles and knowing the bearing or azimuth of the base line, from the corrected angles, the bearings or azimuths of the lines down one side of the chain, across, up the other side, and across again may be found. Knowing the bearings or azimuths and the logs of the lengths of these sides, these values can be substituted in forms B and C and the survey plotted and its area found by the method of latitudes and longitudes. The numerical work of calculating the bearings or azimuths from the angles may be checked by noting, when the base line is again reached after going around the entire outside boundary, whether the bearing then found agrees with the bearing assumed at the start. The bearing of the base line will have been observed during the field-work. It will be interesting also, though not needed as a check, to observe the bearings of some of the other lines and note how they agree with the computed values. Note especially that as the angles have been balanced exactly and the whole work depends on the length measurement of one line, the algebraic sums of the latitudes and of the longitudes, within the limits of the accuracy of the logarithms used, should each be zero, without any further balancing.

It will be found necessary often to run compass traverses between stations. In the case shown in Fig. 174, for instance, if an accurate map of the river is desired, compass traverses can be run between adjacent stations, as 2—4, 4—6, etc., and offsets taken to the bank as often as may be necessary.

The description given above fits especially the case shown in Fig. 174, though the principles, in general, can be applied to any plane triangulation. Much more complex systems are often found in which the problem of angle adjustment becomes more difficult. These, however, belong more properly to the subject of geodetic surveying.

107. Precision in Linear Measurement.—Linear measurements can be made with almost any desired degree of precision. In the latest report of the U. S. Coast and Geodetic Survey,* reference is made to lines in the measurement of which the error is only $\frac{1}{5000000}$ part of the total length. Of course, measurements of this kind are made only in connection with geodetic work of the highest precision. In measurements with the ordinary chain, pins, and plumb-bobs, the chain being leveled and aligned by eye simply, no two measurements of a line should differ by more than about $\frac{1}{1000}$ part of its length, or the "precision" of the measurement should be $\frac{1}{1000}$. Two of the author's students measured a line in the above manner four times with results varying from 831.95 to 832.05, the tenths and hundredths being estimated. The ground was a grassy field, about half the line being measured up the side of a steep hill. The precision of this measurement, about $\frac{1}{8320}$, is probably higher than can generally be expected. With a steel tape leveled by eye and held over a line of stakes previously aligned by eye, the difference between any two measurements should not be greater than $\frac{1}{5000}$ part of the whole length. To secure this precision, however, the ends of the tape and all points of breaking chain should be carefully transferred to the top of the stakes by means of a plumb-bob. This method of linear measurement is probably the best to be employed in general transit surveying.

For work of a considerably higher degree of precision involving a still closer agreement between any two separate determinations of the same line, it is necessary that all the measurements should be made with the tape at, or reduced to, the same temperature, pull, and perfect horizontality. Furthermore, if the *absolute* length of the line is to be determined, there must be known the temperature and pull for which the tape is standard.

For precision of $\frac{1}{17000}$ the following method is followed: † The tape is allowed to rest on stakes carefully lined in with a transit and set on a uniform grade. The pull is measured with

* Report for 1897–8, p. 256.

† See Raymond's Plane Surveying, Art. 23.

spring balances, and is the same at which the tape, by experiment when supported in the same way, has previously been found to be standard. The temperature of the tape is measured by one or more thermometers attached to it or the stakes on which it rests and should be known closely. The lines when measured are reduced to the horizontal and to the standard temperature to avoid the effect of wind. Measurements of this sort should be made in calm weather.

For careful work it is necessary to have a standard of length with which the tapes to be used can be frequently compared. The best way to obtain such a standard is to send a good 100-ft. steel tape to the U. S. Coast and Geodetic Survey at Washington, where, for a small fee, it can be tested and returned marked with the temperature and pull at which it is of standard length, both when supported throughout its entire length and when supported only at the ends. Having obtained such a standard, it is well to preserve it permanently by nailing down on a smooth floor two strips of zinc, stretching the standard tape along the floor between them at the standard tension, and cutting two fine lines just opposite the tape extremities on the zinc plates. The temperature of the tape during this operation should, of course, be standard. Having laid down these marks, to compare any other tape with the standard it will be necessary only to stretch the tape between the marks at the standard temperature and pull, and note the difference of length which can be measured by an auxiliary scale, preferably of wood. If it is difficult to have the tape to be compared at standard temperature, allowance for the difference can be made from the fact that the coefficient of linear expansion for steel is about 0.0000065. To determine the pull at which a tape, when supported in any particular manner, as every fifty feet, every twenty-five feet, etc., will be standard, arrange a line of supports, the required distance apart, on the floor between the zinc plates. Next fasten one extremity of the tape directly over the fine line on one end of the plates. This can be done with the aid of a plumb-bob and a little ingenuity. Next, allowing the tape to rest on the supports, the reading of a spring balance attached at the other end and pulled just

enough to allow the plumb-bob fastened there to hang over the other mark, will give the tension required.

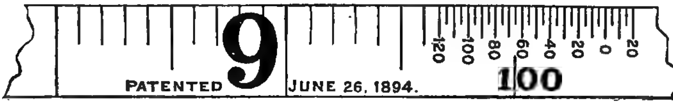


FIG. 175.

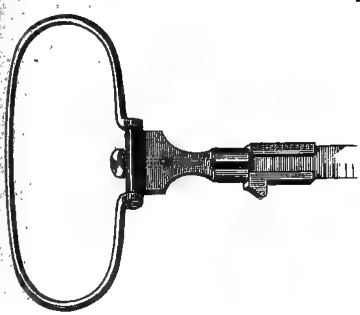


FIG. 176.

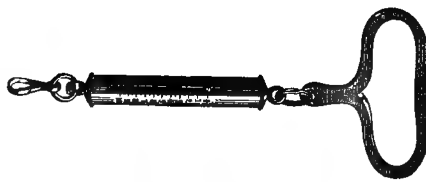


FIG. 177.

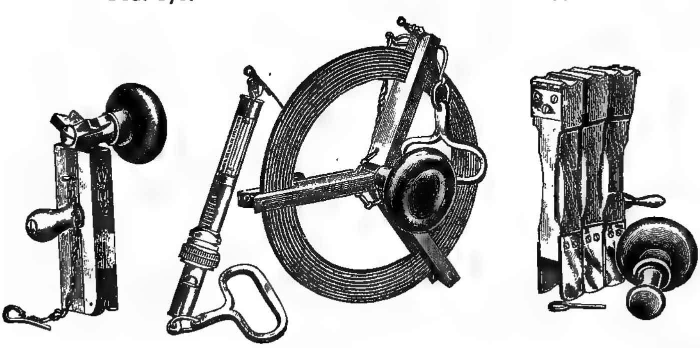


FIG. 178.

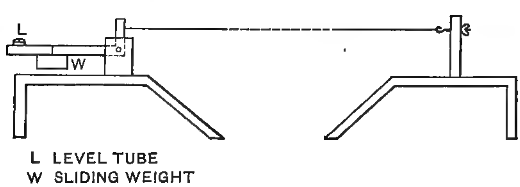


FIG. 179.

Some tapes have temperature scales at their ends, middle points, etc., as shown in Fig. 175. The compensatory handle,

Fig. 176, serves the same purpose. The handle shown in Fig. 177 has an attached spring balance by which the pull can be measured. To aid in holding the tape horizontal these handles sometimes contain also a bubble tube as in Fig. 178. Fig. 179 gives an idea of the tension frame designed by Prof. W. L. Webb of the University of Pennsylvania to obtain the pull in very accurate measurements. Note that the tension is determined by a lever and not by a spring.

108. City Surveying.—City surveying differs from ordinary transit surveying only in the precision necessary to be attained. The linear measurements should be made with a precision of $\frac{1}{10000}$ as described in the last article. The precision of the angular measurements should correspond, and this condition will obtain with a horizontal circle reading to twelve seconds. This circle would necessitate an inconvenient graduation, so that a vernier reading to ten seconds is probably the best for the purpose. Figs. 180, 181, and 182 show some good forms of city transits.

In any particular piece of work, the engineer's judgment will have to be exercised to decide, first, the precision necessary, and, second, the methods to be employed for its attainment.

The methods of keeping the office records, etc., will differ with different engineers. Ordinarily, completeness and readiness of reference are the chief features to be desired.

The marking of street and other city lines by permanent monuments is a matter of the utmost importance. Stone monuments about two and a half feet long by six inches square serve this purpose well. The top can be of circular cross section about five inches in diameter, the exact point being marked by a drilled hole. They should be set deep enough to avoid the disturbing action of frost. A good method is to set the stone with its top two feet below the surface, with a terra cotta or iron pipe leading up. Monuments are generally placed at the intersection of the side, and not the center, lines of streets. In the city of Syracuse, N. Y., the present practice is to set them in the street six feet from the side lines.

In laying out a town site or dividing any tract into blocks

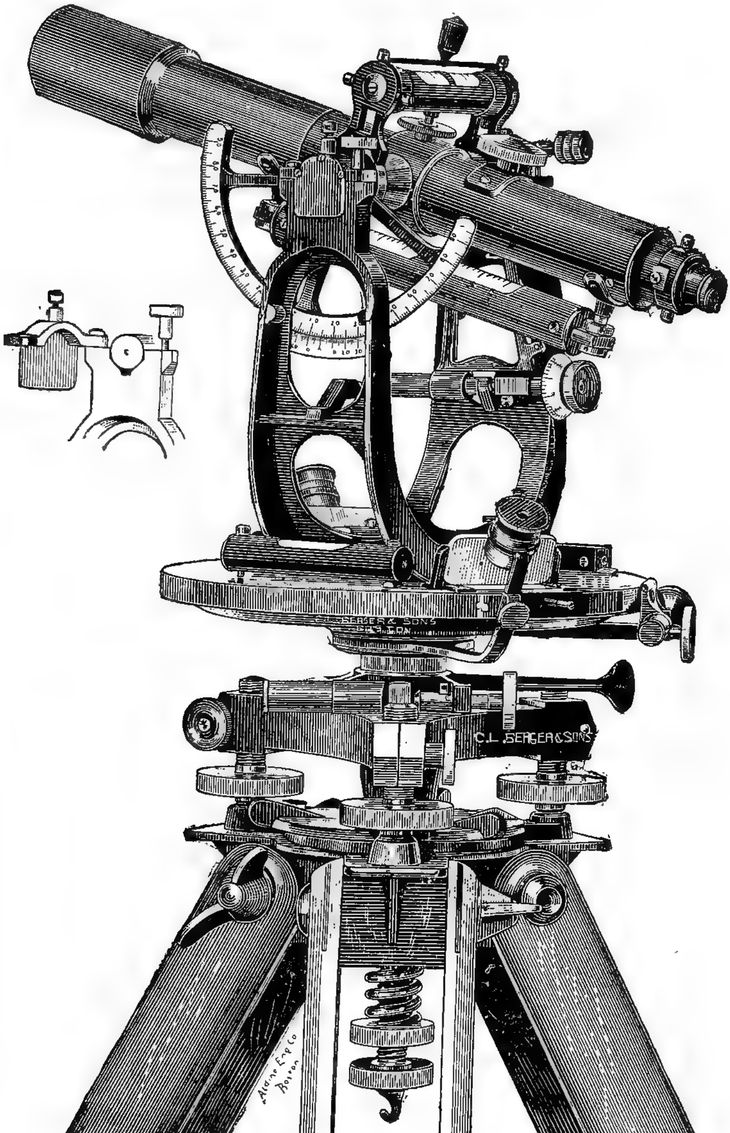


FIG. 180.

and lots, it will be necessary before deciding on the plan for subdivision to make or obtain a map showing the entire bound-

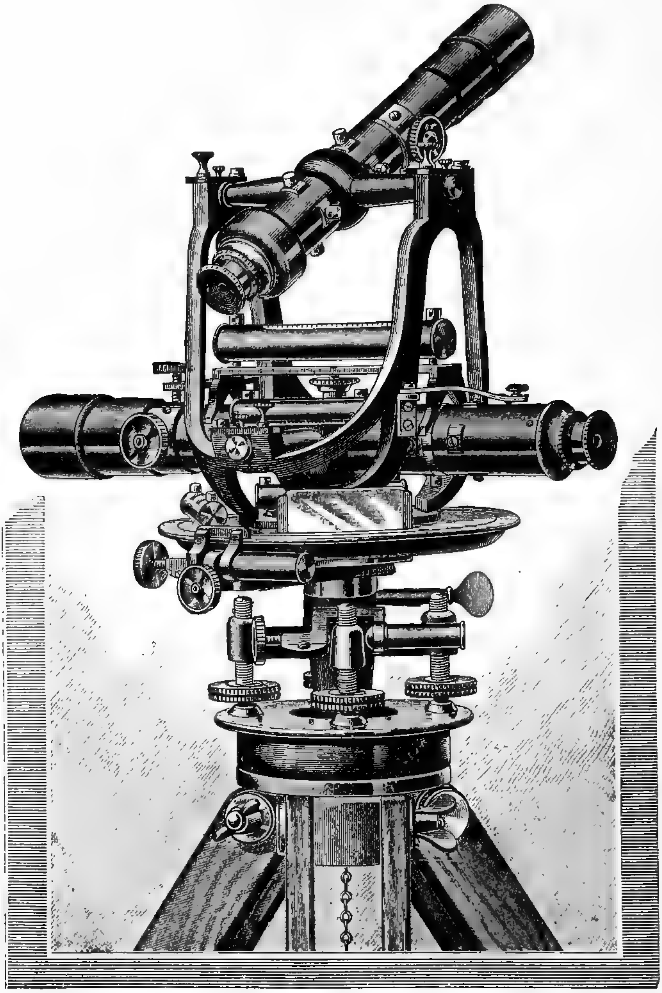


FIG. 181.

ary of the tract. Then, having decided on the plan, run around the boundary lines, setting all monuments and stakes to be located there. The interior lines, from one marked point to another, should be run last.

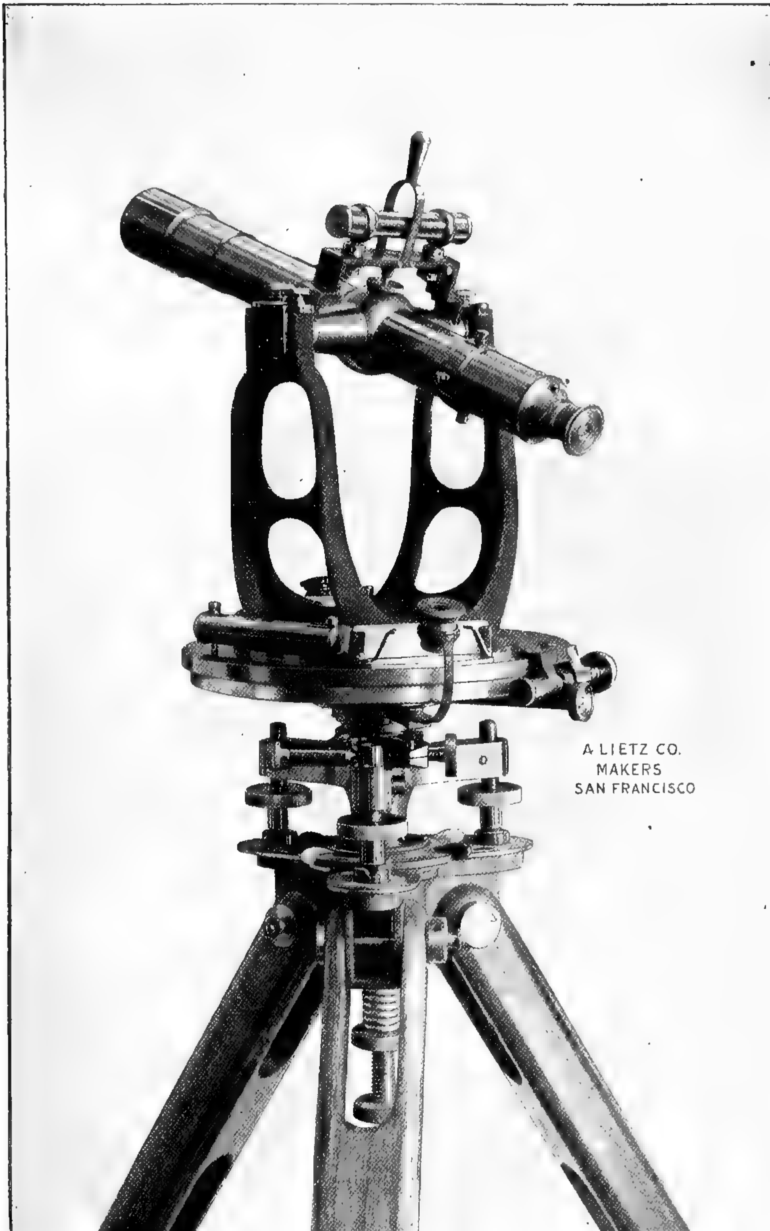


FIG. 182.

109. Conditions Necessary in Transit for Correct Work.—In the "Manual of Engineers' and Surveyors' Instruments," published by Queen & Co., are stated 24 conditions that must be fulfilled before correct work can be done with the transit. It should be noted, however, that some of these conditions are connected with the attached compass and level by means of which the transit can be used in place of those instruments. The further statement is made that these conditions are *some* of the more prominent ones. Generally speaking, for the correct measurement of horizontal and vertical angles, it is necessary that

(1) The line of sight piercing the horizontal and vertical axes at their intersection in revolving around the horizontal axis should generate a vertical plane.

(2) All motions around the horizontal and vertical axes should be truly circular.

(3) The graduated circles should be accurately centered on, and have their planes perpendicular to their respective axes.

(4) The graduation should be uniform.

110. Tests of the Transit.—(1) The test of the graduation. Unfortunately there is no reliable method easily employed by the engineer to test the uniformity of the graduation. The best that can be done is to note whether the entire length of the vernier equals the same number of divisions on the circle at all points around the circumference. While not an entirely satisfactory test, still, if no error can be detected in this way, the graduation is probably accurate enough for plane surveying purposes.

(2) The test of the circles for eccentricity. This corresponds generally to the test preceding the 7th adjustment of the compass. In Fig. 183 let AXY represent a graduated circle with center at C'. The vernier V revolves around the axis of rotation C, and having moved through any angle as VCV' the incorrect angle read is $XC'Y$. To test for eccentricity, in the case of the horizontal circle, note the readings of both verniers, turn them through about 90° and again take the readings. If both indicate the same angular movement there is no eccentricity of the circle. Otherwise the centering has been inaccurate.

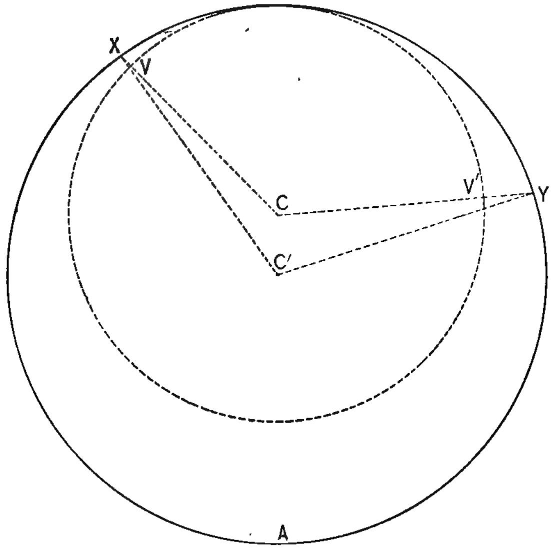


FIG. 183.

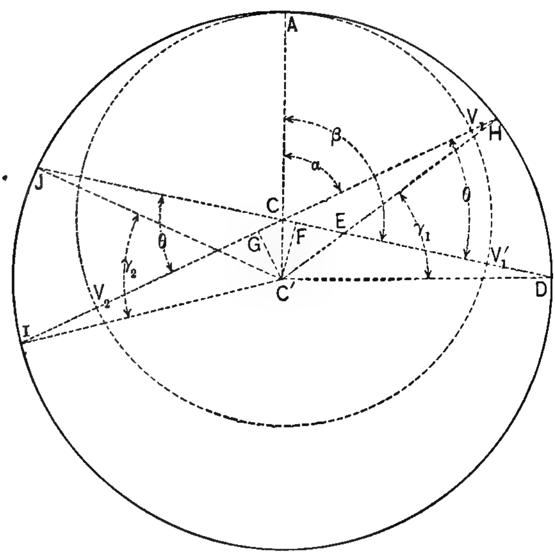


FIG. 184.

In Art. 99 it was stated that reading both verniers eliminated any error due to eccentricity of the circle. This was on the supposition that the two verniers were situated directly opposite each other on a line passing through the center of rotation. The truth of this statement may now be proved as follows: In Fig. 184 call the line AC' passing through the center of the circle C', and the center of rotation C, around which the vernier V_1 revolves, the "reference line." Suppose the vernier moved through any angle $V_1CV_1' = \theta$. Call the angle ACV_1, α, and the angle ACV_1', β, and the angle read on the circle $HC'D$, γ_1.

$$HED = HCE + CHE = \theta + CHE. \quad . \quad . \quad . \quad (78)$$

Also

$$HED = EC'D + EDC' = \gamma_1 + EDC'. \quad . \quad . \quad (79)$$

From which

$$\theta + CHE = \gamma_1 + EDC',$$

and

$$\theta - \gamma_1 = EDC' - CHE. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

Letting R = the radius of the circle DC' and e the eccentricity CC',

$$EDC' = \sin^{-1}\frac{C'F}{DC'}$$

$$= \sin^{-1}\frac{e \sin \beta}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

Also

$$CHE = \sin^{-1}\frac{e \sin \alpha}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

Substituting these values in (80)

$$\theta - \gamma_1 = \sin^{-1}\frac{e \sin \beta}{R} - \sin^{-1}\frac{e \sin \alpha}{R}. \quad . \quad . \quad . \quad (83)$$

In ordinarily good instruments the distance e will be very small, and the angles CHE and EDC' so small that we may

substitute for them their sines. Making this change in (83), we have

$$\theta - \gamma_1 = \frac{e \sin \beta}{R} - \frac{e \sin \alpha}{R} = \frac{e}{R}(\sin \beta - \sin \alpha), \quad . \quad (84)$$

showing that the angular error due to eccentricity of the circle varies directly as the eccentricity, inversely as the radius of the circle and directly as the difference between the sines of the angles β and α. If we have two verniers V_1 and V_2 situated directly opposite to each other on a line passing through the center of rotation the angle read on the circle by V_1 will be

$$\gamma_1 = \theta - \frac{e}{R}(\sin \beta - \sin \alpha), \quad . \quad . \quad . \quad . \quad . \quad (85)$$

and that by V_2

$$\gamma_2 = \theta + \frac{e}{R}(\sin \beta - \sin \alpha). \quad . \quad . \quad . \quad . \quad (86)$$

Adding (85) and (86), and solving for θ, we have

$$\theta = \frac{\gamma_1 + \gamma_2}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

showing that the true angle of revolution is the mean of the two determined by the verniers.

On the vertical circle, generally, there is only one vernier. In such a case the above test cannot be made, nor the correct angle be obtained by taking the average value.*

(3) The test for the verniers being 180° apart. If the tests of the graduation and for the eccentricity of the circle have been fulfilled, a reading of both verniers will show at once whether they are 180° apart. Indeed, if these two tests are fulfilled there is no need for more than one vernier. Suppose, however, that the graduation has been found satisfactory, but a small eccentricity discovered. In such a case the test is similar to the test preceding the 6th adjustment of the compass.

(4) The test for the parallelism of the vertical axes. By the method of the first adjustment of the next article (and the first

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* See Art. 100.

adjustment of Art. 52) make the plate levels or either one of them perpendicular to either the inner or the outer vertical axis. If the test (of the same adjustment) is now fulfilled with reference to the other axis, the axes are parallel. If the axes are not parallel and one is made vertical, then revolution around the other will produce an error in the angle as described in Art. 52, 1.

(5) The test for the perpendicularity of the plane of the horizontal graduation and the vertical axes. The axes having been found parallel as in the last test, this test can be made as in Art. 51, 4.

(6) The test of the magnification in comparison with the verniers, the plate levels, and the level under the telescope. These tests are all similar to the fourth test of the wye level, Art. 82, and can be made in about the same way.

111. Adjustments of the Transit.—(1) The adjustment to make the tangents of the plate levels perpendicular to the vertical axis. This adjustment is the same as the first adjustment of the compass, Art. 52. (Note also Art. 110, 4.)

(2) The adjustment to make the line of sight intersect the horizontal and vertical axes. This adjustment is the same as the first adjustment of Art. 67. If the condition is not fulfilled errors will be introduced as in Art. 51, 2.

(3) The adjustment to make the line of sight perpendicular to the horizontal axis. This adjustment is the same as the second adjustment of Art. 67. If the condition is not fulfilled errors will be introduced similar to those described in Art. 52, 2. Owing to the non-planar motion of the line of sight in revolving around the horizontal axis, errors will be introduced in the measurement of horizontal angles between high and low points.

(4) The adjustment to make the transverse axis truly horizontal. Test. Set up the instrument and direct the line of sight to some high point, as the top of a steeple. Depress the telescope and mark with a pin the point on the ground cut by the line of sight. Next, reverse in altitude and azimuth, again direct the line of sight to the high point, depress the telescope and mark a second point on the ground. If the first and second points coincide, no adjustment is needed.

Adjustment. If not, by means of the capstan-headed screw on one of the standards alluded to in Art. 96, and shown particularly in Fig. 185, raise or lower the end of the horizontal axis until the line of sight, on being directed to the high point and turned down, cuts a point between the two previously marked, conveniently (not *necessarily*) about midway. The work may be expedited here, if the high point has been taken on the corner of the building, by noting whether the intersection of the cross hairs, as the line of sight is turned down, follows the edge of the building. A long plumb-line will serve the same purpose.

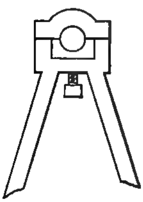


FIG. 185.

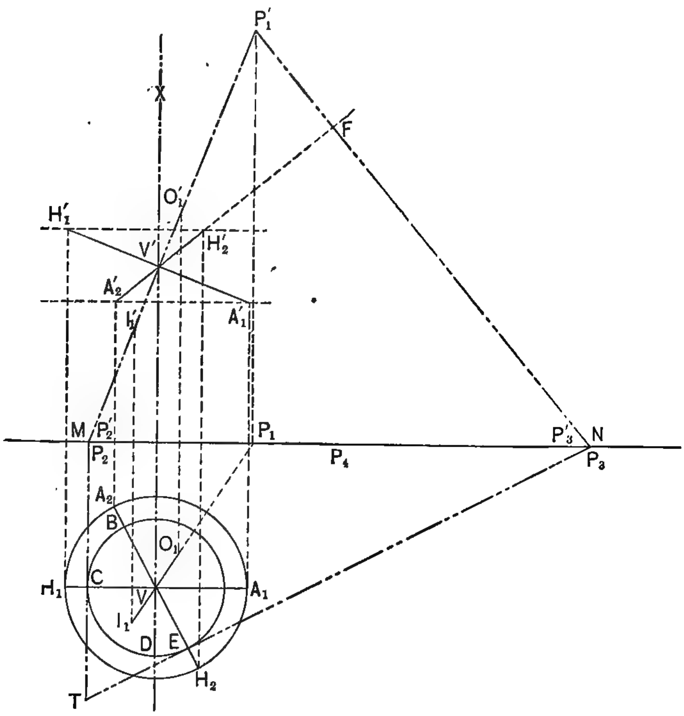


FIG. 186.

Explanation. In Fig. 186, I_1O_1 represents the line of sight in its first position directed to the high point P_1. The hori-

zontal axis is shown parallel with the vertical plane of projection in the position $H_1 A_1$. As the line of sight has, by the third adjustment, been made perpendicular to the horizontal axis, as it revolves around that line, it generates a plane whose horizontal and vertical traces are respectively TM and MP_1'. As the line of sight is turned down, it finally marks the point P_2 whose horizontal and vertical projections are both shown coinciding with M. As the plane generated by the line of sight is perpendicular to HA, its traces will always be perpendicular to the projections of that line.* When, therefore, the instrument is revolved in azimuth and altitude until the line of sight again points to P_1, the traces of the line of sight plane will be TN and NP_1' perpendicular respectively to the new projections, A_2H_2 and $A_2'H_2'$, of the horizontal axis.† As the line of sight is now again turned down, it cuts the vertical projection plane in the line $P_1'FN$, finally marking the second low point P_3 projected both horizontally and vertically at N. Obviously, if the horizontal axis had been truly horizontal, as the line of sight was turned down, it would have cut the vertical plane in the vertical line $P_1'P_1$. P_1 is approximately, in actual practice always much more nearly so than shown in the figure, half way between P_2 and P_3. (If the point P_1 had been chosen in the same horizontal plane with V, or anywhere on the vertical line $V'X$, its horizontal projection would have been *exactly* half way between P_2 and P_3.) Consequently we choose a point P_4 half way between P_2 and P_3 and adjust the horizontal axis until the line of sight on being turned down from P_1 cuts P_4. This will have the effect of making the horizontal axis *more nearly* horizontal, and repeated tests and adjustments will make it practically so. It should be noted that this discussion assumes that the horizontal axis, as it is adjusted, revolves about the point V, whereas in actual practice it revolves

* Church's Descriptive Geometry, Art. 43.

† To construct Fig. 186, first assume the point P_1. Next find the horizontal trace T of the line of sight P_1V and draw the circle V–BCD tangent to MT. From T draw TE tangent to BCD at E and intersecting the ground line at N. Join N and P_1, giving the vertical trace of the line-of-sight plane in its second position. The construction is then completed by drawing $A_2'H_2'$ and A_2H_2 perpendicular respectively to NP_1' and NT.

about either H or A. The difference is too slight to cause material error in the proof.

(5) The adjustment to make parallel with the line of sight the tangent of the bubble tube attached to the telescope. This adjustment is the same as the second adjustment of the level by the second general method, Art. 84, the part relating to the rotation of the telescope in the wyes being omitted.

(6) The adjustment to make the vertical circle read zero when the line of sight is horizontal. There are two methods. The first applies when the telescope has an attached level, as in the preceding adjustment. If such is the case, having made the fifth adjustment, bring the bubble to the center, clamp it in that position, and note whether the vernier of the vertical circle reads zero. If not, having loosened the screws which hold the vernier, slide it until the zero reading is given.

By the second method, proceed just as in the second adjustment of the wye level by the second general method, Art. 84, until the line of sight has been made truly horizontal. Then, with the telescope clamped in this position, if the vertical circle vernier does not read zero, move it until it does, and tighten the screws sufficiently to hold it firm.

In the case where the vertical circle is a *full circle* and not merely an arc, let the student devise a third method of making this adjustment.

In some instruments the vertical circle vernier may not be movable. In such a case, the error can be noted only and applied as a correction to all vertical angles.

112. Remarks on Adjustments.—Instruments accurately adjusted and carefully handled should not lose their adjustment easily. Especial care should be exercised in handling the adjusting screws, as, in fact, all the screws connected with the instrument. They should be turned to a firm bearing only. Otherwise the threads are apt to be stripped and the instrument injured permanently. A valuable discussion of the errors due to general lack of adjustment in the transit may be found in the Manual of Engineers' Instruments, published by Queen & Co., Philadelphia.

CHAPTER VII.

THE PLANIMETER AND THE SLIDE RULE.

SECTION I.

*The Planimeter.**

By C. W. CROCKETT, A.M., C.E., Professor of Mathematics and Astronomy Rensselaer Polytechnic Institute.

113. Descriptive.—The planimeter is an instrument used for measuring areas bounded by irregular curves. It is made in many different forms, but in every case a pointer is moved around the border of the area, causing motion in the mechanism, which results in the rotation of a wheel; from the amount of this rotation we are able to determine the area of the curve.

In the ordinary form, Fig. 187 (which is discussed in Arts.

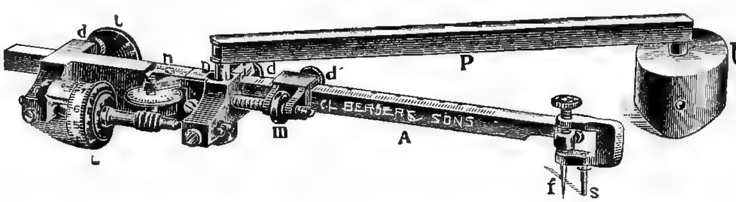


FIG. 187.

114 *et seq.*), the wheel moves over the drawing-paper and it is made to rotate by the friction between it and the paper, while the pointer moves around the curve. In the "suspended disk planimeter," Fig. 188, the wheel R moves on a circular disk S instead of on the drawing-paper, this disk being rotated, as the pointer f and the arm A move around the weight P, by the

* Revised from *The Polytechnic*, Jan. 28, 1893.

gear r which engages in the toothed rim of P. The "suspended ball planimeter" uses a spherical surface instead of a disk, the wheel pressing against the sphere while the latter is rotated by a horizontal axis turned by a gear engaging in a crown-wheel which is fastened to the weight P.

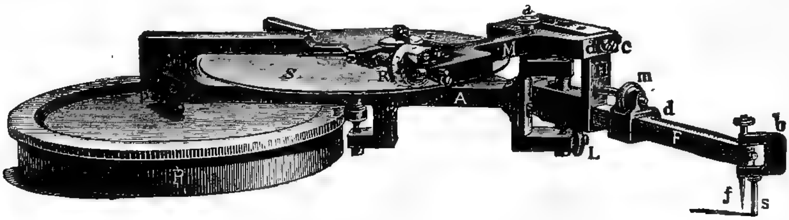


FIG. 188.

In these instruments the mechanism moves around a fixed center (P in Fig. 188) and the path of the pivot of the pointer arm is a circle. If we conceive that P is removed to an infinite distance, the path of the pivot becomes a straight line which may be made in the form of a metallic guide, as in Fig. 189,

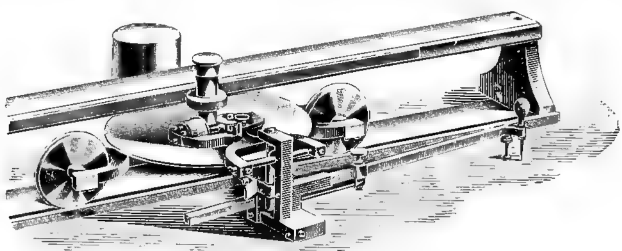


FIG. 189.

which represents the corresponding modification of Fig. 188, the toothed rim of P being replaced by the rack on the side of the top piece of the framework.

The "rolling ball planimeter," Fig. 190, consists of a framework BB resting upon the two equal and coaxial rollers R', which move over the paper as the pointer f moves around the curve, the motion of the left roller being transmitted by gearing to the horizontal axis of the spherical segment K. This segment presses against a cylinder, whose axis is parallel to

the arm F, and by friction causes it to rotate, the wheel in the preceding forms being here replaced by the cylinder. This instrument may also be made with metallic guides similar to

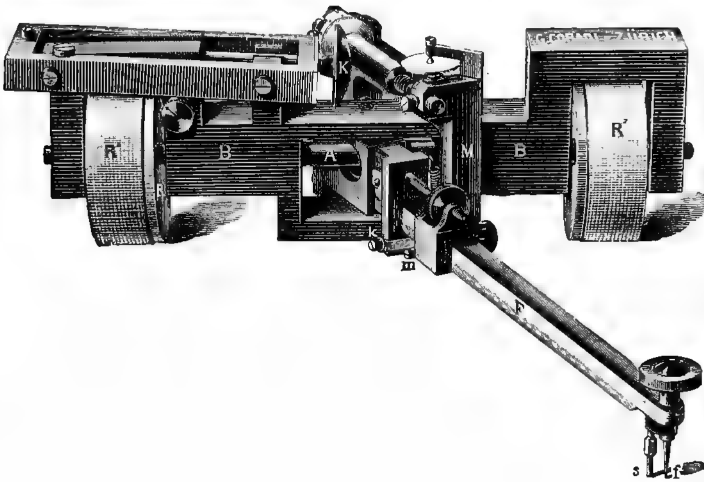


FIG. 190.

Fig. 189, to eliminate the errors due to the inequalities in the drawing-board and paper.

Theory of Amsler's Polar Planimeter.

114. Amsler's Polar Planimeter (Fig. 191) consists essentially of a bar NF, pivoted at F to the frame E carrying a wheel W with a horizontal axis. A second bar AG, that fits in and may be made to slide along the sleeve E, is graduated and provided with a clamp and slow motion screw (m and n) so that a pointer at A may be set at different distances from F. The pointer A, the pivot F, and the axis of the wheel should be in the same vertical plane, and the instrument is constructed so that W cannot pass between N and F. When in use the planimeter rests on the paper at three points—one on the circumference of the wheel and the others at N and A, the point N being fixed. When the pointer A is made to follow an irregular curve drawn on the paper, the wheel will be made to

rotate by its friction on the paper, having both a sliding and a rotary motion. To measure the amount of rotation the circumference of the wheel is divided into 100 equal parts and by

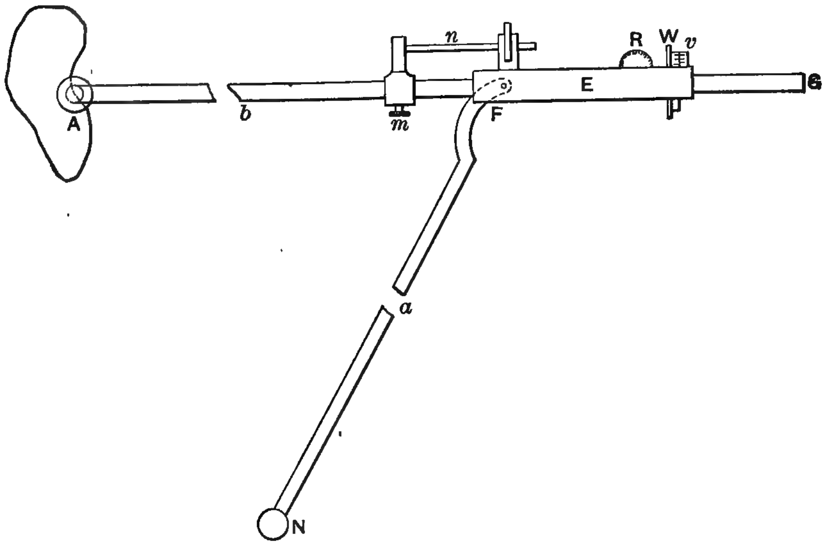


FIG. 191.

the vernier v it is read to thousandths of a turn, the roller R showing the number of complete turns up to ten.

115. The Zero Circle.—If the pointer be made to follow a circle with its center at N, the triangle ANF (Fig. 192) will not be changed, and therefore any point on NF or AW will describe a circle with its center at N. When the pointer moves on the circle such that the plane of the wheel passes through N, the wheel will describe a circle whose tangent will take the direction AW, so that the motion of the wheel will be in a direction perpendicular to its plane and it will not rotate. This circle, described by the pointer without rotation of the wheel, is called the *zero circle*, and its radius is, from geometry,

$$r_0 = \sqrt{a^2 + b^2 + 2bc}, \quad . \quad . \quad . \quad . \quad . \quad (88)$$

where $a = NF$, $b = AF$, and $c =$ projection of a on $b = FW =$ the distance from F to the plane of the rim of the wheel. The

distance $AF = b$ may be changed but a and c are constants; hence the radius of the zero circle will vary with b.

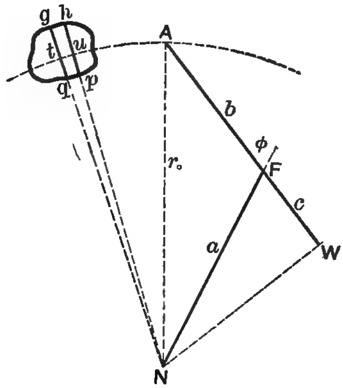


FIG. 192.

116. Direction of Rotation.—In Figs. 193 and 194 let the angle $ANB = d\theta$, AB and CD being circular arcs with their centers at N, and let $NG = r_0$ = radius of the zero circle.

When AB is *outside* of the zero circle (Fig. 193) the pointer moving over the infinitesimal arc AB from A to B (or clock-

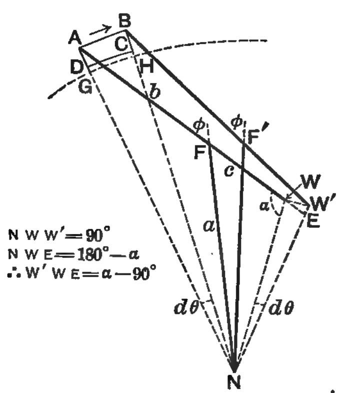


FIG. 193.

wise) will cause the wheel to move from W to W'. This motion may be resolved into WE, perpendicular to the plane of the wheel and not causing it to rotate, and EW', parallel to the plane of the wheel and causing rotation. Moving the wheel

over EW' will cause it to rotate clock-wise (looking in the direction WA) and will cause the reading of the wheel to increase. If the pointer were moved over BA from B to A (or counter-clock-wise) the wheel would rotate by the same amount, but in the opposite direction.

When AB is *within* the zero circle (Fig. 194) the motion over AB will cause the wheel to move from W to W', resolvable into WE and EW'. The motion of the wheel over EW'

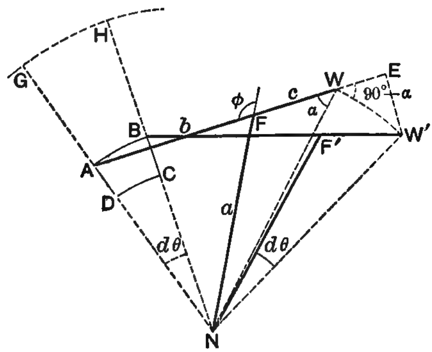


FIG. 194.

will cause it to rotate counter-clock-wise (looking in the direction WA). When the pointer is moved from B to A the wheel will rotate clock-wise.

Hence, when the pointer is moved clock-wise over a circular arc with its center at N,—

> *arc outside the zero circle, rotation is clock-wise;*
> *arc within the zero circle, rotation is counter-clock-wise.*

When the pointer is moved counter-clock-wise the direction of the rotation of the wheel is reversed.

We shall call *clock-wise rotation positive*, since it increases the reading of the wheel, and hence counter-clock-wise rotation will be negative.

117. Connection between Rotation and Area.—Since any area may be considered as being composed of an infinite number of infinitesimal elements similar to $ABCD$, the polar element of area, in Figs. 193 and 194, we shall consider the effect upon the wheel of moving the pointer over the perimeter of

$ABCD$—over the two radial lines BC and DA and over the arcs AB and CD.

1. If the pointer be moved over BC the wheel will rotate by a certain amount, and if the pointer be now brought back over CB to B it will rotate by an equal amount in the contrary direction, thus bringing the wheel into its first position. Also if the pointer be moved from B to C and if the wheel be then lifted from the paper to prevent rotation while the pointer is moved to D, the angle between the arms a and b of the planimeter will be the same when the pointer is at D as it was when the pointer was at C. Hence moving the pointer over DA will have the same effect on the wheel as the motion over CB. Therefore the rotation of the wheel produced by moving the pointer over BC will be neutralized by that produced when the pointer is moved over DA, so that *in considering the rotation of the wheel caused by moving the pointer around the figure ABCD, we may neglect the radial lines and consider only the circular arcs.* Hence AD and BC are not necessarily infinitesimal, so that we may treat any plane area as being made up of an infinite number of infinitesimal parts, each bounded by radial lines from N inclosing an angle $d\theta$ and by arcs with their centers at N, as $ghpq$ in Fig. 192.

2. The arc AB may be *outside* of the zero circle or *within* it. Suppose the motion of the pointer over AB to be clockwise, and let the distance $NA = r$.

(*a*) *The arc outside the zero circle* (Fig. 193).

When the pointer is moved over the infinitesimal arc AB the wheel will describe a circular arc with its center at N and with the radius NW:

$$\therefore \text{ arc } WW' = NWd\theta.$$

The portion of this movement of the wheel that causes rotation is

$$\begin{aligned} EW' &= WW' \sin W'WE \\ &= WW' \sin(\alpha - 90^\circ) \\ &= -\,WW' \cos\alpha \\ &= -\,NW \cos\alpha d\theta. \end{aligned}$$

From the triangles NAW and NFW we have

$$r^2 = (b + c)^2 + NW^2 - 2(b + c)NW \cos \alpha$$

and $$a^2 = c^2 + NW^2 - 2cNW \cos \alpha.$$

Subtracting and reducing, we have

$$NW \cos \alpha = \frac{1}{2b}(a^2 + b^2 + 2bc - r^2)$$

$$= \frac{1}{2b}(r_0^2 - r^2)$$

$$\therefore \quad EW' = + \frac{1}{2b}(r^2 - r_0^2)d\theta, \quad . \quad . \quad . \quad . \quad (89)$$

where $r > r_0$.

(b) *The arc within the zero circle* (Fig. 194). The angle $W'WE$ is here equal to $90° - \alpha$.

$$\therefore \quad EW' = WW' \sin (90° - \alpha)$$
$$= WW' \cos \alpha = NW \cos \alpha d\theta;$$

and this by the method above gives

$$EW' = + \frac{1}{2b}(r_0^2 - r^2)\, d\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

where $r < r_0$.

(c) These values of EW' give the *amount* of rotation. Considering the direction of rotation (Art. 116) we have for the rotation of the wheel when the arc is either outside of or within the zero circle:

*pointer moved C. W.,**

$$\textbf{rotation} = + \frac{1}{2b}(r^2 - r_0^2)d\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

*pointer moved C. C. W.,**

$$\textbf{rotation} = - \frac{1}{2b}(r^2 - r_0^2)d\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

* *C. W.* and *C. C. W.* denote clock-wise and counter-clock-wise.

where the relative values of $\mathbf{r}$ and $\mathbf{r}_0$ will determine the *direction* of rotation.

In Figs. 193 and 194,

$$\text{area } ABHG = \tfrac{1}{2}(r^2 \sim r_0^2)\, d\theta;$$

hence the area between the arc AB and the zero circle is equal to the product of b by the amount of rotation caused when the pointer is moved over the arc AB. Eqs. (91) and (92) are the fundamental equations of the instrument.

3. The rotation produced by moving the pointer over the periphery of any figure will be the sum of the rotations produced by moving the pointer around the infinite number of its infinitesimal portions (*gh pq*, Fig. 192). For in moving the pointer around the figure *abcdhgfe* (Fig. 195), if we represent the rotation due to *ab* by $\overline{ab}$, the total rotation will be

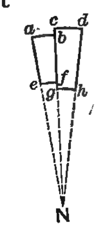


FIG. 195.

$$\overline{ab} + \overline{bc} + \overline{cd} + \overline{dh} + \overline{hg} + \overline{gf} + \overline{fe} + \overline{ea} = (\overline{ab} + \overline{bf} + \overline{fe} + \overline{ea}) + (\overline{bc} + \overline{cd} + \overline{dh} + \overline{hg} + \overline{gf} + \overline{fb}),$$

since $\overline{bf}$ will be neutralized by $\overline{fb}$.

$$\therefore \quad \text{Total rotation} = \overline{abfea} + \overline{bcdhgfb}$$
$$= \overline{ab} + \overline{cd} + \overline{hg} + \overline{fe},$$

the same as if the pointer were moved only over the circular arcs.

118. Area of any Curve.—In using the instrument the needle-point *N* may be placed either outside of the area to be measured or within it, and there are three cases in each position:

(1) Periphery entirely outside of the zero circle.

(2) Periphery entirely within the zero circle.

(3) Periphery partly outside of and partly within the zero circle.

(*A*) Needle-point outside of the area. Considering the pointer as starting from *A* and going around *ABCD* from *A* to *B* to *C* to *D* to *A* we have the following results:

1. The periphery entirely outside of the zero circle (Fig. 193).

Rotation corresponding to AB

$$= +(NA^2 - r_0^2)\frac{d\theta}{2b}.$$

Rotation corresponding to CD

$$= -(ND - r_0^2)\frac{d\theta}{2b}.$$

$\therefore$ Resultant rotation

$$= (NA^2 - ND^2)\frac{d\theta}{2b}.$$

But area $ABCD$

$$= (NA^2 - ND^2)\frac{d\theta}{2}.$$

$\therefore$ **area = b × rotation.** . . . . (93)

2. The periphery entirely within the zero circle (Fig. 194).
Rotation corresponding to AB

$$= +(NA^2 - r_0^2)\frac{d\theta}{2b}.$$

Rotation corresponding to CD

$$= -(ND^2 - r_0^2)\frac{d\theta}{2b}.$$

$\therefore$ Resultant rotation

$$= (NA^2 - ND^2)\frac{d\theta}{2b}.$$

$\therefore$ **area = b × rotation.** . . . . (93)

3. The periphery partly outside of and partly within the zero circle (Fig. 196).

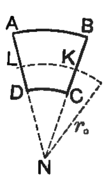


FIG. 196.

Rotation corresponding to AB

$$= +(NA^2 - r_0^2)\frac{d\theta}{2b}.$$

Rotation corresponding to CD

$$= -(ND^2 - r_0^2)\frac{d\theta}{2b}.$$

$\therefore$ Resultant rotation

$$= (NA^2 - ND^2)\frac{d\theta}{2b}.$$

$\therefore$ **area = b × rotation.** . . . . (93)

4. Therefore, in all cases when the needle-point is outside of the area, we have

$$\text{area} = b \times \text{rotation}.$$

If the circumference of the wheel be denoted by s and the number of turns by n, we have

$$\text{rotation} = sn$$

and

$$\textbf{area} \quad = \textbf{bsn}, \quad \ldots \ldots \quad (94)$$

where n is always positive if the pointer is moved clock-wise around the figure.

(*B*) Needle-point within the area.

1. The periphery entirely outside of the zero circle (Fig. 197).

Moving the pointer from A and around $ABLK$, we have

Rotation corresponding to AB

$$= + (NA^2 - r_0^2)\frac{d\theta}{2b}.$$

Rotation corresponding to LK

$$= 0.$$

$\therefore$ Resultant rotation

$$= (NA^2 - r_0^2)\frac{d\theta}{2b}.$$

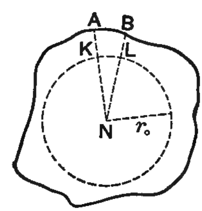


FIG. 197.

But area $ABLK$

$$= (NA^2 - r_0^2)\frac{d\theta}{2}.$$

$$\therefore \quad \text{area } ABLK = b \times \text{rotation}.$$

Hence the area between the curve and the zero circle = $b \times$ rotation, and to get the total area we must add πr_0^2, the area of the zero circle.

$$\therefore \quad \textbf{area} = \pi \mathbf{r}_0^2 + \textbf{bsn}, \quad \ldots \ldots \quad (95)$$

where n will be positive for clock-wise motion of the pointer.

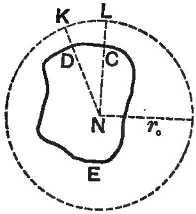


FIG. 198.

2. The periphery entirely within the zero circle (Fig. 198).

Moving the pointer clock-wise around *DCE* (or counter-clock-wise around *DCLK*, we have

Rotation corresponding to *DC*

$$= + (ND - r_0^2)\frac{d\theta}{2b}.$$

Rotation corresponding to *LK*

$$= 0.$$

$\therefore$ Resultant rotation

$$= (ND^2 - r_0^2)\frac{d\theta}{2b}.*$$

But

$$\text{area } DCLK = (r_0^2 - ND^2)\frac{d\theta}{2} = - b \times \text{rotation}.$$

But the rotation $= sn$, where n is a negative quantity, the reading of the wheel having decreased.

$$\therefore \quad \text{area } DCLK = - bsn.$$

Hence the area between the curve and the zero circle

$$= - b \times \text{rotation} = - bsn,$$

and to get the area *DCE* we must subtract the area between the curve and the zero circle from the area of the zero circle.

$$\therefore \quad \text{area } DCE = \pi r_0^2 - (- bsn)$$

$$\therefore \quad \textbf{area} = \pi \mathbf{r_0^2 + bsn}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

where n will be negative for clock-wise motion of the pointer.

* A negative quantity since $ND < r_0$.

3. The periphery partly outside of and partly within the zero circle (Fig. 199).

Moving the pointer from E to C along $EFGABC$ will give a positive rotation equal to

$$\text{area } EFGABCLK \div b.$$

Moving the pointer from C to E along CDE will give a negative rotation equal to

$$\text{area } CDEM \div b.$$

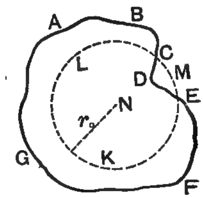


FIG. 199.

The resulting rotation will be positive or negative according as the area $EFGABCLK$ or the area $CDEM$ is the greater; and the area of the curve will be, for clock-wise motion of the pointer,

$$\textbf{area} = \pi \mathbf{r}_0^2 + \mathbf{bsn}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

where n is positive or negative according as the final reading of the wheel is greater or less than the initial reading.

(*C*) Summary.

Hence we have for clock-wise motion of the pointer around the figure:

1. Needle-point outside of the area,

$$\textbf{area} = \mathbf{bsn}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

where n is always positive.

2. Needle-point within the area,

$$\textbf{area} = \pi \mathbf{r}_0^2 + \mathbf{bsn}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

where n is positive or negative according as the reading of the wheel has increased or decreased.

119. Value of b.—The following dimensions of a planimeter were furnished by the makers:

$$s = 2.370 \text{ in.}; \; a = 6.214 \text{ in.}; \; c = 1.343 \text{ in.}$$

In this instrument the zero of the divided arm FA is found at A and the index mark on the frame that carries the wheel is opposite to F, so that the setting of the arm is the value of b.

1. To find b so that the *area of the diagram* may be found to hundredths of a square inch.

Then 0.01 sq. in. will correspond to one unit on the wheel-vernier ($n = 0.001$). Since $A = bsn$ (Eq. 96), we have

$$0.01 = b \times 2.370 \times 0.001.$$
$$\therefore \quad b = 4.219 \text{ in.}$$

Therefore if the bar FA is set at 4.219 in. and if the pointer is then made to pass over the periphery of an area, the rotation of the wheel will give the area in hundredths of a square inch, each vernier-unit representing 0.01 sq. in.

2. Suppose that the *diagram* represents a *figure* on the scale of 1 : 96, and that we wish to find the area of the *figure* in square feet. By setting the arm at 4.219 in., as before, we could find the area of the *diagram* in square inches, and then compute the area of the figure. We may, however, proceed as follows:

1 sq. ft. of the actual area corresponds to $1 \div 96^2$ sq. ft. of the diagram or to $144 \div 96^2$ sq. in. Therefore, since we wish each vernier-unit to correspond to 1 sq. ft. of the *figure*, $144 \div 96^2$ sq. in. of the *diagram* must correspond to one vernier-unit.

$$\therefore \quad 144 \div 96^2 = b \times 2.370 \times 0.001.$$
$$\therefore \quad b = 6.593 \text{ in.}$$

In using the instrument a mark is made on the periphery, the pointer is placed there, and the reading of the wheel is found. The pointer is then moved around the periphery until it returns to the starting point, when a second reading of the wheel is made. Suppose that the first reading was 2.403 turns and the second 5.677; the rotation would then be 3.274 turns, or 3274 vernier-units. The area of the figure would be 3274×144 sq. in. or 3274 sq. ft.

3. Let the scale $= d$, and suppose that we wish each vernier-unit to correspond to y sq. in. of the actual figure. Then y sq. in. of the figure will correspond to yd^2 sq. in. of the diagram.

$$\therefore \quad yd^2 = bs \times 0.001;$$
$$\therefore \quad b = \frac{1000}{s} yd^2. \quad \ldots \ldots \ldots \quad (98)$$

Having found the value of n with this setting, multiply the number of vernier-units by y to get the area of the figure, since 0.001 of a turn corresponds to y sq. in.

Making $y = 0.01$ and $d = 1:1$, we have $b = 4.219$ as before.

Making $y = 144$ and $d = 1:96$, we have $b = 6.593$ as before.

From Eq. (98), since $1000 \div s$ is a constant, we see that b will be the same as long as yd^2 is the same. Hence the same setting may be used for different scales by assigning proper values to y. Thus, if $d = 1:96$ and $y = 144$, we have found $b = 6.593$. Here

$$yd^2 = \frac{144}{96^2} = \frac{1}{64}; \quad \text{or} \quad d = \frac{1}{8\sqrt{y}},$$

and $b = 6.593$ would be the setting for scales found by assigning values to y,—

$$\begin{array}{ll} y = 1 \text{ sq. in.}, & d = 1:8. \\ y = 4 \quad `` \quad `` , & d = 1:16. \\ y = 9 \quad `` \quad `` , & d = 1:24 \\ \ldots\ldots\ldots\ldots , & \ldots\ldots\ldots \\ y = 144 \quad `` \quad `` , & d = 1:96 \end{array}$$

and so on.

4. An application is to steam-engine indicator diagrams, where the mean pressure is the quantity sought. Suppose the length of the diagram is 4.17 in. and that the vertical scale is 40 lbs. to 1 in.

$$\therefore \quad M.\ P. = \frac{40 \times \text{area}}{4.17}.$$

If we wish to find the mean pressure to tenths of a pound, each vernier-unit will correspond to 0.1 lb. The area of the diagram corresponding to a mean pressure of 0.1 lb. will be

$$\frac{1}{10} = \frac{40 \times \text{area}}{4.17}, \quad \text{or} \quad \text{area} = 0.010425 \text{ sq. in.},$$

and we wish each vernier-unit to correspond to this area.

$$\therefore \quad 0.010425 = b \times 2.370 \times 0.001.$$

$$\therefore \quad b = 4.399.$$

5. If the values of s, a, and c are not given, the following method may be used when a trial disk or a circle with a known area is provided:

Let $\varDelta$ = area of circle, d = scale, and y = area of the figure represented by one vernier-unit, the areas being in square inches. The actual area of the figure represented by the circle is $\varDelta \div d^2$, and hence the number of vernier-units that would correspond to it is $\varDelta \div d^2y$.

Thus if the scale is 1 : 300 and y is 10 sq. ft. = 1440 sq. in., the number of vernier-units corresponding to the circle should be

$$\varDelta \div \frac{1440}{90000} = \varDelta \div 0.016.$$

By trial we would then find the setting so that when the pointer is moved around the circle the wheel will rotate through this number of vernier-units.

120. Hints.—The mark at which the pointer is first placed should be so taken that moving the pointer for a short distance along the curve shall cause little or no rotation of the wheel; for in this way the error due to the non-return of the pointer to its exact starting point is reduced to a minimum.

In following straight lines use a ruler.

Move the pointer slowly,—not so fast but that the figures on the wheel may be read,—to avoid effect of inertia.

It is better to place the needle-point outside of the area, as any uncertainty in the area of the zero circle is thus avoided. If the area is too large, divide it up by straight lines and find the area of the portions separately.

If the drawing is to a scale, when the needle-point is within the area, the area of the zero circle must be divided by d^2 in finding the area of the figure represented, the number of vernier-units of rotation being multiplied by y.

Test the planimeter by going around the trial disk in both directions and with the trial disk at different distances from the needle-point. When possible also use different circles for testing.

SECTION II.

The Slide Rule.

121. Description and General Theory.—Fig. 200 represents two divided straight edges, AB and CD, lying adjacent to each other along the line XY.

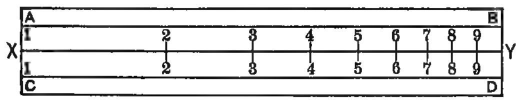


FIG. 200.

On each edge the distance 1—2 is laid off to scale to equal the mantissa of the logarithm of 2, 20, 200, 0.2, .02, etc., the distances 1—3, and 1—4 the mantissæ of the logarithms of 3, 30, etc., and of 4, 40, 400, and so on, the total length XY being equal to the logarithm of 10.

With such a device as this, suppose that it is desired to multiply together mechanically the numbers 2 and 3. Let either of the edges, as CD, be slid along the other AB, Fig. 201, until the division marked 1 on CD coincides with the

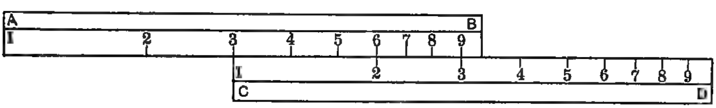


FIG. 201.

division marked 3 on AB. Then, looking along CD to the division 2, we find it in coincidence with the division on AB marked 6, which is the answer sought. We have simply performed the operation of mechanical logarithmic multiplication. Instead of the anti-logarithms, 1, 2, 3, etc., there might have been written at the divisions the logarithms of these numbers, in which case a table of logarithms would have had to be employed. The use of the device here described not

only enables logarithmic multiplication to be performed, but also dispenses with the use of a table of logarithms.

The operation of division, the inverse of multiplication, is as easily performed. Let it be desired to divide 8 by 4. The division 4 on *CD* is made to coincide with 8 on *AB*, Fig. 202.

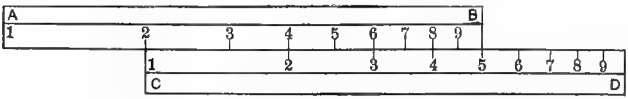


FIG. 202.

Division 1 on *CD* is then found opposite the answer 2 on *AB*. The above examples and illustrations explain the principle of the slide rule. Fig. 203 shows a cut of a form known as the

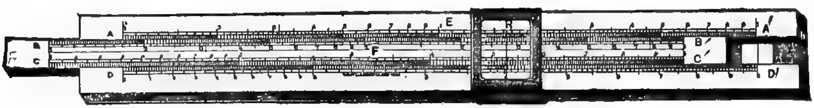


FIG. 203.

"Students' Slide Rule" made by the Keuffel & Esser Co. This form is, in general, similar to the well-known Mannheim rule, the graduations being identical in the two. The principles underlying the construction and use of all slide rules being the same, the remaining discussion will have special reference to this form.

The body of the rule or rule proper consists of two scales, AA' and DD', about 10 inches long, between which the slide BB' CC' runs. The scale AA' is divided into two equal parts, AE and EA', each of these being divided as in Fig. 200 in the directions A to E and E to A'. The scale DD' is like AA', except that its divisions are twice as long, so that 2 on DD' is opposite 4 on AE; 3 on DD' is opposite 9 on AE, and so on. The scales BB' and CC' are, respectively, exactly like AA' and DD'. The divisions on all of the scales are further subdivided than shown in Fig. 200 into ten parts such that the distance from division 1 to the 3d division between 3 and 4, for instance, will equal the mantissa of the logarithm of 33; from 1 to the 7th division between 1 and 2, the man-

tissa of the logarithm of 17 and so on. Where possible, these ten subdivisions are further subdivided in a similar manner. From which it will be seen that the operations of the slide rule are not confined to multiplications, divisions, etc., between quantities represented by single digits. As an illustration of this, let it be desired to multiply 375 by 216, using scales *AE* and *BF*. 1 of *BF* is brought opposite a point half way between the 7th and the 8th subdivisions between 3 and 4 on *AE*. Then, looking along *BF* to a point six tenths of the distance between the first and the second divisions between 2 and 3, this point is found opposite a point on *AE* just about at the first division between 8 and 9, giving the answer 81 + ciphers. Multiplying 375 by 216, in the usual way, we get the result 81,200. This makes evident the further facts that the slide rule gives no information as to the location of the decimal point, and that its results are uncertain in the third place of significant figures. This might have been supposed at first, as the divisions on the slide rule are proportional to mantissæ which have no reference to decimal points, the mantissa of 375, for instance, being the same as that of 37.5, 3.75, 0.375, 0.0375, and so on. This leads to another interesting fact. Let it be desired to multiply 9 by 5, using the scales *AE* and *BF*. 1 of *BF* is brought opposite to 5 of *AE*, but on looking along *BF* to division 9 that point has passed the end *E* of scale *AE* and is found opposite a point on the scale *EA'*. However, the point on *EA'* is found to be half way between divisions 4 and 5, giving the correct answer 45. That this is true in the *general case* results from the fact that the scales are divided proportionally, not to the logarithms of numbers, but to the *mantissæ* of these logarithms, and can be proved as follows:

Let x and y be any two numbers whose product is to be found by logarithmic multiplication. If the mantissæ of their logarithms be added together, there will result a plus quantity ranging in value from zero to anything less than 2. Further, in the case now considered, where the mantissæ, on being mechanically added, extend past the end E of the scale *AE*,

whose total length is equal to 1, the sum of the mantissæ will be 1 plus some decimal. In Fig. 204 AA' and BB' represent

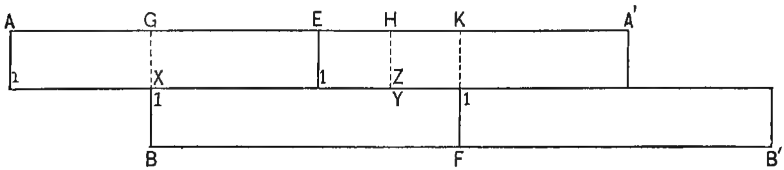


FIG. 204.

the scale and slide as in Fig. 203. Y having passed beyond E, the end of the scale AE is found opposite z, some point on EA'. We desire to prove that the distance EH is equal to the mantissa of the logarithm of the product of x and y. Remembering that the distances AE, EA', BF, and FB' are each to scale equal to 1.

$$
\begin{aligned}
A'K &= EG = 1 - \text{mantissa} \log x. \\
KH &= \quad\quad\; 1 - \text{mantissa} \log y. \\
EH &= 1 - (A'K - KH) \\
&= 1 - (1 - \text{mantissa} \log x + 1 - \text{mantissa} \log y). \\
&= -1 + (\text{mantissa} \log x + \text{mantissa} \log y) \\
&= -1 + 1 + \text{mantissa} \log xy. \\
&= \text{mantissa} \log xy.
\end{aligned}
$$

A similar method applies in cases in division where the result cannot be read on the scale used.

122. Use of the Runner.—The runner R shown in Fig. 203 consists of a piece of celluloid or other transparent material attached to guides running on the outer edges of the rule. A fine line is drawn on it perpendicular to the axis of the rule. By means of this, divisions can be transferred from the slide to the rule and *vice versa*. Thus, consider the following examples:

$$\frac{5 \times 3.5 \times 4}{6} = ?$$

Set 1 on BF opposite 5 on AE. Slide the runner down

until its line covers 3.5 on *BF*. Slide *BF* to the right until its division 1 comes under the runner. Slide the runner to cover 4 on *BF*. Move *BF* to the left until 6 comes under the runner. Read opposite 1 on *BF* the answer 1162, about. Inspection shows that the decimal point should come after the first two figures—11.62.

$$\frac{6 \times 3.1 \times 72}{14 \times 2.7} = ?$$

Set 1 on *BF* opposite 6 on *AE*. Slide runner to cover 3.1 on *BF*. Slide *BF* until 1 comes under the runner. 72 on *BF* is now found to have moved past the end *A'* of *EA'*. Therefore move *FB'* until 1 comes under the runner. Slide the runner to cover 72 on *BF*. Move *BF* until 14 comes under the runner. Slide runner to cover 1 of *BF*. Move *BF* until 2.7 comes under the runner. Opposite 1 of *BF*, read, on scale *AE*, the answer 354. Inspection for the decimal makes this read 3.54. It is sometimes troublesome to locate the decimal point, but it must be borne in mind that in most computations with the slide rule, the computer will know in advance the number of figures that will appear to the left of the decimal point.

123. Powers and Roots.—To find the square of any number, slide the runner to cover the number on the scale *DD'*. The number then covered by the runner on *AE* or *EA'*, as the case may be, will give the answer sought. To find a square root, in general, the reverse operation is performed. That is, the runner is made to cover a number on *AA'*, and the number sought is read on *DD'*. Caution is necessary here, however. Suppose, for instance, that the square root of 3 is to be found. The runner being made to cover 3 on *EA'*, the answer 547 is read on *DD'*. This, however, no regard being had as to the decimal point, is not the square root of 3, .03, 300, but of 30, 0.3, 3000 and so on. But, on the runners being made to cover 3 on *AE*, the correct answer, 1.73, is at once read off on *DD'*. This leads, in connection with the extraction of square roots, to the following

RULE.—*Make the runner cover the number whose square root is to be found on AE or on EA', if the number be whole or mixed, as there is found an odd or an even number of digits to the left of the decimal point; if the number be fractional, as there is found an odd or an even number of ciphers to the right of the decimal point.*

Cubes are easily found from consideration of the fact that $x^3 = (x^2)x$. Thus to find the cube of 4, set the runner over 4 on scale DD' and bring 1 of BF under it. Then opposite 4 of BF read on EA' the answer 64.

To extract cube roots, the reverse process is followed. Suppose it is desired to extract the cube root of 9130. First bring the runner to cover 9130 on the scale AE. Then move the slide down until the same number is found on scale BF under the runner as, on scale DD', coincides with the left end of scales BF and CC'. In the present case try first 2. On bringing this number on scale BF under the runner, the left end of BF is found opposite 214 on DD'. Therefore try again 22. This time the number found on DD' is 204. By a little shifting of the slide, it is found that when 209 of BF comes under the runner, the same number is found on DD' opposite the left end of BF, and it is, therefore, the answer sought, 20.9. The case, however, is similar to that of the extraction of square roots. Suppose, for instance, that the original number 9130 had been taken on the scale EA. The answer found would have been about 45, which is not the cube root of 9130 but of 91,000. In general it will be best to divide the number whose cube root is sought into divisions of three digits each as in arithmetic. The first figure of the root can then be determined by inspection and scale AE or EA' used as may be necessary. In cases where the left end of CC' falls outside the rule, read opposite its *right* end.

124. Cologarithms.—Since the mantissa of the *cologarithm* of a number is one minus the mantissa of the logarithm of the number, the distance from any number on any scale to the right end of the same scale will represent the mantissa of the cologarithm of the number. This furnishes

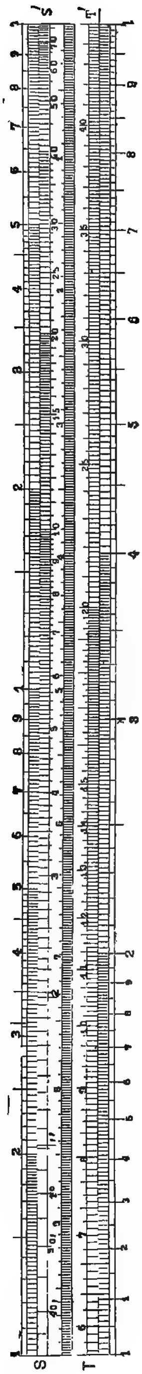


FIG. 205.

another means of making divisions. Let it be desired to divide 164 by 335. Bring the runner to cover 164 on scale AE. Then place 335 of BF under the runner and opposite the right end of BF read on EA' the answer 489.

125. Back of Slide. Scale of Equal Parts. —The *back* of the slide carries three scales as shown in Fig. 205. The scales on the edges will be described in Article 126. The central scale is of equal parts and divided to five-hundredths, its whole length being equal to that of the scale DD'. A piece of celluloid, having drawn on it a line directly opposite the right end of scale DD', is set in the back of the rule as shown in Fig. 203. The numbers on the scale of equal parts increase in an opposite direction to those on the front of the rule. When the ends of the slide scales coincide with those of the rule scales, the zero of the scale of equal parts is found under the line on the celluloid. Now let the left end of CC' be moved until it is opposite any number x, on DD'. The line on the celluloid at the back has then moved over a distance on the scale equal to the mantissa of x, and this mantissa can be read off directly under the line. Let it be required to take out the logarithm of 172. Move the slide until the left end of CC' is found opposite 172 of DD'. Then turn the rule over and under the line read .236, the mantissa required. The rule can thus be used as a three-place logarithm table. Evidently, if the slide is taken out and reversed the logarithms can be read off at once. Thus in the above example let the slide be taken out and replaced with its back face to the front and the numbers on the scale of equal parts increasing in the same direction

as those on DD', the extreme ends of all the scales coinciding. Bring the runner to cover 172 on DD', and on the scale of equal parts we read off the mantissa .236 as before.

126. **Trigonometric Scales.**—The scales on the upper and the lower edges of the slide in Fig. 204 are of logarithmic sines and tangents respectively, the general scheme of scale division being similar to that used on the scales on the front of the rule. The numbers refer to minutes and degrees. Thus, referring to Fig. 205, the first number on the upper scale, SS', counting from the left is 40, indicating that the distance from the left end of the scale to that division equals the mantissa of the log sin 40′. Proceeding to the right the number 50 gives the log sin 50′ and the next marked division the log sin 1°. The remaining numbers to the right on the upper scale indicate degrees. Note that between the degree marks the subdivision is to 12ths instead of 10ths, these subdivisions corresponding to five minutes each. This *sine* slide is drawn to the same scale as AE, EA', BF, and FB' on the front of the rule. The lower scale TT' is one of logarithmic tangents, the scale to which it is drawn being equal to that of CC' and DD' on the front.

All the numbers on this scale refer to degrees, the first on the left being 6. Below 6 degrees the tangents and sines are nearly equal, and the sine scale will serve for both. The extreme angle on the right of the tangent scale is 40°. For larger angles

$$\tan x = \frac{1}{\tan(90 - x)}. \quad \text{(See 5 below.)}$$

The use of the trigonometric scales will be understood from a few examples.

1. Nat. tan 5° 10′ = ?

Take the slide out and turn its back to the front, SS' being adjacent to AA', and bring the left ends of all the scales into coincidence. Bring the runner over the second division from 5 towards 6 on SS' and read on AE the answer 9. The correct answer is .09.

2. Log tan 5° 10′ = ?

Having obtained the nat. tan as above, take the slide out and

return it to its normal position. Then bring the runner to cover 9 on *DD*. Move the slide until its left end comes under the runner. Look at the transparent indicator on the back of the rule and under its line read on the scale of equal parts .956, the mantissa required.

3. $165 \sin 52° \ 15' = ?$

With the slide in the position having SS' adjacent to AA' make the runner cover 165 on AE. Bring the left end of the slide under the runner. Next, on moving the runner to cover $52° \ 15'$ on SS', that point is found to be beyond the right end of EA'. Therefore, set the right end of SS' under the runner and opposite $52° \ 15'$ of SS' read on AE the answer 130 +.

4. $\dfrac{276}{\cos 77°} = ?$

Since $\cos x = \sin (90 - x)$, $\cos 77° = \sin 13°$, and the problem becomes $\dfrac{276}{\sin 13°} = ?$

Bring the runner over 276 on EA' and move under it 13° of SS'. Opposite the left end of SS' read on AE 1225, the answer sought.

5. $18 \tan 60° = ?$

$\operatorname{Tan} 60° = \dfrac{1}{\tan 30°}$. Set 30° of TT' opposite 18 of DD'. Then opposite the *right* end of TT' read on DD', 312. The correct answer is 3117. The decimal point should be placed 31.2.

127. Special Problems and General Remarks.—The foregoing articles describe the general use of the slide rule. Usually a separate setting ofthe slide has been indicated for each multiplication or division, etc. Sometimes, however, this may be avoided. In the first problem of Art. 122, $\dfrac{5 \times 3.5 \times 4}{6} = ?$, note that in the solution there given the slide has to be "set" three times. Consider the following solution: Set 6 of BF opposite 5 of AE (one setting). Make runner cover 3.5 of BF. Bring 1 of BF under the runner. (Two settings.) Opposite 4 of BF read on EA' the answer 1162 as in Art. 122, and the work has been accomplished with one less setting. In general, problems

of the form $\frac{ab}{c} = ?$ can be solved with one setting. Thus $\frac{5 \times 38^2}{29^2} = ?$ Set 29 of CC' opposite 5 of AE and read on AE opposite 38 of CC' the answer 8.58. One of the principal uses

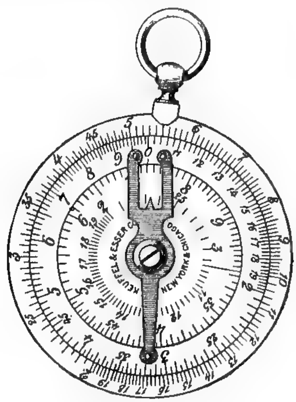


FIG. 206.

of the rule is found in the case where a great many problems *of the same form* are to be solved. In such a case it will be well to decide first on the most advantageous way of setting the slide.

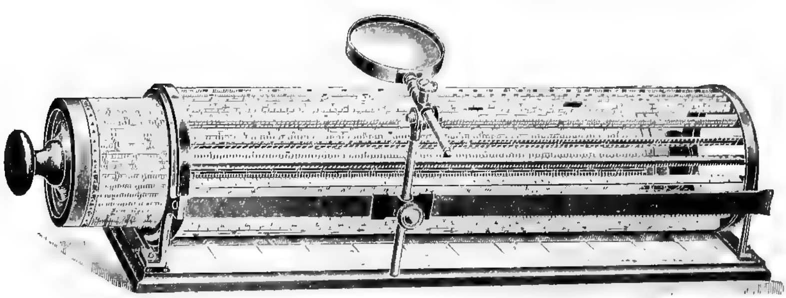


FIG. 207.

If the problem consists of determining the mean value of some constant from a large number of observations, so that the results will all be very nearly equal, the position of the decimal point

may be determined once for all. If a number of problems of the form $\frac{x}{a} = ?$ $\frac{y}{a} = ?$ $\frac{z}{a} = ?$ are given, set a of BF opposite 1 of AE. Then opposite x, y, and z of BF read on AE the numbers sought. So for any number of quantities each divided by a constant number. Many formulæ contain several constants. If such a formula is to be applied a number of times the constant factors C_1, C_2, etc., should be combined to form a constant factor C known as a "gauge point," and this factor used as a whole in each calculation.

128. Other Forms of Slide Rules.—The Charpentier calculator, Fig. 206, consists of a circular slide rule, $2\frac{5}{8}$ inches in diameter, the slide being revolved by means of the handle. On the back are found scales of equal parts and of sines and tangents as in the ordinary form. Fig. 207 shows perhaps the most accurate of all slide rules, the Thacher calculating instrument. It consists of a cylinder revolving in two vertical rings supported on a wooden base. Joining the rings are twenty triangular bars. Inside of these bars the cylinder, whose face they fit accurately, can be moved in the rings with either a longitudinal or a rotary motion. The graduated triangular bars form the rule, and the cylinder, the slide. Quantities calculated by this instrument should not be in error by more than one unit in the fourth place, and indeed the fifth figure can often be found very closely. It is evident that this instrument will perform with sufficient accuracy most of the computations occurring in ordinary engineering practice.

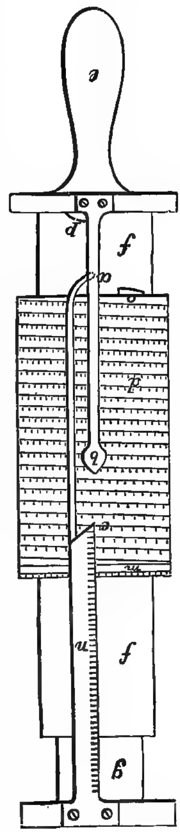


FIG. 208.

Another form known as "Fuller's Slide Rule" is shown in Fig. 208. It "consists of a wooden cylinder which can be moved up or down or around a wooden axis held by a handle. The scale is divided on this cylinder. It is a single logarithmic scale 42 feet long, wound spirally. Ratios are established by means

of a pointer attached to the axis at the handle and another attached to a brass tube sliding in the axis. This latter bears two indexes whose distance apart is the axial length of the complete spiral."*

* Queen & Co.'s catalogue.

CHAPTER VIII.

TOPOGRAPHICAL SURVEYING.

129. Definitions.—A topographical survey is one made to determine the relative elevations of all the points in the area surveyed, and a topographical map is a graphical delineation of the results obtained. These maps are sometimes constructed in

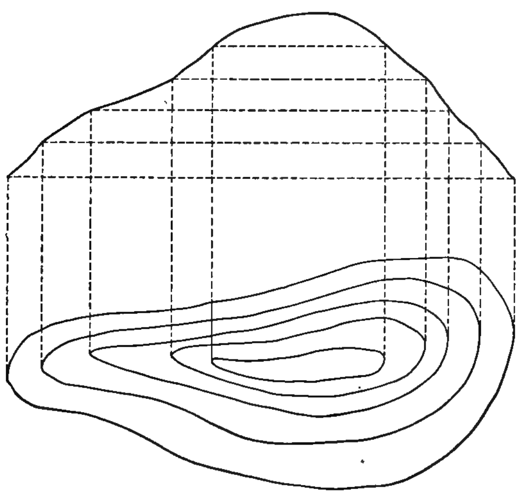
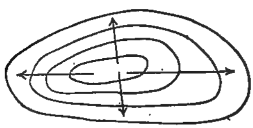


FIG. 209.

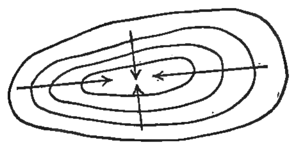
relief of papier maché or other suitable material, and consist, in such cases, of a series of miniature hills and valleys, showing the natural surface exactly, though to a greatly reduced scale.

The most practical method of representing topography, however, and the one universally adopted among engineers is by means of "contour maps." A contour is the intersection of a

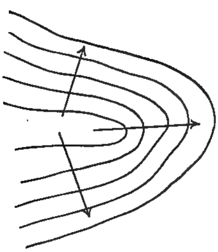
horizontal plane with the surface of the land surveyed. Suppose a hill of somewhat irregular shape, intersected by a number of these planes separated by vertical intervals of say five feet. The plan and elevation of such a case are shown in Fig. 209. The plan forms what is known as a contour map. The lines shown on the map are known as contour lines. It is evident that for a given vertical distance apart, the slope of the natural surface grows steeper as the horizontal distance between adjacent lines on the map decreases. In the case of a vertical wall or cliff the



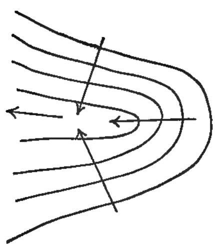
HILL
FIG. 210.



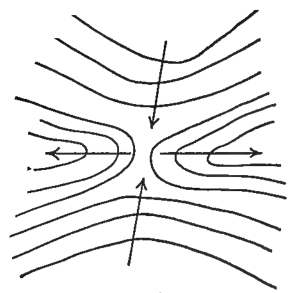
HOLLOW
FIG. 211.



SHOULDER
FIG. 212.



VALLEY
FIG. 213.



SADDLE
FIG. 214.

lines would be shown directly superimposed. Also, except in the case of an overhanging cliff, they can never be shown crossing each other, and provided the area surveyed is large enough, each will form a closed curve. Owing to the fact, however, that this last condition is practically never fulfilled on any contour map, lines will be shown running off the edge. Figs. 210–214 show five arrangements of contour lines to which it will be found all arrangements in nature may be reduced. The arrows show in each case the directions in which water would run.

130. General Method of Survey.—The general method of obtaining the data for a topographical map will be, first, to survey

the boundary of the tract to be mapped, and next to locate and obtain the elevations of a sufficient number of points in the interior to enable the contour lines to be plotted. The interior points are taken more or less at random, but with the effort to choose them such that the slope between any adjacent two shall be uniform. It is evident that it will be desirable to take points along the tops of ridges and the bottoms of hollows. However, as will be shown a little later (Arts. 132 and 138), the choosing of these points depends somewhat on the size of the survey.

131. Instruments.—The Locke hand level, Fig. 215, consists of a small telescope tube containing a horizontal cross hair

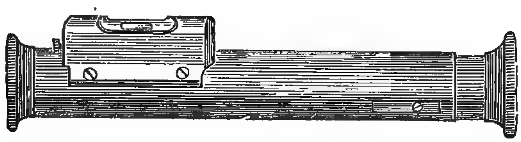


FIG. 215.

and having attached a level vial and prism reflected from which, on looking into the eye end of the tube, the image of the bubble can be seen. The face of the prism makes an angle of 45° with the vertical and the wire is stretched across a hole in the top of the tube. When the level is in adjustment, if the wire is seen bisecting the bubble, the line joining the eye of the observer and the cross hair is horizontal. To adjust this instrument obtain two points of the same elevation and hold the level at one so that on looking through it the cross hair covers the other. If then the hair does not bisect the bubble, move the level vial until it does. This movement is effected by means of the screw shown at the end of the case holding the bubble tube in Fig. 215. Note that there are no lenses in the tube except sometimes a small magnifier at the eye end, so that the instrument has no telescopic line of sight. A serious objection to the Locke level is that any movement of the observer's eye will cause a considerable movement of the cross hair on a rod observed. It is, of course, used only in rough leveling, but there proves a useful instrument.

The pocket level, Fig. 216, is similar to Locke's level except that it has a telescopic line of sight, and hence the movement of the cross hair on the rod is avoided. This instrument can be

used as a hand level or mounted on a tripod or Jacob's staff. In topographical work a convenient adjunct is an attached compass. When the line of sight is horizontal the bubble appears bisected

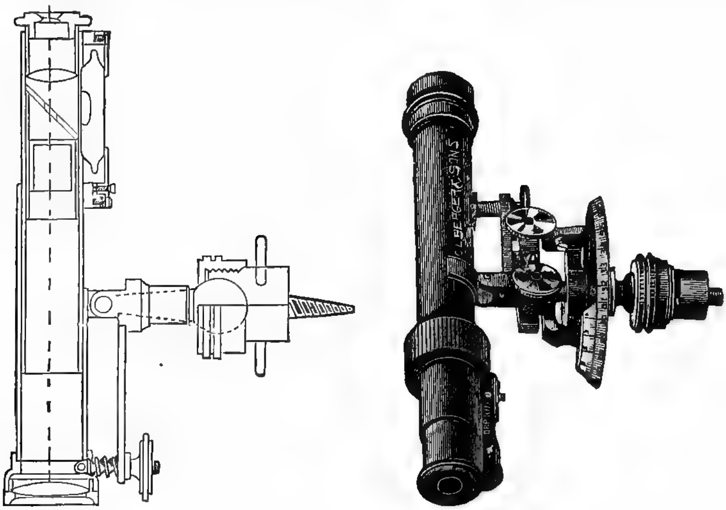


FIG. 216.

by the cross hairs as shown in Fig. 217. The instrument is adjusted in about the same way as the Locke level. The Abney hand level and clinometer, Fig. 218, is similar to the Locke

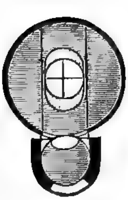


FIG. 217.

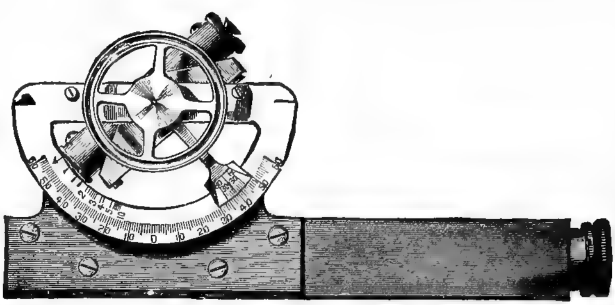


FIG. 218

level, except that its level is attached to a vertical arc as shown in the figure. If the observer looks through this instrument with its tube on any slope, at the same time turning the level vial until

the reflected image of the bubble appears bisected by the hair, the reading of the vertical arc will give the tube's angle of inclination to the horizontal. In practice this instrument is used to determine the slope of the ground. The observer looking through it makes the cross hair cover a point on a rod at the same distance as his eye above the ground. The vertical circle will then give the slope of the ground between the observer and the rod.

132. Method for Small Areas.—If a topographical survey is to be made of a small tract perhaps a quarter of a mile wide, the first step will be to divide one or more sides into even fifty or one hundred feet stations, a stake marked with the proper number being driven at each. The note-book will show a sketch of the outlines of the tract with the locations and numbers of the stakes. Next level over these stakes with a wye level, obtaining the elevation of the ground to the nearest tenth at each, and continuing the work around the entire boundary, leaving bench marks about every five hundred feet. Having finished this work, level over a series of parallel lines with a hand level, starting from the numbered stakes, and running, when the stakes are placed along only one side, across the entire tract. If the stakes have been set along more than one side, the lines will be run far enough for the series to cover the whole area. The direction of these parallel lines is determined by a compass. If the Locke level is used a separate compass will also be taken out. The pocket level, however, may have a compass attached—another advantage possessed by it. The direction of a line having been obtained by the compass at the beginning stake, generally, a point can be chosen on line and far enough ahead, that the line may be run directly towards it without any further determination of direction. If the nature of the ground will not permit this, the compass will have to be employed again at as many points as may be found necessary. Fig. 219 represents a case in point. The side AB of the field is divided into 100-ft. stations. The lines running from these are shown, conveniently, parallel to the side BC. The contours are generally located at vertical intervals of five feet. For this purpose it will be convenient to have the hand level used mounted on a support such that the line of sight will be five feet from the ground. A good support for a Locke level is a plain

board about five and a half feet long with a hole cut to admit the level five feet from the ground. If the pocket instrument is used a blunt Jacob's staff will serve very well. In either case suppose the instrument held on an even five-foot contour. Then looking down-hill at a rod if the level line of sight cuts the ten-foot mark, the rod is on the next lower even five-foot contour. Looking up-hill, if the rod is on the next higher contour, the line of

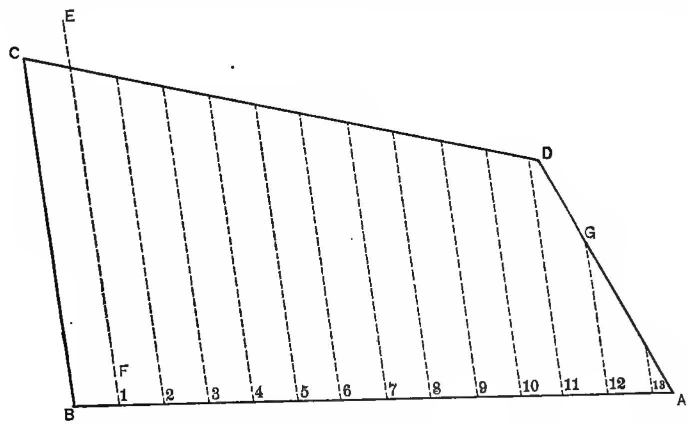


FIG. 219.

sight will cut it at its foot. If the instrument is not on an even contour, points on the next adjacent ones, upper and lower, may be found by the following rules:

Up-hill. Read on the rod the difference in elevation between the point occupied by the instrument and the next lower even five-foot contour.

Down-hill. Read on the rod five feet plus the difference in elevation between the point occupied by the instrument and the next lower even five-foot contour.

Starting then at any stake, as 1, Fig. 219, the observer ranges out his line, notes, if possible, a distant point E on it, and sends the rodman out in the direction $1E$ until the rod is held at F, a point on the next even contour as determined by the proper rule. The distance from the stake to the rod being taped and noted, the observer moves to that point, the rodman moves to the next contour and the work continues. The notes will stand as shown in Fig. 220, representing the right-hand page of the note-book.

The figures on the left give the distances along the line from the starting-point; those on the right the elevations of the corresponding points. With the pocket instrument the distances between points occupied may be determined by stadia methods (Art. 135), and the work thus much expedited. At the end of each line, the elevation of the last contour point should be checked by leveling

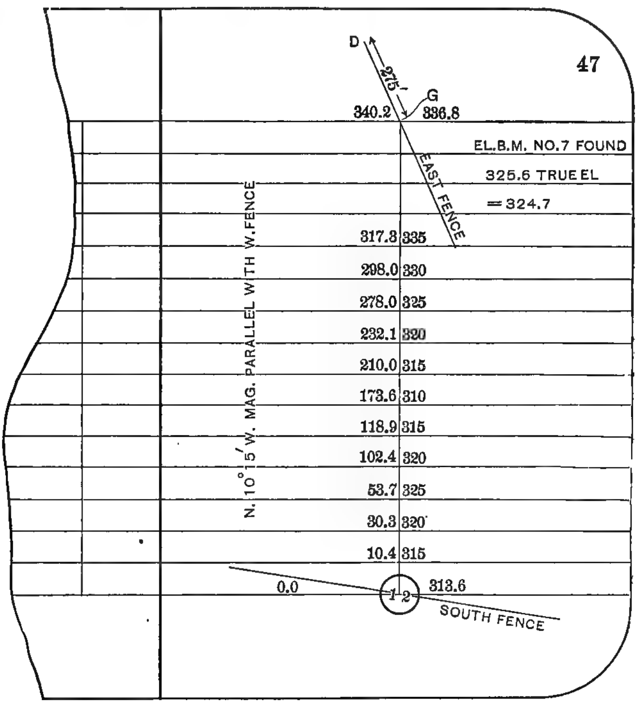


FIG. 220.

over, by ordinary methods but still using the hand level, to the nearest B. M. The elevation as thus determined should not differ by more than a foot from that found previously with the wye level. The position of the line just run should also be checked in some convenient manner. Thus in Fig. 219, supposing that the boundary line *AD* is marked by a fence, the distance from 12 to *G* and from *G* to *D* is noted (see Fig. 220 also). When the work is plotted these distances can be applied as checks, and should be satisfied within 3 or 4 ft. One of the fundamental rules in

engineering operations is never to be satisfied with any work unless some kind of a check can be had on it.

Other methods will suggest themselves. One is to divide the tract surveyed into rectangles and then level over the whole area, obtaining elevations of each rectangle corner and intermediate changes of slope with a wye level. This method is more accurate than the one just described, but also more tedious. Ordinarily the first method is sufficiently precise. It can be extended to larger tracts by dividing them into comparatively narrow strips, a line of wye levels being run down each dividing line.

133. **Plotting the Work.**—The outline is first plotted by the ordinary methods, and the parallel lines with the contour points on them located, not forgetting the check mentioned in the preceding article. Points of equal elevation are then joined by fine even lines in black. It is customary to draw every *n*th line, in a

FIG. 221.

map showing five-foot contours, conveniently every even hundred contour, in red. Leave a sufficient number of breaks perpendicular to the contours to write the elevation of each directly on it. The lines can be drawn with an ordinary right-line pen or with the contour pen shown in Fig. 221. They are best put in free hand. Plate 2 shows a contour map.

134. **The Stadia Rod and Wires.**—A "telemeter"* is an instrument for measuring distances and elevations more quickly than can be done by ordinary methods. The telemeter in most common use is a combination of a rod, and two parallel wires inserted in an ordinary surveyor's telescope. The two wires are known as "stadia wires," the rod as a "stadia rod," and the measurements made by means of the wires and the rod are called "stadia measurements." When the term "stadia" is used, some uncertainty may arise, as the telescope with the wires, or the rod, or both may be meant. However, the use of the terms "stadia wires," "stadia rod," and "stadia measurements" occasions no ambiguity.

* Literally "a far measure."


MAP OF
STADIA SURVEY
HILL
Southeast of
SYRACUSE UNIV. CAMPUS
June 1900 Class of 1901
Scale of Feet
100 50 0 100 200 300
MIDLAND AVENUE
MIDLAND AVENUE
110
115
120
125
135
140
145
150
155
160
165
170
175
180
185
190
195
200
205
210
105

135. Theory of Stadia Measurements.—Suppose that two horizontal wires, separated from each other by a distance i, are placed in an ordinary surveyor's telescope, and that the telescope is turned on a graduated rod. When the telescope is focussed on the rod, the wires, being attached to the ordinary ring, in the plane of the cross hairs will include between them a certain part, or length, of the image of the rod formed by the object-glass. The length of the part of the rod's image included between the wires will grow less or greater as the rod is moved to or from the instrument. This is, practically, the principle of stadia measurement. We proceed to a more detailed discussion.

Fig. 222 represents a longitudinal section of a telescope fitted with stadia wires. The telescope is supposed to be set up over a

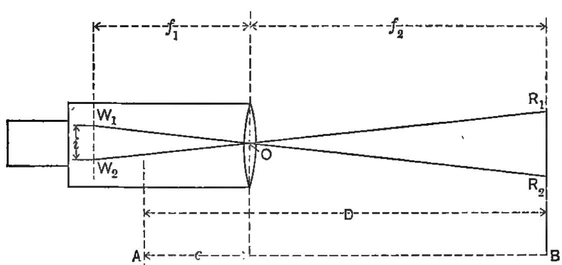


FIG. 222.

point A while the rod is held at B, the telescope being horizontal and the rod vertical. The image of the rod being formed in the plane W_1W_2 of the wires, these will intercept on the image a distance or reading, R_1R_2. O being the optical center of the object-glass, the triangles W_1OW_2 and R_1OR_2 are similar. From which, denoting W_1W_2 by i, and R_1R_2 by S,

$$\frac{i}{S} = \frac{f_1}{f_2}, \qquad (99)$$

where f_2 and f_1 are the conjugate focal distances of the rod and of its image, respectively. Eq. (34) of Art. 55 is

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}, \qquad (100)$$

where f is the principal focal distance of the object-glass. Solving (99) and (100) for $1/f_1$, and putting the results equal, we have,

$$\frac{S}{f_2 i} = \frac{f_2 - f}{f_2}.$$

From which

$$f_2 = \frac{f}{i}S + f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

Eq. (101) gives the distance from the optical center to the rod *when the rod is held perpendicular to the line of sight.* If we wish to obtain the distance to the rod from the center of the horizontal axis of the telescope under which the plumb-bob hangs, we must add to the value of f_2 given by eq. (101), the distance from the optical center to the center of the horizontal axis. Calling this c and denoting by D the total distance desired, we have

$$D = \frac{f}{i}S + (f + c). \quad . \quad . \quad . \quad . \quad . \quad (102)$$

f is a constant for any given telescope, but c varies as the object-glass is moved in or out. The value of the parenthesis $(f + c)$, eq. (102), will range from 9 to 15 inches, so that adding one foot to the distance given by multiplying the reading of the rod by the ratio f/i will give a closely approximate value for D. Eq. (102) holds only when the line of sight is perpendicular to the face of the rod. Generally the rod is held on lower or on higher ground than that occupied by the instrument. The line of sight being thus inclined to the face of the rod, a new formula becomes necessary.

The instrument being at A, Fig. 223, and the rod at B, the horizontal distance AJ is required. Evidently $AJ = D = PG \cos \theta$, where θ is given by the vertical circle of the transit. PG could be determined by eq. (102) if the rod was held perpendicular to the line of sight. This being impracticable, note that the angles CFG and GEI are practically right angles, and, therefore, $EF = CI \cos \theta$.

We may then modify eq. (102) as follows:

$$D = \left[\frac{f}{i}S \cos \theta + (f + c)\right] \cos \theta$$

$$= \frac{f}{i}S \cos^2 \theta + (f + c) \cos \theta. \quad . \quad . \quad . \quad . \quad (103)$$

In a great deal of work the second term in the right-hand members of eqs. (102) and (103) may be neglected. For the purpose of facilitating the computation of $(f/i)S \cos^2 \theta$, tables and diagrams have been prepared. These will be described a little later on.

It is convenient, and generally so arranged, to have the ratio f/i equal to 100. Then if an ordinary leveling rod is held perpendicular to the line of sight of the telescope, and the space included between the wires on the image of the rod is one foot, the

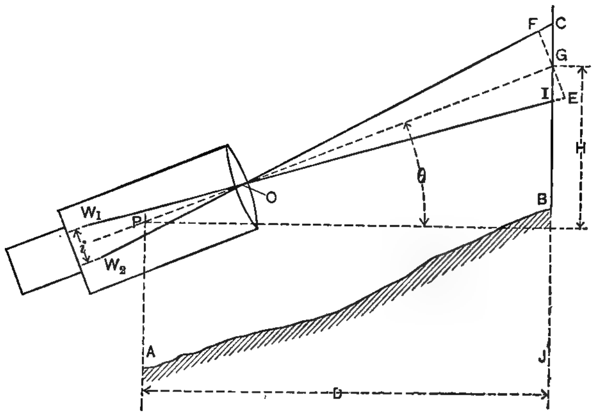


FIG. 223.

rod is 100 feet $+ f$ distant from the optical center. If the reading is two feet, the distance between the rod and the optical center is 200 feet $+ f$, and so on. It is not *necessary* that the ratio f/i should be equal to 100. Evidently, with any other value for this ratio, a rod could be specially graduated so that, at a distance of 100 feet $+ f$ (or 100 units of length $+ f$) from the object-glass, the rod would have one division, no matter what its absolute length, included between the wires.

For instance, suppose that two stadia wires have been inserted in a telescope, and it is desired to graduate a rod for use with them, no knowledge being had of the exact distance between the wires. It will first be necessary to determine the value of $(f + c)$. To find f, focus the telescope on some object at a great distance, as a star. While thus focussed, measure carefully the distance

between the object-glass and the cross hairs. This will give the value of f. Focussing on a rod, say, 300 ft. from the instrument and measuring the distance between the object-glass and the center of the horizontal axis will give a value of c. c changes as the object-glass is run in or out for focussing on objects at different distances from the instrument. The change in c, however, is slight, and focussing on objects 300 ft. away will give a fair average value. Having determined the value of $(f + c)$, set up the instrument on fairly level ground, and point the telescope in a direction which offers easy measurement for over, say, 300 feet. Lay off the distance $(f + c)$ in front of the instrument in this direction, and, from the point thus determined, measure on 300 ft. more. Have the rod held vertically at this last point, and focus on it, keeping the telescope level. The distance included by the wires on the rod, divided into three parts, will give the unit of graduation.* Another method is to measure off distances of 100, 200, and 300 ft. from the point $(f + c)$ in front of the instrument, find a value for the unit of graduation with each of the three points, and take the average of these three values. Owing to the fact that c varies, the distances determined with the rod graduated by either method will generally be in error by a slight amount. The unit of graduation is understood to be the length of the rod, perpendicular to the line of sight, included between the wires when the rod is at a distance from the instrument equal to 100 units of length $+ (f + c)$. This unit of graduation can be further subdivided decimally. Divided to tenths of units, each tenth would correspond to a distance of ten feet between the rod and a point f ft. in front of the optical center. The wires can be put in by the engineer, who will probably experience trouble in getting them adjusted. It is much better to have the work done by an instrument maker, and have the distance between the wires made equal to one one-hundredth of the focal length. A method of placing the wires is described in Baker's "Engineers' Surveying Instruments," Art. 205. If the engineer desires to place the wires himself, he will first have to decide on the distance between them. If this is to be one one-hun-

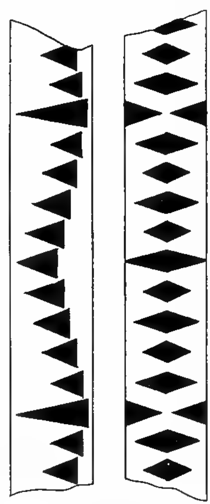
* Supposing the line lengths are to be determined in *feet*. See Art. 136.

dredth of the focal length, f can be measured and divided by one hundred and the wires placed the resulting distance apart. The work should be tested by placing a rod, graduated to feet, 100 feet $+ (f + c)$ from the instrument and noting whether the wires include a distance of one foot.

The two stadia wires should be equidistant from the ordinary horizontal wire, as it is sometimes convenient to make use of the interval between one of them and this last wire.

On some government work, in graduating the rod, the term $(f + c)$ is neglected entirely, the base being measured directly from the point under the center of the instrument. A rod graduated in this way will, of course, give correct results at only one distance from the instrument, but the base being taken at about a mean length, the error is considered small enough to be neglected. Perhaps the best rule for general use will be to have the rod graduation based on the rigorous formulæ in eq. (102), and, in determining distances, neglect the expression $(f + c)$, substituting in its place one foot.

136. Rods.—If, as is almost always the case, the wire interval is one one-hundredth of the focal length, or $i/f = 1/100$, an ordinary level rod can be used. The objection to this is that the divisions do not stand out distinctly enough. On this account special patterns of rods have been designed. Two of these are shown in Fig. 224. The designs explain themselves. For instance in (a) the long triangles mark points on the rod at a distance apart equal to the unit of graduation. The left points of the triangles in between are one tenth of a unit apart, and their vertices on the right mark points midway between the tenths. Between these points the rod can be read by estimation to two one-hundredths of a unit. The rods are generally twelve or fourteen feet long, and three inches wide by one half or one thick, with a cleat down the back to prevent warping. A handle may be cut in the cleat. Strips should be screwed on the edges



a *b*

FIG. 224.

so that the face of the rod will be recessed about a quarter of an inch. This will protect the graduation. Before having the designs put on, the face of the rod should receive several *thin* coats of white paint mixed with enough turpentine to prevent a glossy appearance of the finished surface. To put the designs on, first divide the rod carefully into units of graduation. Then prepare a pasteboard stencil, one unit in length, pencil the designs on each unit length, and fill them in with black. The design nearest to five feet from the lower end of the rod should be marked in some way to attract attention. A good plan is to paint it red. In rods divided to feet it will be advisable to mark in this way both the five and the ten foot marks.

Where the rod is to be used for long distances, it may be convenient to have the units of graduation longer than feet, say yards or meters. The unit of graduation will, of course, be an *exact* foot, yard or meter, only when the ratio f/i equals 100.

137. General Stadia Work.—Stadia methods may be employed to determine the length of any line one of whose extremities is visible from the other and close enough to enable the rod to be read accurately. The state of the atmosphere will have a great deal to do with the possible length of the sight. Readings have been made on lines half a mile long, but this is probably exceptional. No reading should be accepted unless the rod can be read clearly. It is on these long lines that the ordinary horizontal wire becomes useful. Thus with a twelve-foot rod and a ratio $f/i = 100$, at a distance of 1200 feet, the wires would include the whole rod and a reading would be impossible at a farther distance. In such a case, however, the interval between one of the stadia wires and the ordinary horizontal wire can be used, being careful to remember that a reading with these last two corresponds to one double its value with the two regular stadia wires. The horizontal wire also becomes useful in the case where part of the rod is hidden by brush. In a closed survey made with stadia measurements and compass bearings perhaps a ratio of closure of one in three hundred to one in five hundred might be expected. If the angles were measured with a transit the ratio of closure might be reduced to one in one thousand. From which it appears that stadia measurements will suffice in ordinary farm work.

138. Topographical Survey of a Large Area.—Stadia methods, however, find their greatest usefulness in connection with topographical surveys over extended areas. In such work, the first step will be to make a general outline survey of the region whose topography is to be ascertained, determining the positions and elevations of a sufficient number of stations and bench marks, conveniently located for the stadia lines later to be run between them. The outline survey is made by the ordinary methods. Where the size of the tract warrants it, the stations, boundaries, etc., can be located by means of triangulation. Having finished the outline surveys, starting at any station whose location and elevation have been determined, a traverse of stadia lines is run out to connect with some other previously located point. The lengths of the lines in this stadia traverse are determined by means of the wires, the directions by needle bearings or by the methods of Arts. 104 and 105.

Directions given by the horizontal circle of the transit will, of course, be much more accurate than if determined by the needle. If the directions are not determined by needle bearings the method of traversing will be preferable to that by deflection angles. The transit being set up at the station of known position and elevation from which the stadia traverse is to start, when the telescope is turned on the next point, in order correctly to determine its elevation, the horizontal hair must cut a point on the rod at a distance above the ground equal to the distance above the ground of the horizontal axis of the transit. In order to determine this last, a short tape or rod, graduated like the stadia rod, should be carried by the transitman. The marked design on the stadia rod five feet from its lower end will save time in fixing the horizontal hair on the right point, as the telescope axis is about five feet from the ground. The difference in elevation between the point observed and that occupied by the transit will then be given by

$$H=\left[\frac{f}{i}S\cos\theta+(f+c)\right]\sin\theta$$

$$=\frac{f}{i}S\cos\theta\sin\theta+(f+c)\sin\theta$$

$$=\frac{f}{i}S\,\frac{\sin 2\theta}{2}+(f+c)\sin\theta, \quad . \quad . \quad . \quad . \qquad (104)$$

which compare with eq. (103) of Art. 135 and Fig. 223. With the instrument at each station of the stadia traverse enough "side-shots" must be taken to locate the contour lines on each side, and also to determine the positions of such points, as fence corners, etc., as may be desired to appear on the final map, but which have been left out in the original outline survey. In regard to finding the contour lines the directions of Art. 130 apply. The rodman, or rodmen (there should generally be two or more, on different sides of the traverse), will hold rods at all the critical points in the surface contours, in other words, at such points that the slope will be uniform between any two adjacent to each other. The method of making any series of rod positions parallel to the contours will be found convenient. Generally it will be found sufficient to locate contours differing in elevation by about fifty feet. The intermediate ones can then be interpolated. In connection with this, the caution given above, to allow no critical contour point to lie between two adjacent positions of the rod, must not be neglected. Where practicable, occasionally take two "side-shots" from different traverse stations at the same point. This is useful as a check, but takes time; usually the general run of contours will be a sufficient check on the location of side-shot points. The stadia traverse being run out and tied, both in position and in elevation, to two points of the outline survey, the latitudes and longitudes of the stadia lines are computed and balanced to agree with those of the outline survey. In both the horizontal and vertical measurements, a ratio of closure of one in five hundred should not be exceeded. The stations on the stadia traverse are known as "stadia stations," and should be so chosen that from each will be visible the maximum number of critical contour and boundary points. These critical points, etc., on which the rod is held, are known as "side-shot stations." In the notes □ represents a stadia station and △ a station of the outline survey. If a symbol is desired to represent a side-shot station, ⊙ may be used.

In the beginning of a stadia traverse, the transitman will set up his instrument over the △ from which the traverse is to start. If the azimuth method is to be adopted, he will next send a rodman on to the next □ and orient his instrument (Art. 105) on the rod held at that point. While obtaining the azimuth angle,

the rodman can hold the edge of his rod towards the transitman, or, if preferred, he can turn the face of the rod towards the observer, who will then bisect it with his vertical wire. In either case, the center of the rod must be held over the center of the □. Tack points generally are not necessary on stadia stations. If the rodman turns the edge of the rod towards the transit for the azimuth observation, the transitman will have to signal when he is ready to take the distance and elevation observation. The instrument may be oriented with reference to the true meridian, the magnetic meridian, or any other well marked line assumed as a meridian. The outline survey, of course, will have been referred to the same meridian. Enough stadia traverses will have to be run to permit of observations on a sufficient number of side-shot stations for the mapping of the contours and other desired features of the tract surveyed. The stadia traverses should run between outline survey stations. Where practicable, it will be found convenient to locate an outline station, about in the center of the tract to be surveyed. The stadia traverses will then radiate from this point.

To return to the field-work, the transitman, with his instrument on the beginning station, having oriented his instrument on the stadia station occupied by the rodman, checks his azimuth angle by the needle reading, observes the reading of the stadia wires and the vertical angle, and notes all these data in his field-book. In getting the stadia reading, first turn the horizontal hair on the point on the rod at an equal distance from the ground with the horizontal axis of the transit. Then look past the lower stadia wire and note whether it cuts a well-defined division mark on the rod. If not, turn it exactly on the nearest one, which will be generally not more than five one-hundredths of a foot away. Then note the entire stadia reading. It will not have been altered appreciably by the slight turning. Before reading the vertical angle, turn the central horizontal wire back on the point on the rod at the same distance from the ground as the horizontal axis of the telescope.

139. Notes.—The notes are kept, running down the page, on the left-hand page of the field-book, the right-hand page being reserved for a sketch of the line of the stadia traverse, the notes

of which are kept opposite. The sketch runs from the bottom of the page up. As usual, write at the top of the left-hand page the date, names of party, etc. In addition to this, state the station over which the instrument stands, together with its elevation and the height of the instrument above the ground.

The ordinary field-book has its left-hand page ruled in six vertical columns. These should be headed "Station," "Azimuth," "Rod," "Vertical Angle," "Difference of Elevation," and "Elevation." If desired two more columns can be ruled on

25 STADIA SURVEY OF RAYNOR TRACT.
SEP. 23, 1899.
INS'T, CLARK. NOTES, CORNISH. ROD, LOREN.

INS'T ON $\boxed{0}$ EL. = 143.9 H.I. = 5.2

STA.	AZIMUTH	ROD	VERT. ANGLE	DIFF. ELEV.	ELEV.
$\boxed{1}$	0°00′	5.20	2°07′		
C. P.	350 05	5.30	1 10		
C. P.	352 14	5.60	1 02		
C. P.	344 38	4.90	2 45		
N. E. COR. CEM. FENCE	354 26	3.90	0 32		
C. P.	322 36	3.16	1 28		

26

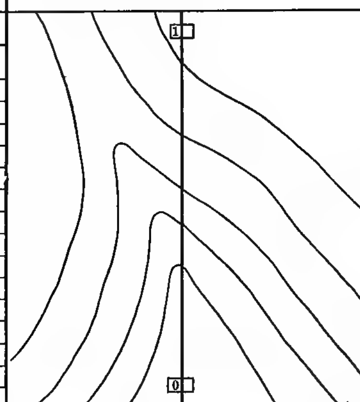


FIG. 225.

the left-hand side of the right-hand page. The third column from the left is then headed "Needle," and the fifth "Distance," the others remaining as above. This ruling of columns on the right-hand page will throw the red line on that page out of center of the available sketching area. A new line can be drawn in to represent the line being run. The sketch is only as an aid to the memory of the observer and often can be omitted entirely. Fig. 225 shows a specimen double page of notes.

The first station observed, unless the instrument happens to be over the first station of the traverse, will be the station last

occupied. This is for the purpose of orienting the instrument, having a check on the length of the last line, etc. Then the stadia station next in line on ahead, after which all the side-shot stations. Stations observed for the purpose of locating a contour simply are marked C. P. in the column headed "Stations." Other side-shot stations, as fence corners, angles in a road, etc., must be described fully in the station column, or the full description may be written on the right-hand page and a reference to this made in the station column. Generally, the notes of two or more lines can be kept on the same page. Crowding, however, must always be avoided.

140. Reducing the Notes.—Having finished the field-work, the next step will be the filling in of the columns headed "Distance," "Difference of Elevation," and "Elevation" in the field-book. The data for these are the rod readings and the vertical angles. Equations (103), Art. 135; (104), Art. 138; and sometimes (102), Art. 135, must be used. Evidently, substituting in (103) and (104) for each observation would require so much time as to render the method valueless. When the instrument and the rod occupy stations of equal elevation, the vertical distance is zero, and the horizontal distance may be taken out at once from (102), adding one foot for $(f + c)$. Such occasions are rare, however, and, to facilitate reductions when the vertical angle is not zero, tables and diagrams have been prepared. In this connection, Table IX has been computed. It gives the horizontal and vertical distance for a unit reading on the rod for every vertical angle, at two-minute intervals, from zero up to thirty degrees. In the body of the tables the expressions $(f + c) \sin \theta$ of (104) and $(f + c) \cos \theta$ of (103) have been neglected, but values of these expressions for $(f + c)$ equal to .75, 1.15, and 1 9 feet are given at the bottom for each degree of vertical angle. Supposing that the vertical angle is $7° 15'$ elevation and the rod reading $2.3'$. Looking in the tables under a vertical angle of $7°14'$, we find in the columns headed "Horizontal Distance" and "Difference of Elevation" the quantities 98.41 and 12.49. Multiply each of these by the rod reading, 2.3, we will have the horizontal and vertical distances, neglecting the expressions $(f + c) \cos \theta$ and $(f + c) \sin \theta$, from the instrument to the point

occupied by the rod. If, in addition to this, we wish to correct for $(f + c)$, look at the bottom of the column headed 7° and take out the corrections, which must then be added to the horizontal and vertical distances already determined. Generally speaking, instead of taking out the $(f + c)$ corrections, simply add one foot to the horizontal distance and neglect the vertical correction unless the vertical angle is as great as 23°, in which case add five tenths of a foot to the vertical distance. The multiplications should be performed with the slide rule. By the letter c in Table IX is meant what we understand by $(f + c)$.

The use of diagrams furnishes another method of reducing the notes. One style of these from which the values of D, eq. (102), and H, eq. (104), can be taken out, the second terms in both equations being neglected, is constructed as follows, Fig. 226: Lay off to any convenient and very large scale as one inch to the foot a horizontal line AB equal to the largest rod reading likely to be had in the course of the work. The nominal length corresponding to this reading would be $(f/i)AB = D'$. Calculate by the first term of eq. (103) or take from the table the values DD corresponding to vertical angles ranging at intervals of 1° from 6° to 30°. Find the quantities $(D' - D)$, $(D' - D)$, and lay them off to scale by the lines BC, BD, BE, etc., on a perpendicular to the line AB at B. Join A with the points C, D, E, etc. Then, for any rod reading AF and vertical angle θ the nominal length will be $(f/i)AF$ and the correction to be applied to this length will be the line FG perpendicular to AB at F and running from that point to its intersection with the line AX corresponding to the angle. The diagram being drawn on cross-section paper, the corrections can be taken directly from it, the vertical scale (for convenience not the same as the horizontal) having been assumed large enough for this purpose.

A diagram for vertical heights can be constructed in a similar manner as shown at $A'B'C'$, etc., Fig. 227. In this case the perpendiculars $B'C'$, $B'D'$, etc., correspond to the first term of eq. (104), Art. 138. With the diagrams ruled for vertical angles at intervals of one degree, interpolation may be used for intermediate values of θ.

A third method of reduction is by the use of a special slide

rule designed by Mr. B. H. Colby, and shown in Fig. 228. The following description is from the pamphlet published by the Keuffel and Esser Co.:

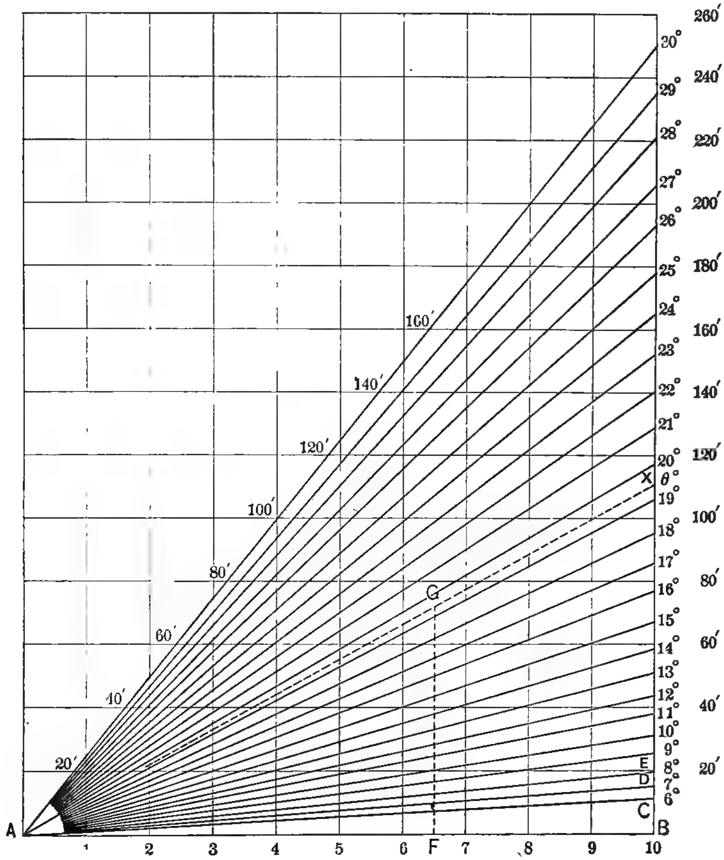


FIG. 226.

"This rule gives differences of elevations between two points when stadia-reading and vertical angle are known.

This slide rule takes the place of tables or diagrams, is much superior to either and is three times as rapid. Taking the notes of any topographic survey with stadia, this rule will give 25 per cent. of the required differences in elevation to the nearest thou-

sandth of a foot, 50 per cent. to the nearest hundredth of a foot, and all differences within the probable error of the field notes.

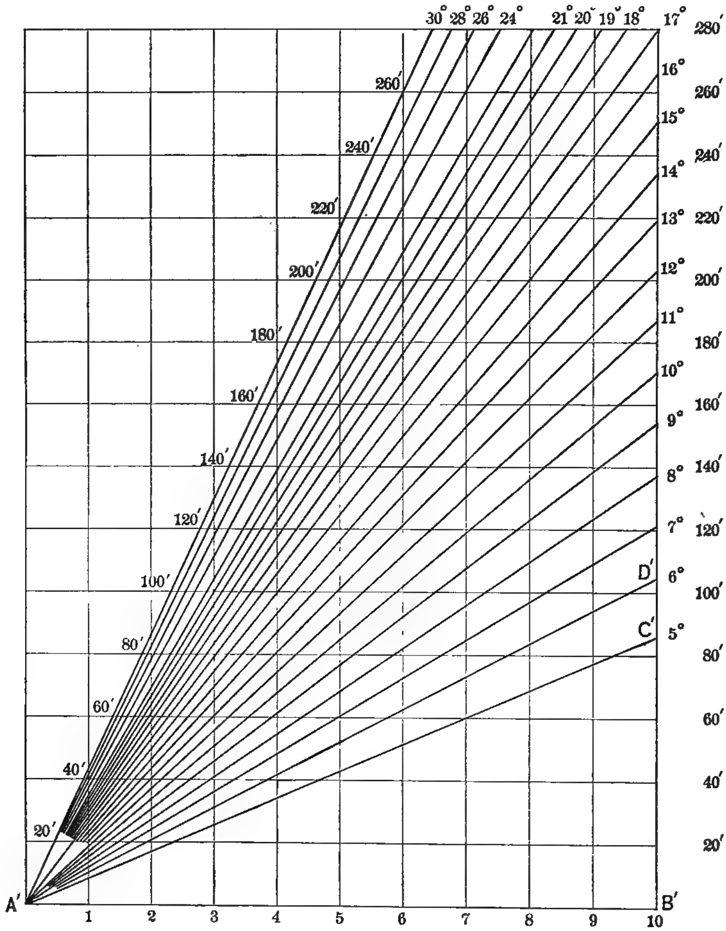


FIG. 227.

It can be used by any one after spending one minute in reading the directions.

The length of this slide rule (50 inches) makes it impossible to show a cut of the complete instrument. The cut shows about $17\frac{1}{2}$ inches of the slide rule near the middle and reduced more than one-half. The scales meet at an obtuse angle for convenient

reading. The arc scale (see cut) slides easily in the groove and will always work easily and never 'pinch.' The slide rule has three indexes, allowing distances to be read in feet, yards or meters, as desired, thus meeting all requirements.

Directions for Using.--Suppose the distance read between two points is 340 feet (see cut) and the vertical angle is 30 minutes; slide the arc scale until the same unit index is opposite 340, the given distance; then upon the logarithmic scale, at a point opposite 30 minutes, the given vertical angle, read 2.97 feet, the difference of elevation sought. If the vertical angle were 1 degree, the difference in elevation for 340 feet would be 5 93 feet (see cut). This simple operation of setting an index opposite a number corre-

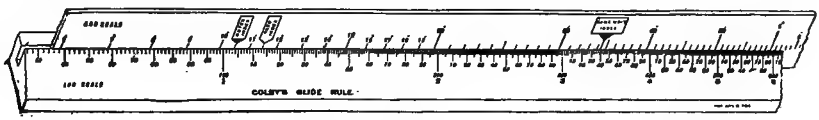


FIG. 228.

sponding to a distance, and then reading a number opposite a given graduation of arc, is all that is necessary in using this slide rule."

141. Plotting Stadia Work.—Having plotted the outline survey by the method of latitudes and longitudes, the stadia traverses running between stations of the outline survey can be put in in the same way. Before this is done, however, the latitudes and longitudes of the stadia traverse lines must be balanced in a manner similar to that described in Art. 47. Often it will suffice to plot the stadia traverses by the second method of Art. 44, checks being applied as there described. The stadia traverses having been laid down, the side-shots are plotted with the assistance of a protractor and scale. If the ordinary semi-circular protractor is used, it is first oriented at the □ and dots made around its circumference to correspond with the azimuths of the different side-shots from that station. The directions having thus been determined, the distances can be scaled off and the points located. Various protractors have been designed for use in this connection. Fig. 229 shows one of the best of these. It is known as "Colby's Protractor." Quoting from the catalogue of Messrs. Keuffel and Esser:

"This instrument can be used for all kinds of protracting, but it is especially designed for plotting notes of surveys made with the stadia. For speed and accuracy in this work it is without a rival.

The limb is graduated from 0° to 360°, 15-minute divisions. Scale on cross-arm has zero mark in center, and is graduated in

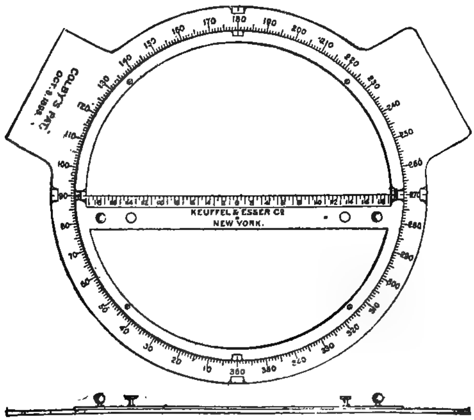


FIG. 229.

both directions in any unit desired. The revolving inner circle with the cross-arm, is raised to prevent rubbing on the paper.

To use this instrument, draw right lines through the station, place zero of scale over station, indexes of limb over the right lines, turn off required angle and plot point at required distance by scale on cross arm.

To hold instrument in position, leaden paper-weights are used on the two ear-pieces."

The arrangement is such that the scale can be unscrewed from the protractor and different ones used in its place.

"Ockerson's Protractor" is somewhat similar to Colby's, but semi-circular in shape. It revolves about a pivot at its center, and the graduation is such that, when centered over any station, if the azimuth division is revolved into the meridian the diametral edge of the protractor will coincide with the line to be drawn. The line may then be laid off along the edge (graduated for the purpose) just as in the Colby form.

Figs. 45 and 46 show other and cheaper forms. A paper protractor may be used, in which case it will be necessary to cut a small opening at its center that the protractor may be oriented over the station on the map. Sometimes also the center of a large paper protractor is cut in the form of a flap. This contains the geometric center of the protractor and is turned down while the instrument is being oriented over the station, and weighted down. It is then turned back to admit of plotting side-shots which do not extend out as far as the edge of the protractor.

Having plotted all the side-shots, each being shown by a dot and circle with the elevation written near, all being penciled in very lightly, the next step will be to put in the contour lines. To do this, first, points on even contours are obtained, by estimation or interpolation. For instance, suppose that the elevations of two adjacent contour points are are 53 and 77.7 feet and that the map is to show even five-foot contours. The 55, 60, 65, 70, and 75 contours will intervene between the points. Ordinarily these may be put in by estimation, making the horizontal distances equal between succeeding contours. If more precision is desired, one of the two following methods of interpolation may be employed: By the first method obtain a sheet of profile paper having, let us say for the sake of illustration, four divisions to the inch horizontal and twenty vertical. In the case mentioned above the difference in elevation of the two points is $77.7 - 53 = 24.7$ feet. Assume a vertical scale suitable to the paper used, in this case, twenty feet to the inch. With a pair of dividers transfer the horizontal distance between the two points from the map to a horizontal line of the profile paper at a distance, to the vertical scale, below one of the horizontal lines on the paper equal to the difference in elevation between the lower point and the next higher even contour, and making one extremity fall on a vertical line as at *AB*, Fig. 230. At *B* take off, to the vertical scale, the perpendicular *BC* equal to the difference in elevation of the two points and join *C* with *A*. Calling the two points *X* and *Y*, the latter being the higher, the horizontal distance from *X* to the next higher even five-foot contour will be given by the line *AD*. Having obtained this point, the succeeding contours will be spaced apart by the equal distances *DE*, *EF*, *FG*, etc. The second method employs a sector similar to

that shown in Fig. 231. In the case just described the horizontal distance between the two points would be taken off on a pair of dividers and the sector opened until the distance between the points on its arms numbered to correspond with the difference in elevation of the two points X and Y is just spanned by the dividers. The sector is now set for interpolation between the points X and Z. For instance, to obtain the horizontal distance from

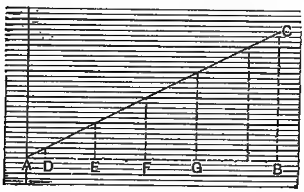


FIG. 230.

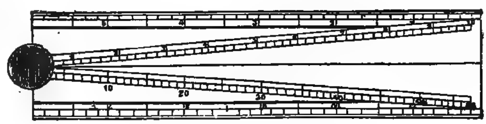


FIG. 231.

X to a point 11 feet higher it is only necessary to spread the dividers between the points marked 11 on the arms of the sector and take off that distance on the map from X towards Y.

The further remarks of Art. 133 apply to topographical maps in general.

142. The Plane Table.—The plane table is another instrument extensively used in topographical work. Its essential elements are a portable drawing table and a combination of a line of sight and a straight edge or ruler situated in the same vertical plane. This is the strict requirement, but if the line of sight and the ruler lie in vertical planes parallel and about an inch or an inch and a half apart, or even making with each other a small angle, the error introduced will generally be inappreciable. Usually they are parallel. The line of sight is generally telescopic, though it may be determined by the plain compass standards. If telescopic it has a vertical motion of rotation measured by a graduated arc, or a complete circle. An accessory is a compass and two level vials attached to a square brass plate. The level vials are sometimes attached to the straight edge, which together with the line of sight is known as the "alidade." The box containing the compass is called the "declinator." There are a number of styles of plane tables differing mostly in the method of attachment of the table or board to the tripod. Fig. 232 shows one of

these. The table proper, about 30 by 24 inches, is fastened to a flanged spherical fitting *b*. This rests in the cup or socket *a*

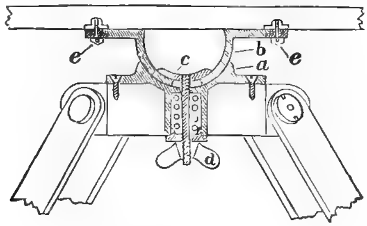


FIG. 232.

screwed to the head of the tripod. When the board is leveled, it is held in that position by tightening the nut *d* and thereby

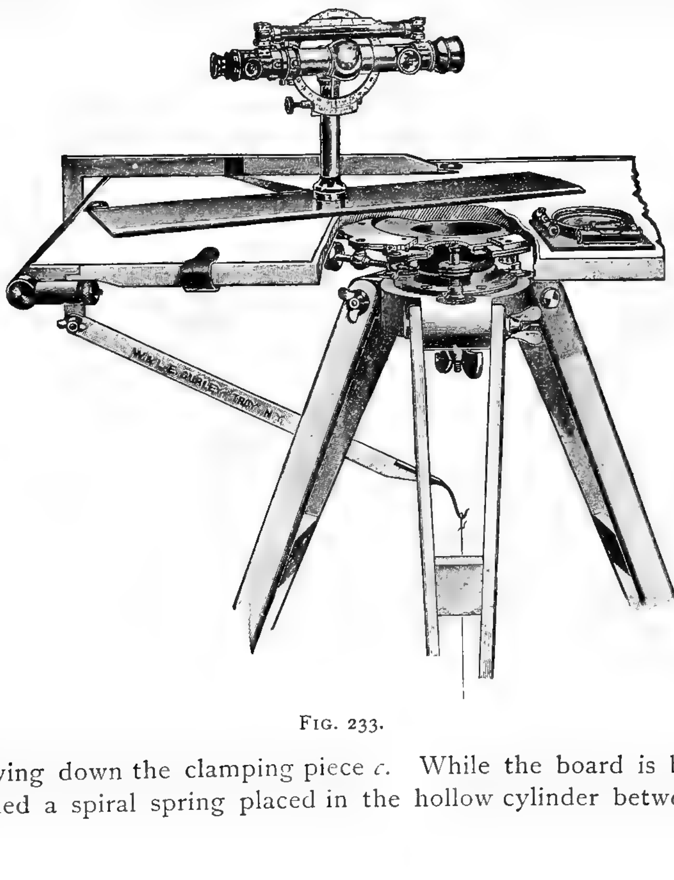


FIG. 233.

drawing down the clamping piece *c*. While the board is being leveled a spiral spring placed in the hollow cylinder between *c*

and *d* insures sufficient tightness. The segments *ee* by which the board is attached to *b* are shaped to permit of revolution of the table in azimuth. One of the segments has passing through it a milled head screw. By means of this screw the azimuth motion can be clamped in any position. A more complex arrangement is illustrated in Fig. 233, which also shows cuts of the compass and level plate, and of the alidade. Here we have a set of leveling screws and a tangent screw similar to the corresponding features in the transit. Fig. 234 shows Johnson's plane-table

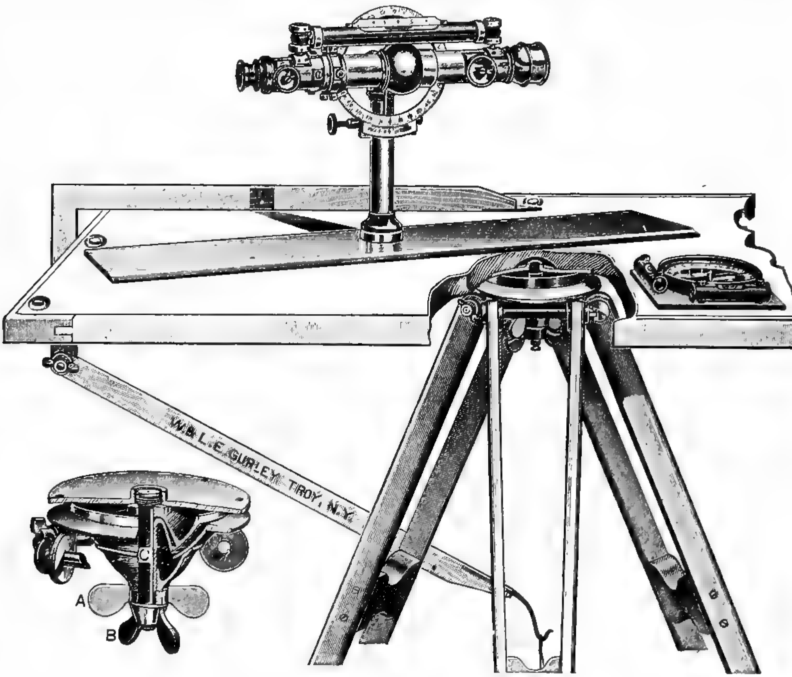


FIG. 234.

movement. This is essentially the same as the movement in Fig. 232, with the addition of the revolving center *C* and the wing nut *B*, shown in the separate cut of the movement in the lower left-hand corner of Fig. 234. The table is leveled and then fastened firmly by the wing nut *A*. To turn the board in azimuth, the nut *B* is loosened. Johnson's movement does not permit as

accurate work as the one described just previously. It is, however, a simpler arrangement. With the plane table maps are drawn directly in the field on paper, in the form of a sheet or roll, laid on the board. The paper is held flat on the board by

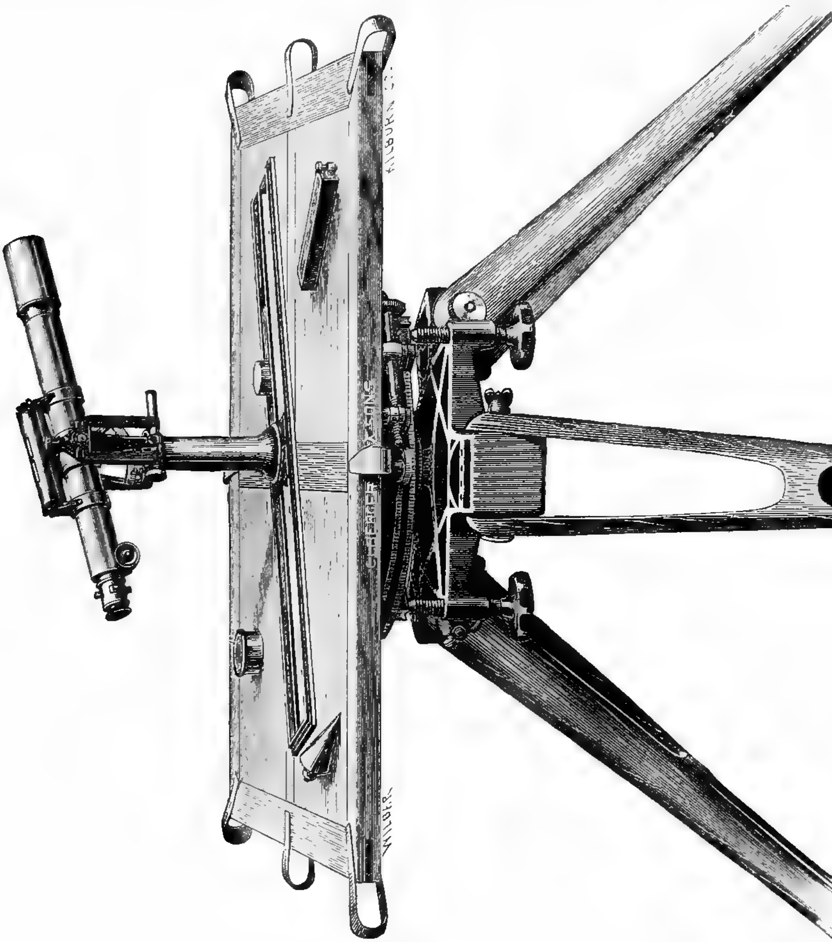


FIG. 235.

clips at the sides as shown in Fig. 233. If the paper is longer than the board, the ends are wrapped on wooden rollers similar to that shown at the left side of the board, Fig. 233.

The base of the alidade consists of a flat rectangular piece of brass or bronze, sometimes with an opening in the center

(Fig. 235) that one of the inner edges may lie in the vertical plane of the telescopic plane of sight. From the center of the base plate or ruler rises a column which supports the telescope supplied with stadia wires, and a vertical arc.

The plumbing arrangement shown in Figs. 233 and 234 is so arranged that if the end of the upper arm is brought in contact with a point on the paper the plumb-bob will hang over a point exactly below on the ground.

143. Orienting the Instrument.—In Art. 105 a definition was given of "orienting" when applied to the transit. The plane table is said to be oriented when, a line being drawn on it to represent a line on the ground, the line on the plane table lies in the same vertical plane with the line on the ground and any one point in the one is situated directly over the corresponding point in the other. Thus suppose the case of a closed traverse, a plot of which is drawn on the plane table. For the instrument to be oriented at one corner of the traverse it would have to be set up at that corner with the corresponding point on the plot directly over it. The two lines intersecting at that corner would be vertically below the corresponding ones on the plot, and all the other lines of the plot would be parallel and proportional to the corresponding lines on the ground. Or suppose the case of any number of lines radiating from a point. For the instrument to be oriented at that point, the corresponding point on the plot must be directly over it and each one of the lines on the plot must be found in the same vertical plane with the corresponding line on the ground. From this the method of orienting will easily be understood. The table is first set up with the point on the plot and the line (or lines) running from it about over the corresponding point and line on the ground. Calling the point P and the farther extremity of a line running from it X, let the plotted line be px. The edge of the ruler is next brought into coincidence with px and an observation taken to determine how far and to which side the line of sight misses X. The table is then revolved on its vertical axis, the tripod legs moved, etc., until p being directly over P, and the edge of the ruler coinciding with px, the line of sight is found directed to X. The operation will be found difficult except in the case where p lies in the vertical axis of the

instrument. A shifting center (Art. 96) would be found convenient here, but would be difficult of construction. A somewhat similar device is attached to a German plane table. Fortunately it is not always necessary for the instrument to be oriented *exactly* over the station occupied. Like the compass, the plane table finds its greatest field of usefulness in the less precise operations where exact orienting would prove a useless refinement.

All of this and the further discussion supposes the line of sight and the edge of the ruler to lie in the same vertical plane. This requirement, though rigid, is often not fulfilled. However, the discrepancy is so slight generally, that, for all practical purposes, the definitions and explanations here given will still hold.

The purpose of the plane table is to furnish a means of making a map at once, in the field. Its general use is best explained by descriptions of its use in special cases. These are divided into two classes as the point to be plotted is found removed from, or directly at, the station occupied by the instrument.

144. Problems of the First Class.—(1) The method of radiation. Set the instrument at any point from which are visible all the points desired to be plotted. Assuming any point on the paper, keep the edge of the ruler in contact with it while the line of sight is directed in succession to each of the desired points. A line being drawn along the ruler in each case from the assumed point towards the desired point, and the proper distance, determined by direct measurement or stadia methods, scaled off, the points are located on the paper. By joining these by lines, a plot of the entire tract can be had.

(2) The method of intersections. As before, set up at a point from which all the other points are visible, and rule indefinite lines towards them but without scaling off the distances, or, necessarily, determining them at all, except in the case of one, which may be either the distance to one of the points to be plotted, or to any other conveniently assumed point. Next move the instrument to this point, orient it there and draw lines towards the other points as before. The intersections of these lines with those drawn from the station first occupied will locate the points on the plot. Care should be taken to have the intersections if possible at angles between 30 and 150 degrees. If desired, the instrument can now

be moved to a third station and the points located again as a check on the first determination. If the lengths of the lines have been already found by stadia methods, another check can be had by comparing these with the lengths as scaled from the plot. The whole operation is illustrated in Fig. 236.

(3) The method of traversing or progression. If a traverse is to be plotted by this method, set up the instrument over the first

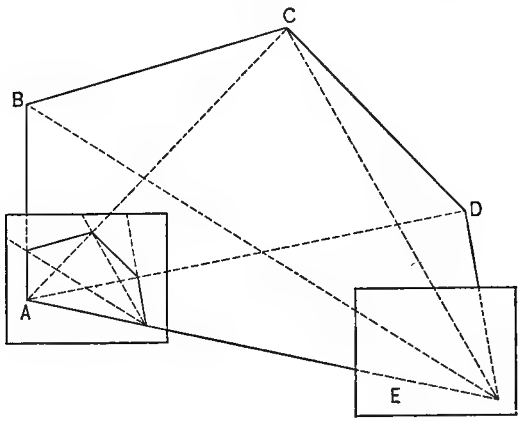


FIG. 236.

station, bring the ruler into contact with the point over the station, direct the line of sight to the second, and plot to scale the line joining the two, its length having been found by stadia methods or by direct measurement. Next, set up the instrument at the second station, orient, plot the line to the third station, and so proceed. If the traverse is closed, the plotting will finally return to the first station, which should be located at the point assumed for it at first, thus furnishing a check.

(4) The method of radio progression. The method of traversing requires the instrument to be oriented at each station occupied. In practice this will be found tedious. In the method now to be described, by a combination of the methods of radiation and traversing, this difficulty is avoided. Fig. 237 represents a tract *ABCDE* to be plotted by the method of radio progression. The instrument is first set up at any corner as *A*, with the *center of the table*, which should be marked by a small plate of brass, let in

flush, and having a pin-hole to indicate its exact intersection by the vertical axis of the instrument, directly over the station occupied. The center of the table being indicated by p, the lines $p(ea)$ and $p(ab)$ are next drawn in the directions EA and AB, and equal to scale to those lines. Moving the instrument to B, it is set up with its center over that point, the lines already drawn are brought parallel to the corresponding ones on the ground, and the line $p(bc)$ drawn in a similar manner, and so on, occupying

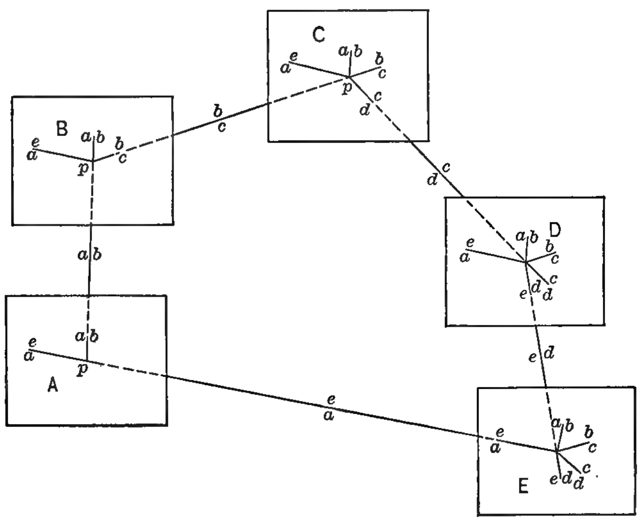


FIG. 237.

each station. When the last station is occupied a check is had by the coincidence of the line drawn in the direction of the first station, with that previously drawn from the first station in the same direction. Indeed a check of this kind can be had at any time during the progress of the work by sighting across the tract from any station X to any other station Y from which a sight has previously been taken to X. All the stations having been occupied, there will appear on the table a series of lines radiating from the center p and numbered (ab), (bc), (cd), etc. Then by starting at any point and drawing a line ab equal and parallel to $p(ab)$, from b drawing a line bc equal and parallel to $p(bc)$, etc., a plot of the field will be made. Note that the lines should all be drawn in the

directions from p towards the lettered ends. As a final check, the finished plot should close.

145. Problems of the Second Class.—(1) To locate a point not plotted on the table, but from which are visible three plotted points. The "three-point problem." Set up the instrument about over the point and fasten a piece of tracing paper on the table. Assume a point p on the tracing paper directly above the point P on the ground which it is desired to locate on the plot, and bring the edge of the ruler over it, at the same time turning the line of sight on one, as A, of the three points A, B, and C, plotted on the table at a, b, and c. Draw an indefinite line pa' along the ruler.

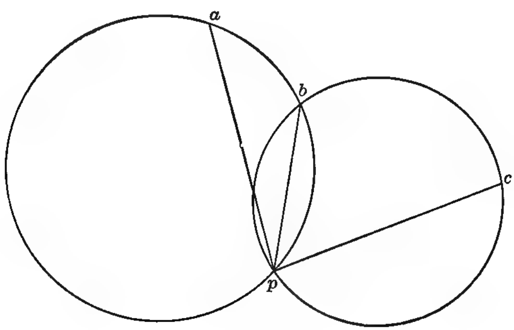


FIG. 238.

Proceed in the same way with the points B and C, when there will appear on the tracing paper three lines, pa', pb', and pc', radiating from the point p. Remove the alidade and shift the tracing paper until the lines pa', pb', and pc' pass through a, b, and c respectively. While in this position prick the point p through to the paper beneath. It is now plotted, and the table may be oriented and the work proceed as in Art. 144.

Evidently, generally speaking, the point p has been correctly plotted with reference to a, b, and c. For, suppose the plot which already shows a, b, and c completed by the addition of p. The lines ap and bp, and bp and cp must make with each other the angles $a'pb'$ and $b'pc'$ drawn on the tracing paper. But suppose lines drawn through a, b, and c making with each other these angles and

meeting in any common point p. A circle, Fig. 238, can be passed through the three points, a, b, and p and as long as the angle apb remains constant, the point p must lie on its circumference.* In like manner p must lie on the circumference of a circle passed through it, b, and c. But these two circles, unless they coincide, meet at only two points, b and one other. Therefore, unless the point P lies on the circle through A, B, and C, the lines passing through a, b, and c and making angles as given by the tracing paper, can locate only one point which coincides with the true position of p.

(2) To locate a point not plotted on the table, and from which are visible only two plotted points. The "two-point problem." Let the two visible points be A and B plotted at a and b. C, Fig. 239, being the point to be located, select a fourth point D such that

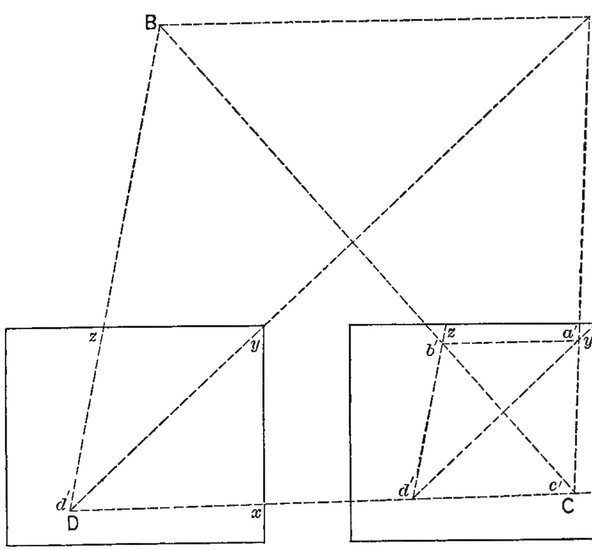


FIG. 239.

the angles CAD and CBD will lie between 30° and 120°. Set up over D, plot it on the paper at d', and draw indefinite lines $d'x$, $d'y$, $d'z$ in the directions of C, A, and B. On $d'x$ assume a point

* Beman & Smith, Geom., Bk. I, Th. 31, Cor. 2, and Bk. III, Th. 11, Cor. 3.

c' such that the distance $d'c'$, to the scale to which a and b have been plotted, equals the *estimated* distance from D to C. Next, assuming C to be correctly plotted at c', set up at C and orient by sighting to D. Draw lines $c'a'$ in the direction of A, and $c'b'$ in the direction of B, intersecting the lines $d'y$ and $d'z$ in the points a' and b'. Join a' and b'. The quadrilateral $a'b'c'd'$ is similar to $ABCD$. Construct on the tracing paper another similar quadrilateral $abcd$, having the side ab equal to the line ab as plotted on the plane table map. Place the tracing paper on the table so that the points a and b on it coincide with a and b on the plot beneath. By pricking through the point b, its location on the plot is finally obtained. By making the construction on paper the correctness of this method can easily be seen.

If an assistant with a stadia rod can be sent to A and B, with the table set up at C, the distance AC and BC can be determined. Then by swinging arcs from a and b on the plot, c may be located at once.

146. General Remarks.—The plane table is used most extensively by the U. S. Coast and Geodetic Survey to map the topographical features of areas whose outlines have previously been determined by other methods. If as many as two stations of the outline survey are plotted on the plane table sheet, the instrument can be oriented at any point by the two-point problem and the work carried on. In practice no one method of work is adhered to exclusively, combinations of methods peculiarly suited to the work in hand being adopted. For ordinary topographical work, stadia, in preference to plane table, methods will usually be adopted, though advantages attach to the latter owing to the mechanical way in which they may be carried out and the facility with which the plot may be prepared. The plane table should be set up low enough to enable the operator to reach any part of it with ease. When a point is once occupied and the instrument oriented, care should be exercised not to disturb the position of the table before leaving the station. As a check on this, after all the points have been plotted at any station, the orienting should again be tested. The bearing of any plotted line can be determined by bringing the edge of the declinator into coincidence with it and reading the needle. Sometimes the declination is in

the form of a "trough compass," that is, an oblong box permitting the needle to swing only 10° or 15° on either side of the zero line. In any case the outside edge of the box is parallel with the line joining the zeros of the graduation. At the beginning of any work, it will be found convenient to draw a meridian line near the center of the sheet. This will aid in orienting the instrument approximately at any new station. From the finished plot, angles, bearings, and distances can be determined by the aid of a protractor and scale, and with these data, areas calculated.

147. Tests of the Plane Table.—(1) The test of the table. The surface of the table should be a perfect plane. This condition may be tested by a straight edge applied in all directions. If the surface proves warped or uneven, it must be made perfectly flat and smooth or a new table supplied.

(2) The test of the edge of the ruler. This should be a mathematically straight line. Test the condition by placing the ruler on the table and drawing a line along its edge. Then reverse the ruler end for end, place it in coincidence with the ends of the line and draw another line along its edge. If the two lines coincide, the edge is straight, except in the very improbable case where are found a concavity and a convexity of exactly the same shape and size and at the same distance from the ends.

148. Adjustments.—In some forms of alidade (Figs. 233 and 234) the telescope can be transited. In others (Fig. 235) it cannot. For a reversal in altitude (Art. 67, 1, Test) with the latter form, the telescope, with its transverse axis, will have to be lifted from the bearings and turned end for end, care being taken that the transverse axis is not reversed during the operation. The reversal in azimuth is easily accomplished by revolving the whole table, with the alidade resting on it, around its vertical axis. In connection with this, in the further discussion of the adjustments it will be understood that, while a reversal in azimuth is taking place, the vertical axis of the instrument intersects the axis of figure of the telescope tube. This condition can be made to exist closely enough by placing the alidade properly on the table.

(1) The adjustment to make the tangents (Art. 52, 1, Ex.) of the bubble tubes parallel to the base of the declinator or of the alidade. The tubes may be attached either to the declinator or

to the alidade. We assume that they are found on the declinator.

Test. By means of the leveling screws bring one bubble to the center, the declinator being placed at the middle of the table. Draw lines around the edges of the declinator, reverse it end for end, so that it will have the same position as before, and note whether the bubble settles again in the center. If so, the test is satisfied.

Adjustment. If not, by means of the adjusting screws attached to the level, bring the bubble half-way back to the center. Repeat the test and adjustment until the test is satisfied, treating the other tube in the same way.

Explanation. By Art. 51, 4, Figs. 77 and 78, it is shown that if a bubble tube, with the bubble in *any* position, be placed on a plane, and then turned end for end, if the bubble remains in the *same* position the plane is horizontal. Now, if the position of the bubble be *at the center*, and remain so after being turned around end for end, it would show not only that the plane was horizontal, but also that the tangent of the tube was parallel to it, and consequently parallel to its base resting on the plane. That the bubble should be brought *half-way* back to the center can be proved in a manner similar to that of Art. 52, 1, Fig. 82.

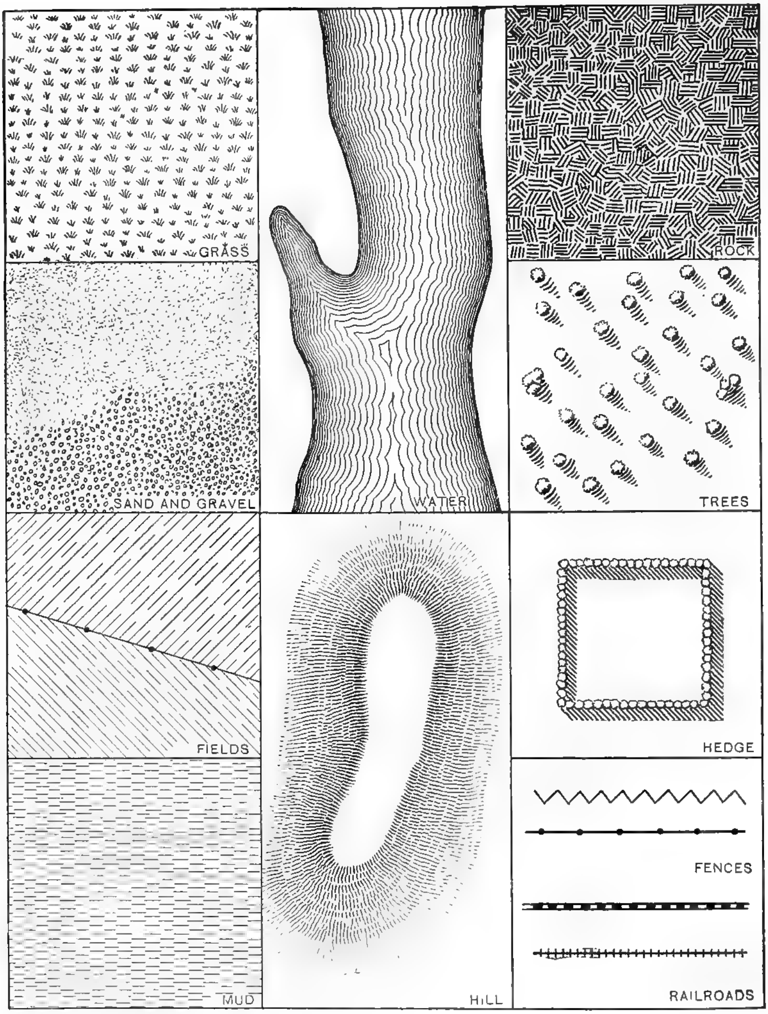
(2) The adjustment to make the surface of the table perpendicular to the vertical axis of the instrument.

Test. Having found the first adjustment satisfied, place the declinator on the table, and, by means of the leveling screws, bring both bubbles to the center. Revolve the table in azimuth 180° and note whether the bubbles remain at the center. If so, the test is satisfied.

Adjustment. If not, by the insertion of washers between the table and the support to which it is screwed, correct half the error. Test again, adjusting if necessary until the test is satisfied.

Explanation. From the similarity of this adjustment to others previously described, it is thought that the student can construct a sketch and explanation. Instead of the levels attached to the declinator or base plate of the alidade, the level under the telescope may be employed. In this case, it will be necessary to

PLATE III.


GRASS
WATER
ROCK
SAND AND GRAVEL
TREES
FIELDS
HEDGE
MUD
HILL
FENCES
RAILROADS

make the test with the bubble in two positions at right angles to each other.

(3) The adjustment to make the line of sight intersect the horizontal and vertical axes.

This adjustment is the same as the second adjustment of the transit, Art. 111. It will, however, probably be more convenient to make a special pair of wooden wyes and adjust the instrument as in Arts. 65 and 84, 1.

(4) The adjustment to make the line of sight perpendicular to the horizontal axis. This adjustment is the same as the third adjustment of the transit, Art. 111.

(5) The adjustment to make the transverse axis truly horizontal. This adjustment is the same as the fourth adjustment of the transit, Art. 111, the supports being raised or lowered in different ways according to the make of instrument.

(6) The adjustment to make parallel with the line of sight the tangent of the bubble tube attached to the telescope. This adjustment is the same as the fifth adjustment of the transit, Art. 111, and the second adjustment of the level by the second general method, Art. 84.

(7) The adjustment to make the vertical circle read zero when the line of sight is horizontal. This adjustment is the same as the sixth adjustment of the transit, Art. 111.

149. Conventional Symbols.—Some of these are given in Plate III. Under "hill" is shown the method of representing slopes by means of "hatch lines," the lines growing heavier as the slopes grow steeper. Practically, this method is much inferior to, and also much more tedious than, that by means of contour lines. In fact, in this connection, the latter method is now used almost entirely.

CHAPTER IX.

HYDROGRAPHIC SURVEYING.

150. Definition and Purposes.—A hydrographic survey is a survey of a stream, lake, or any body of water. Its purpose is twofold, to obtain the necessary data from which may be drawn, first, an outline map of the body of water surveyed, and, second, a contour map of the channel or basin in which it is contained. The calculation of reservoir and lake capacity, or of stream velocity and discharge, is here omitted as pertaining more properly to the science of hydraulics.

Hydrographic and topographical surveys are seen to be entirely similar. In the present case, however, the contours will correspond to distances measured *downward* from the surface of the water assumed as a datum plane.

151. The Outline Survey.—The outline survey may be made by any of the methods described in the preceding chapters. In the case of a small stream a compass or transit traverse run along one bank will serve the purpose, offsets being taken to the edge, and the width of the stream noted, at sufficiently close intervals. This method of survey is known as "meandering." It may be extended to the case of a small lake by running the traverse entirely around, closure being had on the beginning point. The usual method of obtaining the outline of a body of water, however, is by means of a triangulation survey, the bank being meandered between adjacent stations, all as described in Art. 106. This meandering may often be omitted, and the shore, between triangulation stations, sketched in by eye simply. The observer will be aided in this by noting the *relative* lengths of

different lines. To illustrate, let Fig. 240 represent a sketch of a shore line between two triangulation stations A and B. The observer will note that, approximately, $AC = \frac{1}{2}AB$, $CD = DE = EB = \frac{1}{6}AB$, $CF = \frac{1}{4}AB$, and so on, these relative dimensions

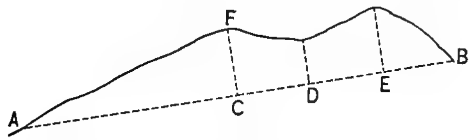


FIG. 240.

being sketched in the field-book. Bearings taken with a hand compass will prove of assistance here.

152. Soundings.—The outline survey having been completed, it next becomes necessary to determine the elevations with respect to the surface of the water of a sufficient number of points on the bottom to enable the contour map to be drawn. This requires soundings. They may be made, up to depths of fifteen feet, with a pole graduated to feet and tenths. For depths over fifteen feet a lead line is used. This consists simply of a stout well stretched line, graduated in convenient units and having a sufficiently heavy weight attached. If only the depth of the water is desired, this weight may be of any convenient shape, preferably long and slender. Ten pounds will be found sufficiently heavy for depths up to about 40 feet. For greater depths a weight of fifteen to twenty pounds should be used.

It is recommended that the line should be of hemp three eighths of an inch in diameter, and stretched by winding tightly around a smooth tree, fastening in this position, wetting thoroughly, and allowing to dry. This operation is repeated until the line shows no appreciable slack, though care should be exercised not to carry the stretching too far. When finally stretched the line is graduated by attaching suitably marked leather tags at proper intervals. They should be fastened into the strands. Up to depths of 100 feet the tags may be marked with points and notches similar to those found on the ordinary chain, though if preferred the numbers may be stamped on. It is usual to give the depths in feet up to four fathoms (24 feet) and from that on in fathoms. To insure agreement with the standard

the line should be frequently tested. In cases where some knowledge of the character of the bottom is desired, the sounding lead is made as shown in Fig. 241. The rod *R* has moulded around it the weight *W*, and attached to its lower end the cup *C* covered by the leather washer *L*. As the lead sinks, the washer rises. When the bottom is reached the cup penetrates far enough to collect some of the material, which is kept in by the washer, as the lead is being pulled up.

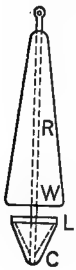


FIG. 241.

The method of using both sounding rod and lead line will come easily with a little practice. It must be borne in mind that the observation must be taken with the rod or line vertical. If the soundings are taken while the boat is in motion, this will require the lead to be thrown out considerably in advance so that the line will become vertical just as the boat is over the point.

153. **The Sextant.**—Before discussing the methods of locating contour points, an instrument used in this connection will be described. The sextant, Fig. 242, is a hand instrument for measuring angles. Its essential elements are a telescopic line of sight, a graduated arc, and two plane mirrors. The graduated arc LL', Fig. 242, forms part of the frame of the instrument. The arm A rotates around an axis C, perpendicular to the plane of LL' at its center. The mirror M_1 is attached rigidly to A and lies in a plane perpendicular to that of LL'. The vernier V indicates on LL' the amount of revolution of A. M_1 is known as the "index glass." The second mirror or "horizon glass" M_2 is attached rigidly to the frame of the instrument and in a plane perpendicular to that of LL'. Only the lower half of M_2 is silvered, its upper half being transparent. The telescope T, also permanently attached to the frame, is directed towards M_2. On looking through it, objects can be seen, both in the direct line of sight through the upper part, and reflected from the lower part, of M_2. When the vernier reads zero the planes of the mirrors M_1 and M_2 are parallel. Fig. 243 shows a diagram of the instrument with the mirrors in this position. Note that, though the sextant is often so constructed, the line joining C with the zero of the vernier does not neces-

sarily lie in the plane of M_1. The same statement may be made in regard to the axis C. From Fig. 243 it is evident that, provided the arc LL' is graduated in the ordinary manner, as A is revolved along LL', V will read the angle between the planes of M_1 and M_2. Now suppose, Fig. 244, that the line of sight of the telescope,

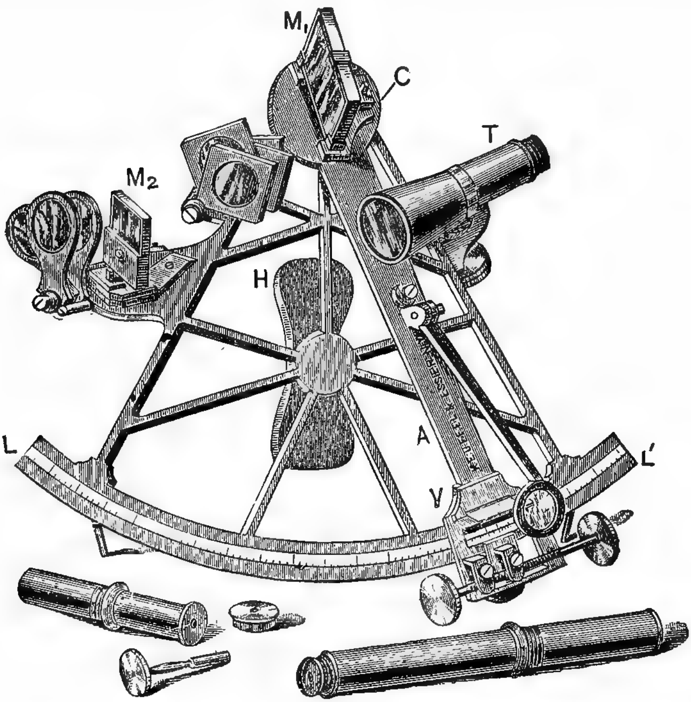


FIG. 242.

through the upper and unsilvered part of M_2, is directed to a point P, and that a point Q is seen through the telescope, coinciding with P, but reflected from the lower half of M_2. The ray of light, QB, proceeding from Q, will have impinged on M_1, been reflected from there to M_2 and from M_2 along the line of sight of the telescope. From the principles of Optics, if the planes of the mirrors make with each other the angle θ, the rays QB and $B'E$ will meet at the angle $QSP = 2\theta$. Now, on the arc LL', *half* degrees are numbered as *whole* degrees, etc.

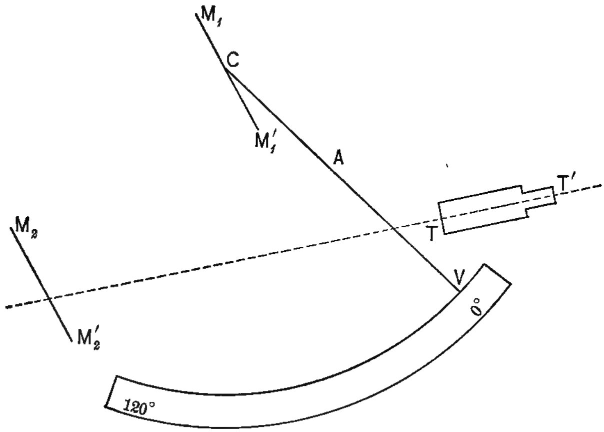


FIG. 243.

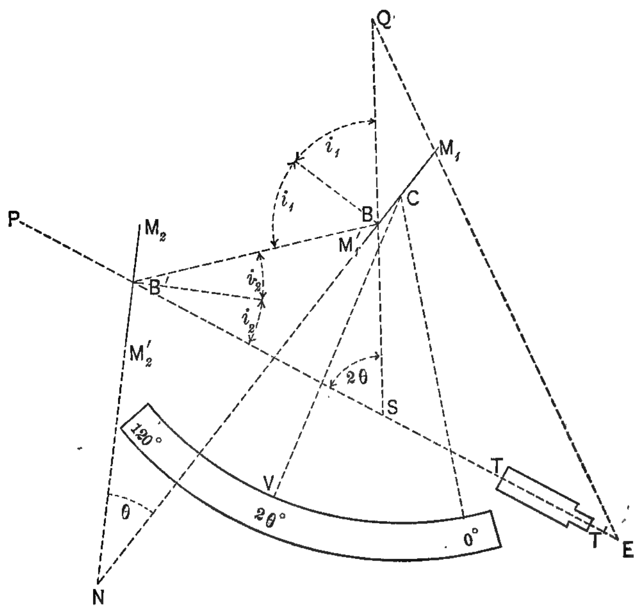


FIG. 244.

Consequently, if the observer holds the sextant with the arm A in such a position that on looking through the telescope any point P appears to coincide with any point Q, the vernier will read the angle between the two points. The angle read is QSP. This is practically equal to QEP, as SE is very small in comparison with QE and PE. Actually a cone or pencil of rays proceeds from Q and impinges on M_1, as shown in Fig. 245. The ray QB

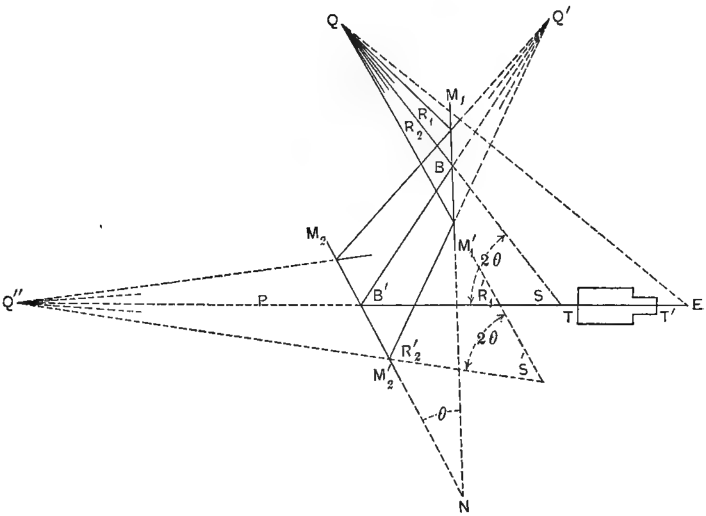


FIG. 245.

Fig. 244, is the particular ray R_1, Fig. 245, which, after the double reflection, passes through the optical center of the telescope object-glass. However, any other ray, R_2, would make the same angle 2θ with its position R_2' after the double reflection, so that the exact position of the telescope is not a matter of great importance, provided its line of sight is parallel to the plane of LL' (Art. 159).

In measuring an angle the sextant is held by the wooden handle H, Fig. 242. It is a particularly useful instrument when the observer is stationed in a boat, though possessing, under other circumstances, the advantages of fair accuracy and great portability.

154. Wood's Double Sextant.—This instrument is a combination of two sextants in one. With it two angles can be read at the same time as AOB and BOC, Fig. 246. Under certain circumstances, it becomes particularly useful. (Art. 155, 1.)

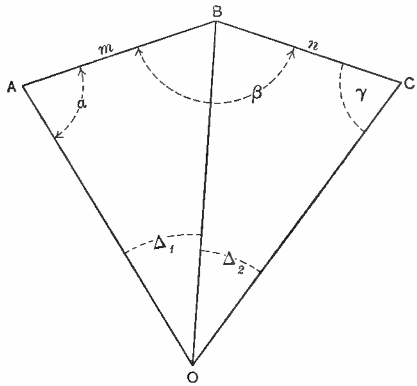


FIG. 246.

155. Locating the Soundings.—This can be done in a number of ways:

(1) By determining from the boat two angles between three fixed points on shore. All things considered, this is probably

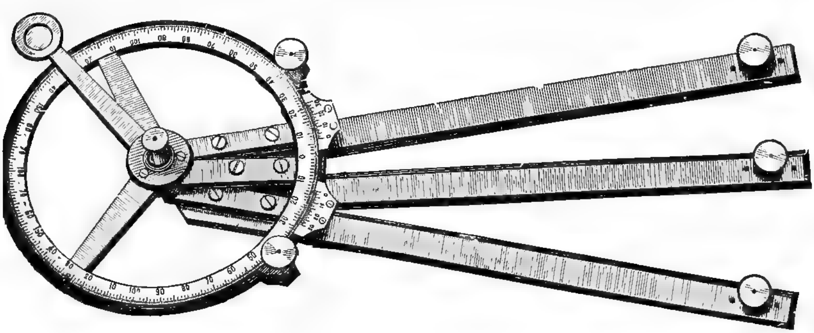


FIG. 247.

the best general method. At the time the sounding is taken in the boat, two observers stationed there with sextants, or one observer with a double sextant, measures two angles between three fixed points on shore, as the angles AOB and BOC be-

tween the points A, B, and C, from the boat O, Fig. 246. If these two angles be plotted on tracing cloth, and the lines OA, OB, and OC be made to pass through the points A, B, and C previously plotted on the outline survey, O will indicate the position of the boat. (See Art. 145, 1.) An instrument which serves the purpose better than the tracing cloth is the three-arm protractor shown in Fig. 247. The verniers attached to this instrument read to single minutes. The edges of the three arms shown in the cut intersect at the center of the protractor and correspond to the lines OA, OB, and OC of Fig. 246.

The method with the three-arm protractor is the most practical and the one commonly employed. However, the following analytical method may sometimes prove useful: In Fig. 246, if the distances AO and CO are known, by swinging arcs of these lengths from the points A and C of the outline survey, their intersection will give O, the point desired. The distances m and n and the angle β are known from the previous work.

$$OB = \frac{m \sin \alpha}{\sin \Delta_1} = \frac{n \sin \gamma}{\sin \Delta_2}, \quad . \quad . \quad . \quad . \quad . \qquad (105)$$

$$\alpha + \beta + \gamma + \Delta_1 + \Delta_2 = 2 \times 180° = 360°, \quad . \quad . \qquad (106)$$

and

$$\alpha + \gamma = 360 - (\beta + \Delta_1 + \Delta_2) = S, \quad . \quad . \quad . \quad . \qquad (107)$$

from which

$$\gamma = S - \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (108)$$

and

$$\sin \gamma = \sin S \cos \alpha - \cos S \sin \alpha. \quad . \quad . \quad . \quad . \qquad (109)$$

Substituting this value of $\sin \gamma$ in (105),

$$\frac{m \sin \alpha}{\sin \Delta_1} = \frac{n (\sin S \cos \alpha - \cos S \sin \alpha)}{\sin \Delta_2}, \quad . \quad . \qquad (110)$$

from which, after reduction,

$$\cot \alpha = \cot S \left(\frac{m \sin \Delta_2}{n \sin \Delta_1 \cos S} + 1 \right) \quad . \quad . \qquad (111)$$

The value of cot γ can be determined in the same way. With α and γ known, the distances OA and OC may be computed by the law of sines.

The location of the soundings by means of angles measured from the boat is seen to be a case of the three-point problem (Art. 145, 1). As there noted, the method fails when the point O lies on the circumference of the circle passed through A, B, and C. If the boat is anchored at each sounding, one observer may do the work, with the plain sextant, reading first one angle and then the other.

(2) By compass bearings read from the boat to two fixed points on shore. The bearings being taken in this manner, the intersection of lines having the reverse bearings and laid off on the map from the fixed points will give the points desired This method is rather inaccurate, but a check may be had by an observation on a third point.

(3) By stadia methods. The objection to this method is the difficulty of handling the stadia rod in the boat and the fact that long sights cannot well be taken. Also, no check is offered unless a second instrument is used, in which case method (4) would probably be employed.

(4) By the measurement, with two observers and instruments at two fixed points on shore, of the angles made by the lines from the instruments to the boat with any fixed lines of the outline survey. Usually the observers will be stationed at the opposite ends of a line connecting two adjacent outline survey stations and the angles will be measured with zeros on this line. Either sextants or transits may be used, preferably the latter. The intersection of the two lines whose positions are determined by the fixed points and angles made with the fixed lines will give the location of the boat. A third instrument used in a similar manner will furnish a check.

(5) By soundings, taken along a fixed line, the angle made by two lines radiating from a fixed point on the shore, one to the boat and the other to a second fixed point, being measured by an intrument at the first fixed point. No check is offered

unless a second instrument is used, in which case (4) is again a better method.

(6) By taking soundings along a fixed line at equal intervals of time, the speed of the boat being kept uniform.

(7) By taking soundings along a fixed line marked by a graduated wire. This is an accurate method, but adapted only to narrow channels.

(8) By taking the soundings on the ice, the points being located by any of the methods of the preceding chapters. This is the most accurate method of all, but generally impracticable.

156. Remarks on the Different Methods.—As above noted (1) is about the best general method. With methods (3), (4), (5); and, perhaps, (8), some scheme will have to be employed to insure the proper soundings and corresponding shore observations being used together. With method (4), for instance, it is usual for the sounding party and shore observers to set their watches together. The observers then read an angle every even minute, while the sounding party measure a depth every quarter minute, the time being recorded with the observation in the note-book in each case. When the parties join, the proper soundings are entered in the observers' field-books opposite the corresponding angles. The soundings are taken along regular lines as far as may be practicable. A steam launch will be found particulary useful here. When a rowboat is used, if the current is sufficient, it will sometimes be convenient to start at a point up stream, and take soundings as the boat drifts down. At all events, it should be borne in mind that more points will be needed than in the corresponding case on a land topographical survey. The depths should, of course, all be referred to the same datum, low-water level, high-water level, etc. This is particularly important in the case of tidal waters, where the datum plane is generally that of mean tide. In such a case a "tide gauge" consisting of a plank painted white and graduated to feet and tenths is fastened in a vertical position near the shore. Enough readings should be had on this, as the soundings are being taken, to insure their reference to the same datum.

157. Notes.—The style of notes of the soundings will vary with the methods employed. With method (1) the form of notes shown in Fig. 248 may be employed.

If method (4) is employed three sets of notes will have to be kept, one by each of the shore observers and one by the sounding party. The sounding party's notes will be headed as in Fig. 248, giving the name and date of the survey, etc.

SURVEY OF HUDSON RIVER
JUNE 25, 1900
SEXTANT NO 1, A LEADSMAN; C
" " 2, B NOTES D
TIDE CORRECTION = + 0.6

SEXTANT NO. 1		SEXTANT NO. 2		CORRECTED DEPTH
STAS.	ANGLE	STAS	ANGLE	
7—9	49° 13	9—11	48° 06′	23.4
"	47 10	"	50 02	24.6
"	44 23	"	51 50	24,9

FIG. 248.

The notes proper will stand in two columns as shown in Fig. 249:

Time.	Corrected Depth.
A.M. 10.30	20.4
	21.0
	21.2
	21.3
31	21.8
	21.6
	22.2
	22.7
32	23.0

FIG. 249.

Fig. 250 shows the notes kept by observer No. 1 when completed by the addition of the notes of observer No. 2 and of the sounding party.

SURVEY OF HUDSON RIVER,
June 17, 1900.
Observer No. 1, Smith on sta. 5, zeros on sta. 7.
Observer No. 2 on sta. 7, zeros on sta. 5.

Time.	No. 1.	No. 2.	Corrected Depth.
A.M.			
10.30	13° 00	12° 10′	20.4
			21.
			21.2
			21.3
10.31	21° 05′	18° 00′	21.8
			21.6
			21.2
			22.7
10.32	30° 15′	29° 32′	23.0

FIG. 250.

With the other methods, suitable forms of notes will suggest themselves.

158. Drawing the Map.—This is done in accordance with the principles stated in previous articles, the only distinguish-

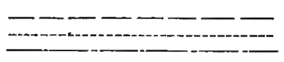


FIG. 251.

ing feature being the lines drawn to indicate the depths. A separate style of line may be drawn for each contour, as shown in Fig. 251. This, however, is not absolutely necessary. If the topographical features of the shore are to be shown it may be done by the methods of Art. 149.

159. Adjustments of the Sextant.—(1) The adjustment to make the plane of the index glass perpendicular to the plane of the graduated arc.

Test. Set the vernier near the middle of the arc, and, with the eye about in its plane and near the index glass note whether the part of the arc towards the zero division, seen directly, appears to form a continuous plane surface with the part towards the other end as seen reflected from the glass. If so, the adjustment is correct.

Adjustment. If not, adjust by loosening the screws which attach to the arm the plate holding the index mirror, and inserting thin slips of paper. With some instruments, the adjustment can be effected by the use of the screws merely.

(2) The adjustment to make the plane of the horizon glass perpendicular to the plane of the graduated arc.

Test. The index glass having been adjusted, direct the line of sight through the unsilvered part of the horizon glass, to a star, and move the arm with the index glass until the reflected image of the star appears to pass the direct image. If the images pass exactly over each other, the adjustment is correct

Adjustment. If not, the adjustment is effected in a manner similar to that of the first adjustment.

(3) The adjustment to make the line of sight of the telescope parallel to the plane of the graduated arc.

Test. Place the sextant on a table, resting it on the three legs which project from the back of the frame. Obtain two peep sights, as shown in Fig. 252, and place them on the

FIG. 252.

graduated arc. These sights should come with the instrument. Sight through them and mark a point, say on a wall, not less than 15 or 20 feet distant. Sight through the telescope in the direction of the mark on the wall and note when its line of sight cuts the wall with reference to the mark. If the height above the plane of the graduated arc of the peep-sight line of sight is the same as that of the center of the ring into which the telescope is screwed, as should be the case (the height of the ring is adjustable), the telescope line of sight should intersect the mark. Otherwise it should miss it by a distance equal to the difference in elevation of the two sight lines.

Adjustment. If the test is not fulfilled the ring into which the telescope is screwed is attached to the frame of the instrument by means of two screws, fastening it to a second ring, which, in turn, is fastened to the frame. By means of these screws the position of the telescope with reference to the frame

may be varied slightly and the telescope brought into proper adjustment.

(4) The adjustment to make the vernier read zero when the mirrors are parallel.

Test. Sight at a very distant object and bring the direct and reflected images into coincidence. If the vernier now reads zero, the adjustment is correct.

Adjustment. If not, and one of the mirrors is supplied with screws for the purpose, it can be revolved around a vertical axis until the test is satisfied. Usually, however, the error is merely noted and applied as a correction to all observations.

CHAPTER X.

MINE SURVEYING.

By WILLIAM S. HALL, C.E., E.M., M.S., Professor of Mining Engineering, Lafayette College.

SECTION I.

Definitions of Mining Terms.

160. The following are brief definitions of a few mining terms that occur in this chapter:

Bed.—A mineral seam that occurs in stratified rocks and which is conformable to the stratification of the country rock.

Vein or Lode.—A mineral deposit that fills a fissure in the rocks. Veins are usually highly inclined.

Strike.—The strike of a bed or vein is the direction of the intersection of a horizontal plane with the bed or vein. A level in the deposit takes the direction of the strike.

Dip.—The dip of a bed or vein is the angle which the most highly inclined line in it makes with the horizon. Therefore, the dip is the inclination below the horizontal of a line in the bed or vein perpendicular to the strike.

Adit.—A horizontal passage from the surface into a mine and driven in the mineral deposit.

Shaft.—A vertical pit sunk from the surface.

Slope or Incline.—An inclined opening, usually driven from the surface in the seam.

Tunnel.—A tunnel is a horizontal passageway through a hill with both ends open to the surface. But in mining it is frequently employed to designate a horizontal passage through

the country rock, one or both of whose ends may be in the mineral deposit.

Level.—A horizontal passage in a mine. Mines are usually worked by running levels at different depths in the deposit.

Gangway.—A principal level that is commonly used as a haulage road.

Hanging-wall, Roof or Top.—The wall of rock above the bed or vein.

Foot-wall, Floor or Bottom.—The wall of rock below the bed or vein.

SECTION II.

Importance and Difficulties of Mine Surveying.

161. There is no branch of surveying which requires more careful methods and more systematic prosecution of the work than mine surveying. Some reasons for insisting upon accurate work in mine surveys are as follows: (*a*) The great value and limited extent of mineral deposits. (*b*) The frequent necessity for leaving barrier pillars to separate adjacent mines, and the necessity for supporting the roof and protecting the passage-ways by pillars of definite size. (*c*) Royalties are often based upon the underground surveys. (*d*) The best location for a shaft, slope or tunnel, and the best methods of exploitation, drainage, underground transportation, hoisting, ventilation, etc. depend on the knowledge of the deposit as shown on a good map based on careful surveys. (*e*) Much litigation is caused by conflicting claims that result from poor locations on U. S. mineral lands. (*f*) Accurate surveys and careful mapping are often required to avoid breaking into old workings where there may be great quantities of gas or water. (*g*) Many mine survey problems occur that demand great exactitude; for instance, to determine a point on the surface directly above a given point underground, in order that a bore-hole or a shaft may be sunk to connect the former with the latter.

162. Some of the difficulties that characterize underground surveying are as follows: (*a*) Survey lines are frequently run

in low narrow places where it is difficult to set up a transit, locate stations, take sights, and measure distances. (*b*) Artificial light must be used which gives but poor illumination. When a sight is taken, a light must be placed to show the target and another must be held so as to illuminate the cross hairs. And in gaseous mines, safety lamps are required, which give a very feeble light. (*c*) A smoky atmosphere often necessitates very short courses and sometimes compels the postponement of the survey. (*d*) Vertical angles are commonly taken and horizontal distances and differences of level calculated from the vertical angles and inclined distances. (*e*) It is often impossible to make a closed survey, so that the advantage of having a closing check is lost. (*f*) Ordinarily stations must be located in the roof, and it is sometimes difficult to establish them permanently. (*g*) It is sometimes necessary to stop the work of mining while a survey is being made, consequently it becomes important to push the survey as rapidly as is consistent with good results.

SECTION III.

Instruments and their Adjustments.

163. Transit.—The mine transit has almost altogether supplanted the compass in underground surveying. The compass is too inexact and unreliable for such work. The instrument should be a complete transit or tachymeter. Ivory reflectors are placed over the verniers on the horizontal limb.

The cross wires are illuminated by an elliptical silvered plate inclined at an angle of 45 degrees with the ring which carries it, and by means of which it is fitted to the object end of the telescope. This reflector is shown in Fig. 254.* The hole in the center of the reflector permits the use of the telescope, while a light held near the inner surface illuminates the cross wires.

A diagonal prism eye-piece, as shown in Fig. 255, is very convenient in upward sighting when high vertical angles are

* Also convenient when a transit is used for Polaris observations. See Art. 50.

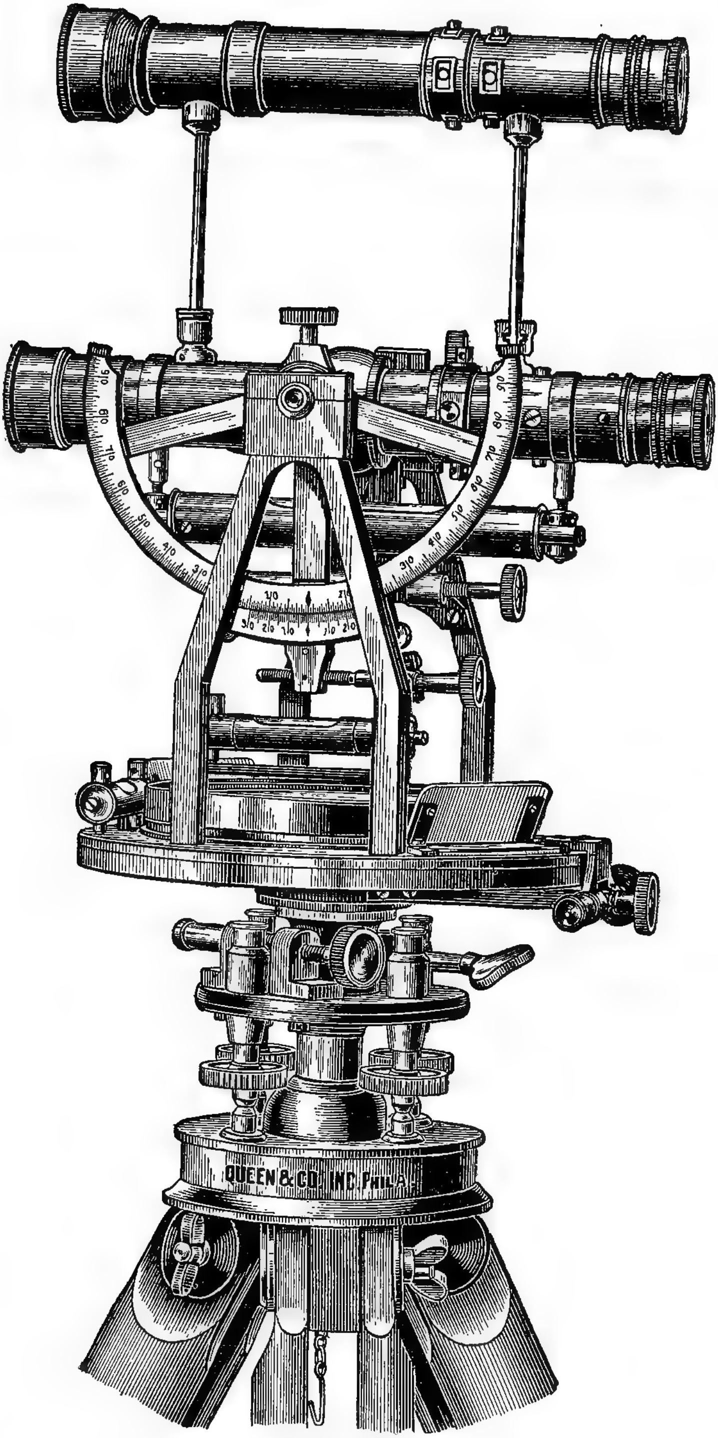


FIG. 253.

taken. A semi-circular vertical arc that can be set to any reading, such as is shown in Fig. 253, is probably the best kind for vertical angle reading. A long sensitive level tube should be attached to the telescope.

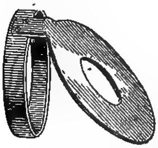


FIG. 254.

Vertical sighting necessitates a detachable side or top telescope, which projects beyond the transit plates when the telescope is in a vertical position. In Fig. 253 a mining transit is shown with a top telescope in position. In Fig. 256 a side telescope is shown attached at one end of the horizontal axis of the main telescope, with a counterpoise at the other end of the axis.

The mine transit should be provided with a solar attachment (Chapter XI) in order that the direction of the meridian may be determined.

The mine transit should be mounted on an extension tripod.

164. All the adjustments of a transit as previously given* must be made before undertaking a mine survey. Not only should the adjustments that are important in measuring horizontal angles be accurately made, but also the adjustments that cause the line of collimation to revolve in a vertical plane when the telescope is turned on its horizontal axis, and great care should be taken to have the zero of the vertical arc to read zero when the line of sight is horizontal.

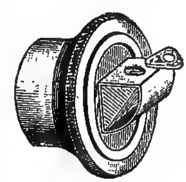


FIG. 255.

The lines of collimation of the main and extra telescopes should be parallel, and with the side telescope should lie in a horizontal plane and with the top telescope in a vertical plane. In the best transits the auxiliary telescope is so attached to the main telescope as to be in good adjustment when placed in position. But the mining engineer should be able to satisfy himself that the auxiliary telescope is in accurate adjustment, so the adjustments of the detachable telescope will now be given.

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* Art. 111.

The auxiliary telescope can be removed and replaced readily without disturbing the adjustments of the main telescope.

First adjustment of side telescope. To make the lines of collimation of the two telescopes to lie in the same plane with the horizontal axis.

The horizontal limb of the transit is leveled and a point, several hundred feet distant is sighted through the main telescope, and the telescope is clamped on its horizontal axis. Then the same point is sighted through the side telescope,

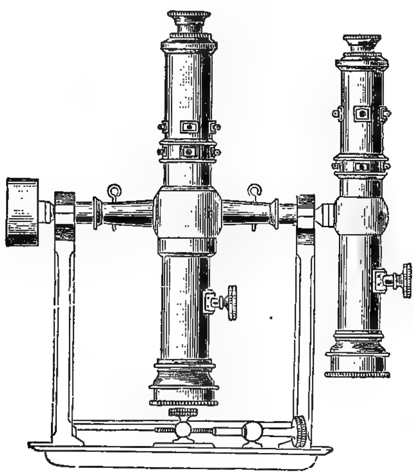


FIG. 256.

which is then clamped. Now, with both telescopes clamped on the horizontal axis, a vertical line a few feet in front of the transit is sighted first through one and then through the other telescope. If the two lines of sight cut the vertical line at the same point the adjustment is made. But if the two lines of sight cut the vertical line at different points, the horizontal cross wire of the side telescope must be shifted vertically in a similar manner to the method given in the next adjustment for shifting the vertical cross wire.

Second adjustment of side telescope. To make the two lines of collimation parallel.

The horizontal distance between the two lines of sight is measured at a few feet from the transit and also at a distance of several hundred feet. If these two measures are equal the side telescope is in adjustment. But if not, the vertical cross wire of the side telescope is shifted until the distance between the two lines of sight at the greater distance from the instrument is equal to $CD = x \pm d_2 \dfrac{(x-y)}{d_2 - d_1}$ as derived from Fig. 257.

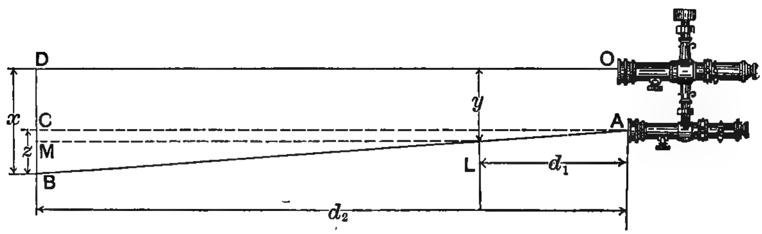


FIG. 257.

In Fig. 257, x is the distance between the lines of sight at a distance d_2 from the transit, and y is the distance between the lines of sight at a distance d_1.

$$AC : BC :: LM : BM,$$

or

$$d_2 : z :: d_2 - d_1 : x - y;$$

$$\therefore \quad z = d_2 \frac{(x-y)}{d_2 - d_1},$$

hence

$$CD = x - z = x - d_2 \frac{(x-y)}{d_2 - d_1}.$$

First adjustment of top telescope. To bring the lines of collimation of the two telescopes into the same vertical plane.

A distant point is sighted with the main telescope and the horizontal limb is clamped. Then the telescopes are turned about the horizontal axis until the same point can be sighted with the top telescope. If the intersection of the cross wires of the top telescope comes to the same point, the adjustment is

made. If not, the cross wires of the top telescope must be moved until the intersection comes to this point.

Second adjustment of top telescope. To make the two lines of collimation parallel.

This adjustment is made in the same manner as the second adjustment of the side telescope.

165. In using the side telescope a correction must be applied to each reading of a horizontal angle.

This correction may be readily obtained, as shown in Fig. 258. Let it be assumed that A is at the top and B at the

FIG. 258.

bottom of a shaft or slope. Then setting the transit at A and sighting on B, the side telescope takes the position A'. If the sight is taken through the side telescope, the correction for the horizontal angle is evidently the angle ABA'.

$$\text{Tang. } ABA' = \frac{AA'}{AB},$$

or the tangent of the angle at B is equal to the distance between the axes of the telescopes divided by the horizontal distance between the stations.

The horizontal angle may be read immediately by sighting on a point to one side of the station and at a distance from it equal and parallel to the distance between the telescopes.

When using the top telescope a similar correction must be made for the vertical angle.*

166. Steel Tapes.—Distances in mine surveying should be measured with steel tapes. The steel tape is convenient and accurate. It is made of a ribbon of tempered steel and is graduated into feet with brass or copper sleeves at every five or ten feet, on which the distances are marked by countersunk figures. The first five feet should be graduated into feet and

* See also Art. 51, 2.

tenths of a foot. Flat wire tapes are probably to be preferred to ribbon tapes, as they do not kink and break so easily. In surveying extensive mines like coal mines, tapes of three hundred or five hundred feet in length, wound on suitable reels, are used. An ordinary chain, a small box tape, or an offset rod, may be used for offset measurements.

167. In mine surveying a distance is usually measured along or parallel to the line of sight. The horizontal distance between two stations is obtained by multiplying the measured distance by the cosine of the vertical angle, and the difference of level between the two stations is found by multiplying the measured distance by the sine of the vertical angle. The distance between two consecutive stations should not be measured in segments, but the tape should be long enough to reach between the stations. If the distance between two regular consecutive stations is greater than the length of the tape, a secondary station should be established between the two given stations on the line connecting them. In measuring the length of a course, the end of the tape is held at the rear station and the tape is unwound from the reel to reach to the fore station. The whole length of the tape reeled off should be examined to see that no obstruction deflects it from a straight line. The tape can be held in a line parallel to the line of sight by stretching it between points at proper heights on plumb-lines hanging at the stations. The pull should be such as to cause the error from stretching to approximately balance the error from sag. A good method of reading the exact distance is as follows: when the tape is first held in position between the stations, the fore chainman calls out the distance to the next foot-mark on the tape beyond his station, the tape is then pulled back until this foot-mark comes to the fore station, when the rear chainman reads off the number of tenths to be deducted from the exact number of feet first given. For instance, suppose that the distance between the stations is 87.3 feet. When the tape is first held in place, the fore chainman calls out 88. The tape is then pulled back until the 88th foot-mark comes just to the fore station, when the

rear chainman gets the reading .7, which deducted from 88 gives the required distance.*

168. Devices for the Illumination of Stations.—Sights are frequently taken directly on a plumb-line. The line is made visible by putting the flame of a lamp behind the line, or by putting a piece of oiled paper just back of the line and by holding a light back of the paper.

169. Plummet lamps, hung from stations in the roof, are often used for sights. This lamp, as shown in Fig. 259, is a large plummet, which is hollow to contain oil. The vertical axis of the plummet passes through the center of a small cylindrical wick. The lamp is made of brass, and is hung in gimbals by two chains, so that when the lamp is hung up, the point of suspension, the center of the wick, and the point of the plummet are in a vertical line. The flame illuminates the wick, which is bisected by the vertical cross wire of the telescope in taking a sight. These lamps are used in pairs for fore and back sights.

FIG. 259.

170. In the three-tripod system, a special target lamp is used. The target and lamp are supported by a standard which is carried by a leveling head similar to that of a transit, and is attached to the tripod head in the same way. By this device, the intersection of the illuminated slits of the target can be accurately centered over the station, at the same time being at the height above the station that the horizontal axis of the telescope would be if the transit were placed upon the tripod. The target and transit are thus interchangeable on three tripods which are placed at three consecutive stations. The transit is placed on the intermediate tripod and the observation is taken by sighting on the targets placed on tripods at the rear and forward stations. When this observation is finished, the rear tripod is the only

* See also Art. 9.

one that is moved, and it is taken forward to the station in advance of the one last sighted on. The transit is placed on the tripod which carried the target on which the fore sight was just taken, and the target from this tripod is moved back to the tripod just vacated by the transit. This is a rapid and accurate method, as the vertical angle for each course is obtained directly and the distances between stations can be conveniently measured parallel to the line of sight.

SECTION IV.

Stations.

171. The stations of an underground survey are usually located in the roof. In order that the stations may be as permanent as possible, they should not be placed on the floor in traveling-ways. If the top is firm rock which is not liable to roof-falls, a small hole about one-quarter of an inch in diameter and one inch deep is drilled in the roof. Then a dry wooden plug which just fills the hole is driven in, and a nail with a small hole through its flattened head is driven into the plug. The wooden plug expands as it gets wet and there is no danger of its pulling out. The nail is commonly called a spad. The spads should be made of copper or of some metal that is not easily corroded by the mine water. If the roof is timbered, the spads are sometimes driven in the collars. But the timbers may decay rapidly, or may be moved, so that the practice of placing the stations in timber is to be condemned.

172. The stations should be numbered consecutively, and no numbers should be repeated in the same mine. Marks should be put at or near the stations to distinguish them. A circle may be made with white paint having the number of the station within it, or a triangle may be chiseled in the rock or cut in the timber with the number of the station within it. In some locations white paint marks are soon obliterated by the smoke from the lamps.

173. When a sight is taken on a station a plummet may be suspended from the spad as shown in Fig. 260.

When the transit is to be set up under a station, a plummet is used to transfer the point from the roof to the floor. Thus a point is found on the floor vertically under the station. A good device for centering may be made by boring a large hole in a block of wood and filling it with molten lead, having a

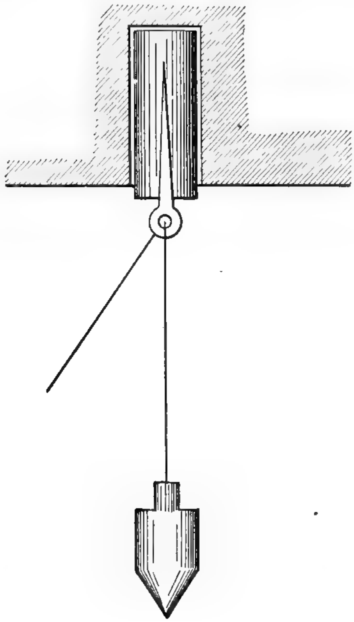


FIG. 260.

sharp steel point projecting above the surface of the lead. This device is small but heavy. The plummet is let fall from the station so as to hang a few inches above the floor and the block is moved about until its steel point comes directly under the point of the plummet. The transit is then centered above the steel point in the block.

SECTION V.

Traversing Underground.

174. A mine surveying corps should contain at least three men. The mining engineer in charge of the survey should

locate the stations, as it is important to have as few stations as possible, and to have them in favorable positions. It is a good plan to have the stations located before the transit work is commenced.

Mining engineers have different methods for the instrumental work and different methods for recording the notes of a survey. Details will be given of one of the most approved methods.

Fig. 261 shows the first three or four courses of a survey. The transit is first placed at Sta. 17, the last station of a pre-

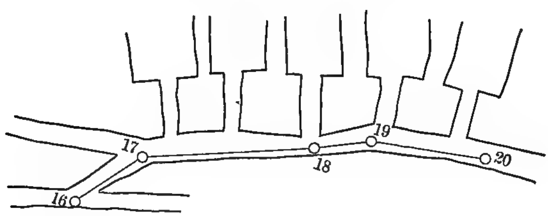


FIG. 261.

ceding survey, and Sta. 16 of the same survey is used for a back sight. Sta. 18, or the first new station, is located as far in advance as can be seen from Sta. 17, care being taken to insure a good sight ahead from Sta. 18. After the transit is centered at Sta. 17 and leveled, the needle is unclamped and the north end of the dial is pointed in the direction from Sta. 17 to Sta. 18. The vernier plate is clamped at zero, and a back sight is taken on Sta. 16, and the limb is clamped. The compass bearing is now read. The telescope is then revolved on its horizontal axis, the vernier plate is unclamped, and the telescope is turned until Sta. 18 is sighted. Both verniers are read for the deflection angle and recorded. The compass bearing is also read and recorded. From the back sight bearing and the deflection angle the fore sight bearing is calculated, and compared with the observed fore sight bearing. If the two agree closely the deflection angle reading is verified. The vertical angle is now obtained by sighting on a target at Sta. 18 placed at the same height from the station that the axis of the telescope is from Sta. 17. The distance from Sta. 17 to Sta. 18 is

now measured as previously explained and the offset measures are taken and recorded. The transit is then removed to Sta. 18, a back sight is taken on Sta. 17 and the same operations are repeated.

In this method the needle is simply used as a rough check. The existence of local attraction at Sta. 17 evidently will not affect the reliability of the needle in this work, as both compass readings will be affected in the same way. Some mining engineers check the horizontal angles by taking multiple readings. This is certainly to be preferred where very accurate results must be obtained, and the only objection is that it requires more time.

A copper or brass lamp should be used when the needle reading is taken.

175. The notes for this survey may be arranged on the left-hand pages of a note-book as follows:

SURVEY OF CRANBERRY SLOPE NO. 1.

Beginning at top of No. 2 Plane, March 29, 1900.

Sta.	Deflection Ang. R = Right. L = Left.		Bearing on Fore sight.	Vertical angle. + for elevation − for depression.	Distance	Remarks.
	Ver. A.	Ver. B.				
						Corps: Transitman, J. Smith. Back sight, S. Jones. Fore sight, W. Black. Chairman, C. Bell. Survey started by setting up at Sta. 17 and back sighting on Sta. 16 of survey in west gangway.
21	4° 04′ L	175° 57′	N 51° 00′ E	+ 10° 22′	61.24	
20	3° 32′ L	176° 28′	N 55° 00′ E	+ 15° 45′	38.85	
19	11° 05′ R	168° 54′	N 58° 35′ E	+ 10° 47′	59.1	
18	0° 11′ L	179° 49′	N 47° 30′ E	+ 20° 08′	29.8	
17	25° 28′ R	154° 31′	N 47° 40′ E	+ 3° 20′	90.75	
16						

The right-hand pages of the note-book contain sketches and side measurements.

Fig. 262 shows a common method of recording the side notes between two stations.

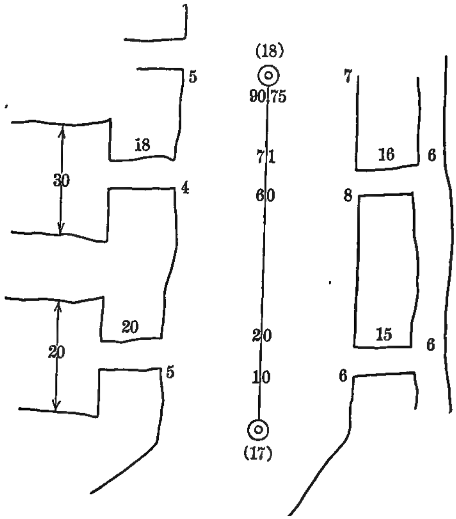


FIG. 262.

At the distance of 10 feet along the course offsets of 6 feet and 5 feet are taken right and left to the sides of the gangway. On the right a cross-cut 15 feet long opens into an airway which is 6 feet wide. On the left a passageway 20 feet long opens into a breast which is 20 feet wide. At a distance of 20 feet along the course the offsets are the same as before and so are not repeated.

Fig. 262 shows the method of sketching and putting in the measurements and does not require any further description. The side notes are sometimes kept in a separate book.

176. The directions of the courses are sometimes determined by the method of continuous azimuth angles. If a previous survey is to be extended by this method, the transit is first placed at the last station of the previous survey and the vernier is set to the azimuth reading of the last course on this survey. If a new survey is started the transit may be first set up at Sta. 2 and the first course may be taken as the meridian of the survey; or the transit may be set up first at Sta. 1, the

vernier plate clamped at 0°, and the transit turned until the *N* point on the dial comes exactly under the *N* end of the swinging needle; then the limb is clamped and the angle turned off to Sta. 2; thus the bearing of the course will be taken as its azimuth. If *A*, *B*, and *C* are any three consecutive stations, the reading of the vernier when the transit is placed at *B* will be the azimuth of the course *AB*; then a back sight is taken on *A*, the telescope is revolved on its horizontal axis, the vernier plate unclamped, and the telescope is directed to *C*, when the reading of the vernier will be the azimuth of the course *BC*.

SECTION VI.

*Calculations and Mapping.**

177. An accurate method of plotting should be employed. The protractor should only be used in unimportant work. A rectangular coordinate method, called the method of sines and cosines, or latitudes and longitudes, is commonly employed. The meridian of the survey is taken as the axis of ordinates and a line perpendicular to it is the axis of abscissas. From the azimuths or horizontal angles, as recorded in the note-book, reduced bearings of all the courses are calculated with reference to the established meridian. Then from the inclined distances and vertical angles the horizontal distances and differences of level are computed. Next the latitude and longitude of each course, and then the total difference of latitude and total longitude for each course are found. The results may be tabulated for convenience in plotting, as in the following table:

Station.	Defl. Angle.	Course.	Vertical Angle.	Distance.			Latitude.		Total Latitude.		Longitude.		Total Longitude.	
				In-clined.	Hori-zontal.	Ver-tical.	North.	South.	North.	South.	East.	West.	East.	West.
16														
17	25° 28′ R	N 47° 40′ E	+ 3° 20′	90.75	90.6	5.3	61.	....	61.	...	67.	....	67.	
18	0° 11′ L	N 47° 29′ E	+20° 08′	29.8	27.97	10.26	18.9	...	79.9	....	20.6	....	87.6	
19	11° 05′ R	N 58° 34′ E	+10° 47′	59.1	58.05	11.05	49.5	...	129.4	...	30.3	...	117.9	
20	3° 32′ L	N 55° 02′ E	+15° 45′	38.85	37.39	10.54	21.4	...	150.8	....	30.6	....	148.5	
21	4° 04′ L	N 50° 58′ E	+10° 22′	61.24	60.24	11.02	37.9	....	188.7	....	46.8	...	195.3	

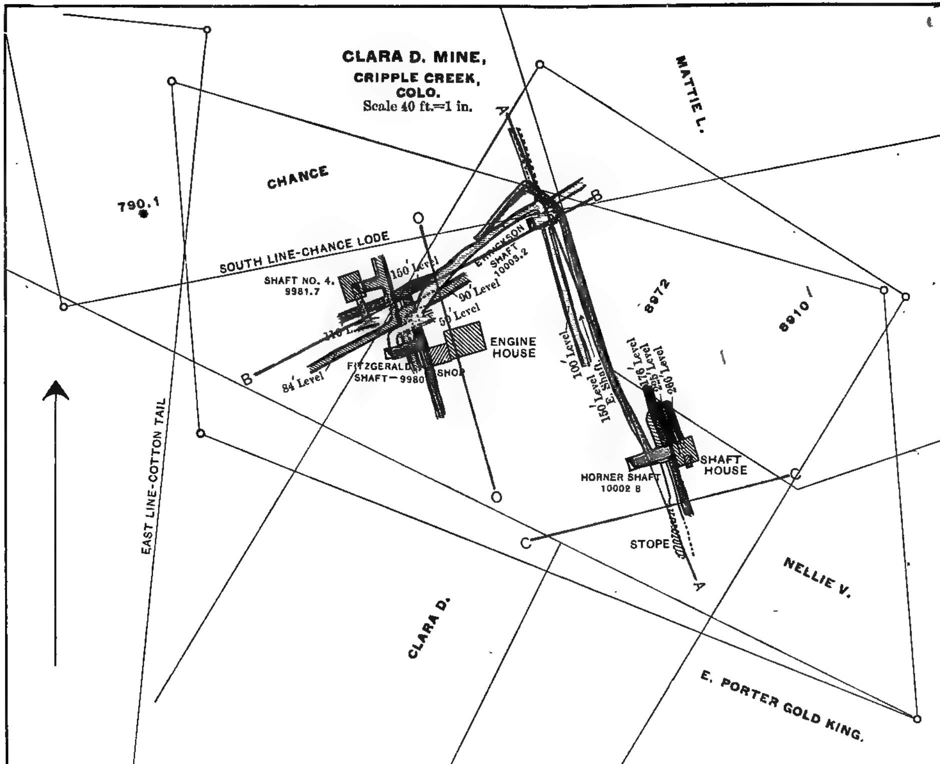
* See also Art. 44.

The work of plotting is started by drawing two lines perpendicular to each other. Then using these lines as axes of coordinates, a station, as for instance station No. 20 in the table, is located on the paper by taking the distances 148.5 and 150.8 as abscissa and ordinate of the point. Thus every station is plotted by using its total latitude and total longitude as coordinates and hence is determined independently of all others, so an error in plotting any one station will not affect the others. Latitudes and longitudes may be rapidly calculated by using a traverse table showing differences of latitude and longitude for every minute of the quadrant and to several decimal places.

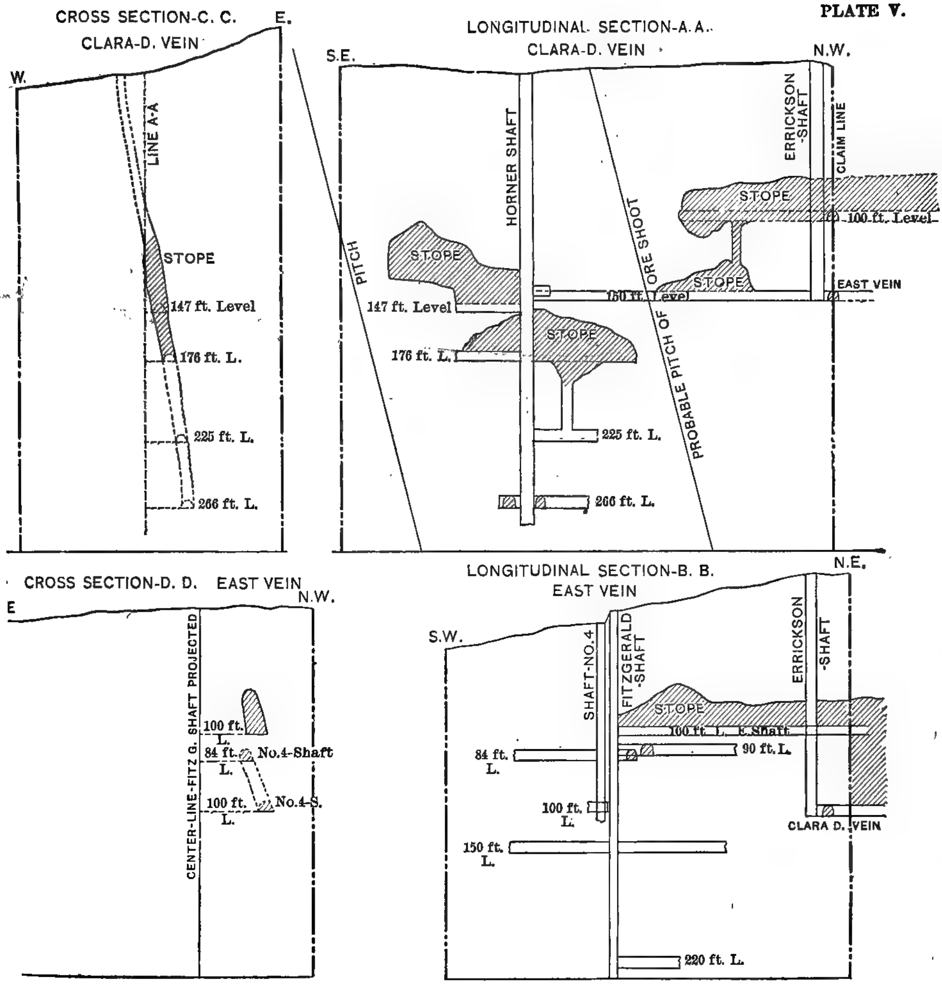
178. Three drawings are usually required to show the mine fully: a plan, a longitudinal section, and a transverse section. The plan is a horizontal projection of the mine workings and corresponds to the map of an ordinary surface survey. If the mine is in a bed that is nearly horizontal, as is the case with some coal mines, a plan is often sufficient. A longitudinal section is the projection of the mine workings on a plane parallel to the longitudinal direction of the mine, and a transverse section is taken perpendicular to the latter direction. If the vein is highly inclined, the longitudinal section is usually vertical and parallel to the strike, but if the vein is quite flat, the longitudinal section is sometimes taken parallel to both dip and strike. Plate IV shows the plan of a metaliferous mine and Plate V shows longitudinal and transverse sections of the same. Plate VI is part of a plan of an anthracite coal mine.

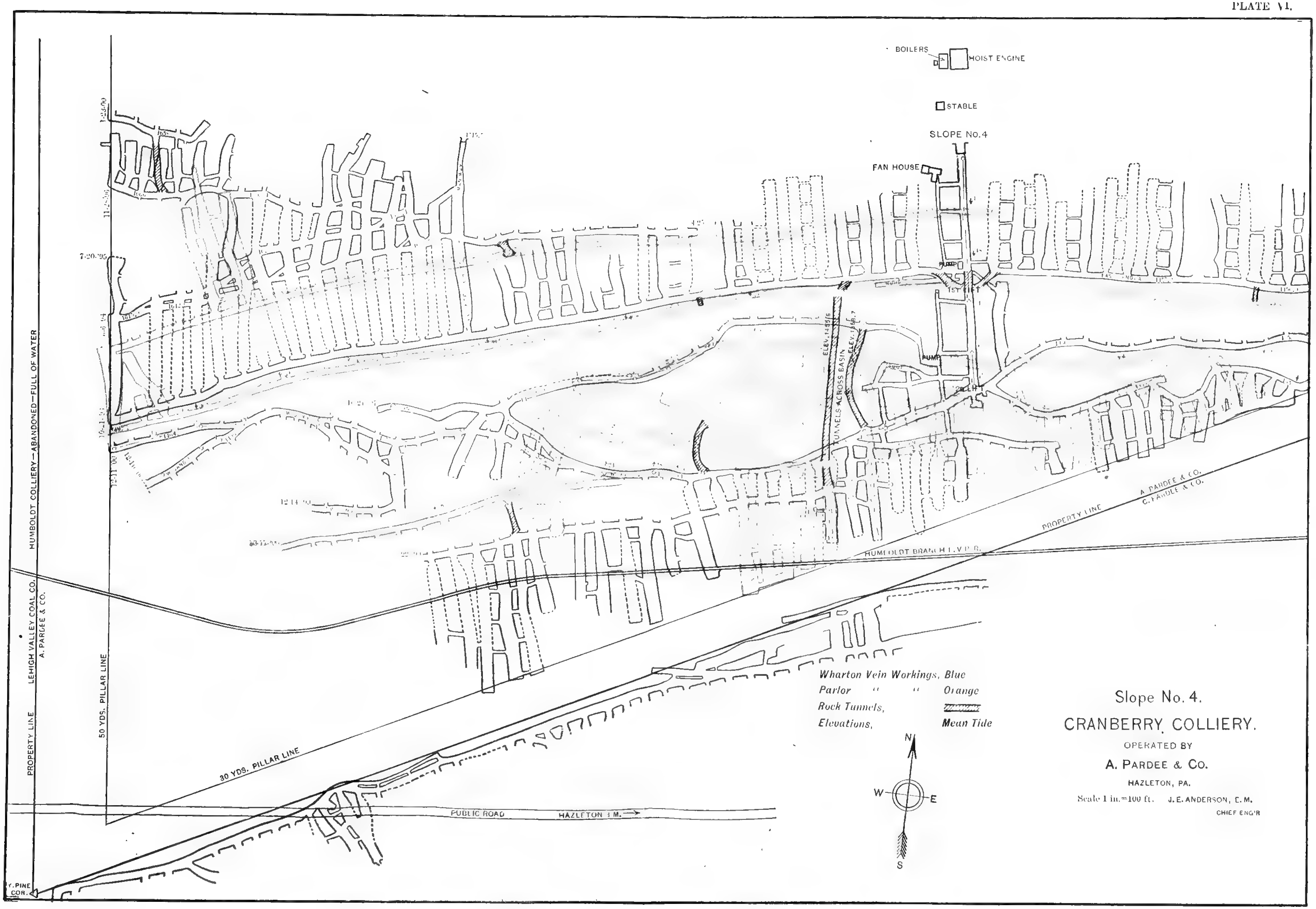
179. The workings in the different levels of a mine are usually shown on one plan, the workings at different levels often being tinted in different colors. Arrows are used to denote the direction of an air current. Elevations of principal points and the dip of the bed or vein at different places should appear on the map. Notes of all surveys should be carefully preserved, and maps should be made in duplicate and the two sets kept in different places. It is the practice with large mining companies to have the maps, survey notes, working drawings of machinery, etc., kept in a fire-proof vault.

PLATE IV.



CLARA D. MINE,
CRIPPLE CREEK,
COLO.
Scale 40 ft.=1 in.
MATTIE L.
CHANCE
790.1
SOUTH LINE-CHANCE LODE
SHAFT NO. 4.
9981.7
150' Level
90' Level
50' Level
ERICKSON
SHAFT
10003.2
ENGINE
HOUSE
FITZGERALD
SHAFT—9980
SHOP
84' Level
8972
8910
100' Level
150' Level
E. Shaft
176' Level
260' Level
SHAFT
HOUSE
HORNER SHAFT
10002 B
STOPE
EAST LINE-COTTON TAIL
CLARA D.
NELLIE V.
E. PORTER GOLD KING.
A
B
C


CROSS SECTION-C. C.
CLARA-D. VEIN
W.
E.
LINE A-A
STOPE
147 ft. Level
176 ft. L.
225 ft. L.
266 ft. L.
LONGITUDINAL SECTION-A. A.
CLARA-D. VEIN
S.E.
N.W.
PITCH
HORNER SHAFT
PROBABLE PITCH OF ORE SHOOT
ERRICKSON -SHAFT
CLAIM LINE
STOPE
STOPE
STOPE
STOPE
100 ft. Level
EAST VEIN
150 ft. Level
147 ft. Level
176 ft. L.
225 ft. L.
266 ft. L.
CROSS SECTION-D. D. EAST VEIN
E
N.W.
CENTER-LINE-FITZ G. SHAFT PROJECTED
100 ft. L.
84 ft. L.
No.4-Shaft
100 ft. L.
No.4-S.
LONGITUDINAL SECTION-B. B.
EAST VEIN
S.W.
N.E.
SHAFT-NO.4
FITZGERALD -SHAFT
ERRICKSON -SHAFT
STOPE
100 ft. L. E. Shaft
84 ft. L.
90 ft. L.
100 ft. L.
150 ft. L.
220 ft. L.
CLARA D. VEIN


BOILERS
HOIST ENGINE
STABLE
SLOPE No.4
FAN HOUSE
1ST LIFT
PUMP
2ND LIFT
ELEV. 1455.6
ELEV. 1468.7
TUNNELS ACROSS BASIN
HUMBOLDT COLLIERY—ABANDONED—FULL OF WATER
LEHIGH VALLEY COAL CO.
A. PARDEE & CO.
PROPERTY LINE
50 YDS. PILLAR LINE
30 YDS. PILLAR LINE
PROPERTY LINE
A. PARDEE & CO.
C. PARDEE & CO.
HUMBOLDT BRANCH L.V.R.R.
PUBLIC ROAD
HAZLETON 1 M.
Y. PINE COR.
Wharton Vein Workings, Blue
Parlor " " Orange
Rock Tunnels,
Elevations, Mean Tide
N
W
E
S
Slope No. 4.
CRANBERRY COLLIERY.
OPERATED BY
A. PARDEE & CO.
HAZLETON, PA.
Scale 1 in.=100 ft.
J. E. ANDERSON, C. M.
CHIEF ENG'R

State mining laws usually contain explicit directions with reference to surveys and maps. Some of the provisions of the mining laws of Pennsylvania for the anthracite coal mines are as follows: Accurate maps shall be made on a scale of 100 feet to the inch; maps shall show all the workings and passageways, give inclinations of seams, shafts, slopes, etc., show tidal elevations of principal points, give data of each survey and show number of lost survey station in gangway or workings, show boundary lines of colliery and proximity of workings thereto, give location of dams and accurate data concerning them; a true copy of the mine map must be deposited with the inspector of mines for the district and another copy kept at the colliery; periodical surveys must be made and a plan of the extensions made in the mine must be placed on the inspector's map at least once in every six months; whenever any excavation is approaching workings filled with water or when any part of the mine is being worked out preparatory to abandonment, the inspector must be notified, and in the latter case duplicate surveys must be made that practically agree and certified copies filed with the inspector. The mine map may be in use for a number of years, many different surveys being plotted on the same map as the underground workings advance. Thus the map will become very much soiled unless special precautions are taken. When working on the map the hands should not rest directly on the map surface, but a piece of paper should be interposed, and the map should be well covered when not in use.

Frequently occasion will arise for the use of a map underground. It should be the rule never to take the main mine maps out of the office. When the engineer, foreman, or any other person needs a map for underground use, a tracing should be made.

SECTION VII.

Calculation of the Contents of Beds and Veins.

180. The area of the bed may be calculated with a fair degree of accuracy by taking a sheet of tracing paper divided into one acre squares, drawn to the same scale as the plan of the

mine. Then the tracing paper is placed over the map and the number of squares covering the plan are counted. From the area, thickness, and inclination of the bed, its cubical contents can be readily calculated in cubic feet. Then the specific gravity of the mineral multiplied by 1000 ounces, the weight of a cubic foot of water, will be the weight of a cubic foot of the mineral. For moderately pitching coal beds the quantity is usually estimated at 100 tons per acre for each inch of thickness in the bed.

In a metaliferous mine the amount of vein matter that remains is spoken of as the *ore reserves. The amount of ore in sight* may be considered as a rectangular block limited by the outcrop, the depth of the shaft, and the extreme points of the levels, diminished by the amount already extracted. This, however, is excessive, and deductions should usually be made.

SECTION VIII.

Connecting Surface and Underground Surveys.

181. Case 1. When the mine is opened by a shallow shaft, tunnel, adit-level, or slope.

In this case no difficulty presents itself. The surface survey can be extended through the mine opening and directly connected with the underground survey. The last course of the surface survey will end at a station at the mouth of the mine opening. The transit is set up at this station, and a back sight is taken on the preceding surface station and a fore sight on the first station underground. Then the transit is set up at the first station underground, and a back sight is taken on the station at the mouth.

182. Case 2. When the mine is opened by two shafts.

The method in this case will be explained by reference to Fig. 263. Let M and N represent the two shafts. A plumb-line is suspended in each shaft reaching from top to bottom. Any convenient traverse, as M-a-b-c-d-e-N, is made on the surface from the plumb-line in M to the plumb-line in N. Then regarding the first course Ma as the meridian course, the bearing of MN or angle NMa is found by the method for supply-

ing a lost course. Likewise a traverse underground is made, as *M-r-s-t-v-w-N* connecting the two plumb-lines, and treating *MN* again as a lost course, the angle *NMr* is calculated. Then

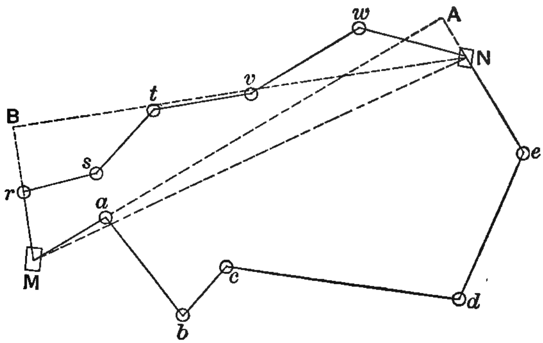


FIG. 263.

the angle *rMa*, or the difference between the two computed angles, will be the difference between the azimuths of a given line at the surface and a given line underground.

183. Case III.—When the mine is opened with one shaft.

This is the most difficult case. Several methods are in use, and two of the most approved ones will be given.

1st Method. This method will be explained by referring to Fig. 264. *MN* represents the cross-section of the shaft, and

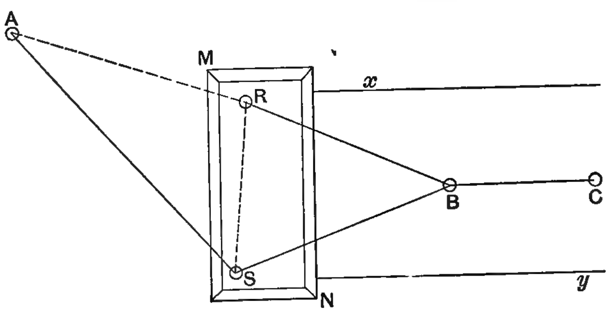


FIG. 264.

xy a mine gangway leading from the bottom of the shaft. Two plumb-lines at *R* and *S* are suspended from the top nearly to the bottom of the shaft and as far apart as they can be placed without touching the sides of the shaft and so as to be seen from

the same point as B in the gangway. A line as AS is connected with the surface survey and BC is a line in the gangway and connected with the underground survey. The transit is set up at A and the angle SAR is measured. The distances AS and AR are accurately measured. Then by trigonometry the unknown parts of the triangle SAR are calculated, and knowing the azimuth of the line AS and the angle ASR, the azimuth of the line SR is determined. Another transit which is set up in the gangway at B measures the angles RBS and RBC. Then the distances from B to the plumb-lines R and S are measured carefully. Then the angle BRS in the triangle BRS is calculated. Knowing the azimuth of RS and the angles BRS and RBC, the azimuth of BC can be determined.

2d Method. In this method four plumb-lines are used.

In Fig. 265 the plan of a shaft with two hoisting compartments is shown. A point O is assumed on the surface which is known to be vertically above an open space at the foot of the shaft, and the transit is set up at this point. At points A and B on the opposite side of the shaft from the point O, plumb-

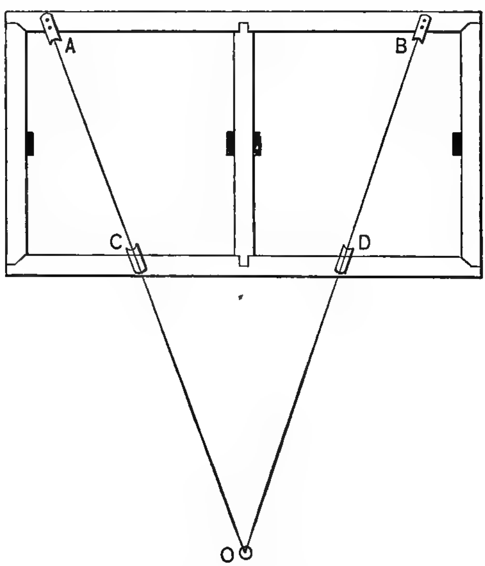


FIG. 265.

lines are suspended from hangers placed as far apart as possible and which project a little beyond the edge of the timber. A plan of one of these hangers is shown in Fig. 266. Then plumb-lines are let down from hangers on the other side of the shaft at C and D, being carefully lined in by the instrument with the lines at A and B. The plumb-lines A, B, C, and D

FIG. 266.

are in the clear spaces between the ends of the cages and the sides of the shaft, so that they can be examined to see if they are hanging freely by causing the cages to pass slowly up and down the shaft.

The point O is connected with the surface survey, determining the azimuths of OA and OB. Then the distances from O to C, A, D, and B and also the distances from C to D and from A to B are carefully measured.

The transit is now taken underground and set up at a point vertically under O. This point is found approximately by getting the intersection of a string connecting plumb-lines A and C with a string connecting plumb-lines B and D. The angle AOB is now measured. Distances are measured underground corresponding to those measured at the surface.

The angle and distances measured underground are compared with the corresponding angle and distances measured at the surface, and the transit is moved until the two sets of measurements agree closely. Now by back sighting on either A or B, the azimuth of a course underground may be determined with reference to the same meridian used on the surface.

184. The lines for plumbing deep shafts should be made of piano wire and the plummets should be quite heavy, sometimes weighing as much as 25 pounds. The line is first lowered with a small weight, which is removed at the bottom and a heavy plummet attached. The plummet is often suspended in a bucket of water or some thicker liquid to diminish the vibrations. Precautions must be taken to be sure that the plumb-line hangs freely. If a light passed around the plummet at the bottom can be seen in all positions from the top, it is probable

that no obstruction deflects the line. If the voice cannot be heard from top to bottom, there should be a method of signaling and a code of signals. Signals are frequently given by taps on an iron pipe that runs down the shaft.

SECTION IX.

Driving a Passageway to Connect Two Points of a Mine.

185. One of the most important problems and one that frequently occurs in mining is to drive an opening to connect two points underground or to connect a point on the surface with a point underground. This is a problem that arises in holing from one excavation to another, and in sinking a shaft to strike certain workings. Any possible traverse is made starting at the first given point and ending at the other. The azimuth, vertical angle, and length parallel to line of sight are found for each course of this survey. Then the direction and distance between the two given points can be calculated.

If a new road is to be put in to connect one part of the mine with another, the road is first excavated for a few yards approximately in the right direction. Then the transit is set up at the point where the road is to begin and the calculated horizontal and vertical angles are turned off. A plummet is then suspended from the roof and in the line of sight as far from the instrument as possible, and another is lined in midway between that one and the instrument, and a third is put near the transit. These three lines are used as range lines for the azimuth direction. When the road has advanced sufficiently the last line is taken forward and lined in by the other two, and thus the work proceeds. When the road has advanced to some distance the transit is brought in again and the range lines are set accurately by it, and the proper slope is given by turning off the correct vertical angle.

SECTION X.

Measuring the Depths of Shafts.

186. It is sometimes necessary to measure accurately the depth of a vertical shaft. Several methods have been used

with good results. Fig. 267 illustrates a successful direct method. A steel piano wire is unwound from a reel at *A* and passed over a pulley at *B*, dropping down into the shaft and held taut by a heavy weight at the end. There is a horizontal

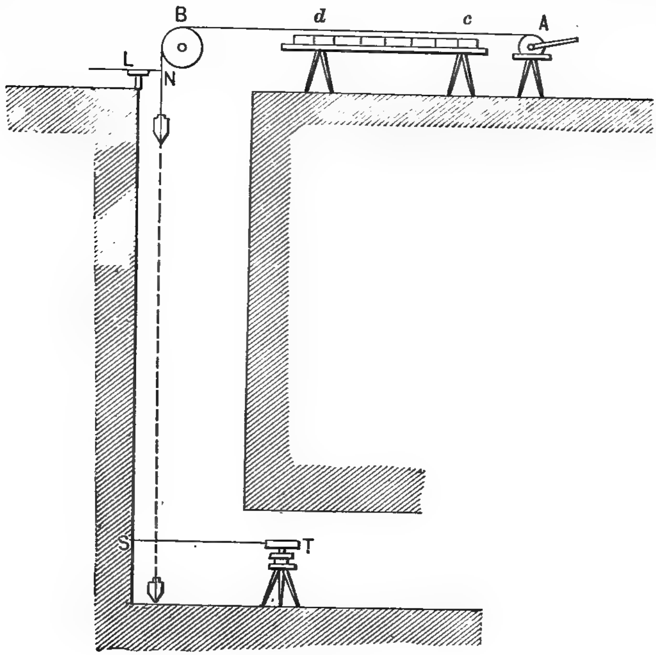


FIG. 267.

scale at *cd* over which the wire passes. The measurement is begun by lowering the weight a short distance and placing a mark on the wire at the point *N*, which is at the height of the point from which the depth is to be measured. At the same time a slight mark is made in the wire opposite the zero point of the scale at *c*. The wire is then unwound until this mark moves to a point of graduation at *d*, when this distance *cd* is read and recorded. Then another mark is put on the wire at the zero point and another part is measured, and thus the measurement continues by segments until the weight has nearly reached the bottom of the shaft. Then a level line from an instrument at *T* will determine a point *S* on the side

of the shaft at the height of the first mark made on the wire. Then the last portion of the wire that was unwound is meas-

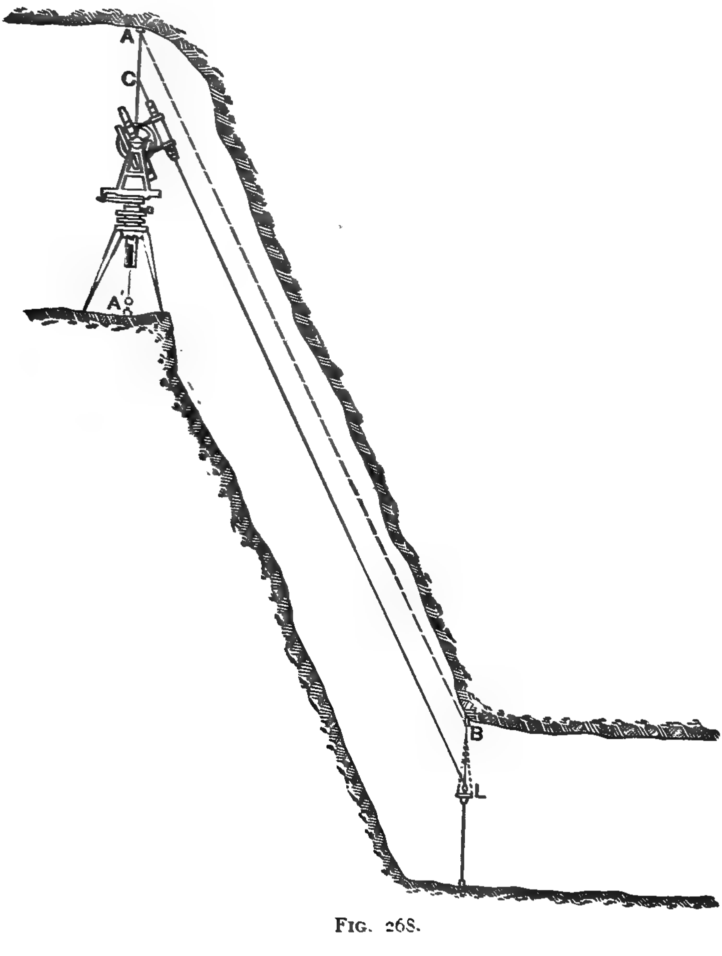


FIG. 268.

ured on the scale, and the total depth from N to S is equal to the sum of all the parts measured on the scale. The depth may also be measured when the reel is wound up and the two results compared.

SECTION XI.

*Correction of Vertical Angle when Top Telescope is used.**

187. When the top telescope is used, the observed vertical angle requires correction. This may be seen by reference to Fig. 268. Let it be required to find the vertical angle from Sta. A to Sta. B. A point A' on the floor vertically under A is found and the transit is centered over this point. A plummet lamp is now suspended at Sta. B, and a sight is taken on the flame through the top telescope. Evidently the vertical angle given by the transit is not equal to the required angle unless the line of collimation of the top telescope intersects the line AA' at a distance below A equal to the distance from Sta. B to the flame.

Thus a number of different and yet similar problems arise. The solution in one case is shown by reference to Fig. 269. A

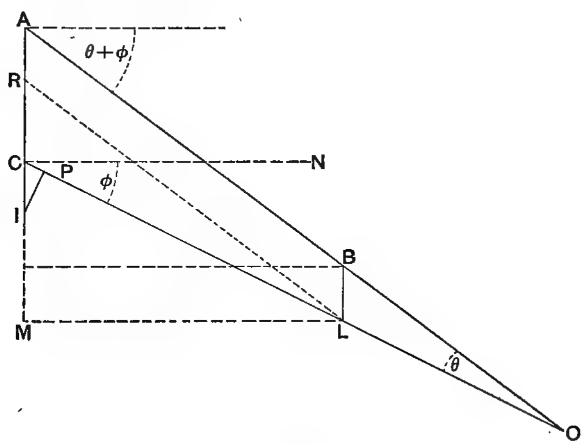


FIG. 269.

and B represent stations in the roof, L is the position of the lamp hung at B, CL is line of sight of top telescope, I is the position of the horizontal axis of main telescope centered under Sta. A, and IP is the perpendicular distance between the

* See also Art. 51, 2.

two telescopes. Lines AB and CL are extended to intersect at O, LR is drawn parallel to AB, and horizontal lines are drawn through A, C, B, and L. Angle $NCO = \phi$ is the observed vertical angle. It is required to find the vertical angle from A to B and also the horizontal and vertical distances between these stations. The correction for the vertical angle is evidently the angle $AOC = \theta$.

In the triangle RLC,

$$RL : RC :: \sin RCL : \sin RLC,$$

or

$$RL : AI - AR - CI :: \cos \phi : \sin \theta;$$

hence

$$\sin \theta = \frac{AI - AR - CI}{RL} \cos \phi.$$

The distance RL is found by suspending plumb-lines from A and B and measuring the distance between points on the plumb-lines equally distant from A and B.

In the right triangle CIP, the angle at I is equal to ϕ, hence

$$CI = \frac{IP}{\cos \phi}.$$

AR is equal to the distance from Sta. B to the flame of the plummet lamp, and distance AI is measured. Therefore, the value of θ may be determined.

$$\text{Hor. distance between stations} = RL \cos (\theta + \phi)$$

and

$$\text{Ver. distance between stations} = RL \sin (\theta + \phi).$$

SECTION XII.

U. S. Mining Claims.

188. When a prospector makes a location on a piece of U. S. mineral land, he must stake off his claim, putting in corner monuments and posting a notice on the deposit. This survey may be made by any surveyor. Before a patent will

be issued by the Land Office, an accurate survey of the claim must be made by a U. S. Deputy Mineral Surveyor with a solar transit or similar instrument, as all courses must be referred to the true meridian. According to the U. S. mineral laws, the claim may be 1500 feet in length along the outcrop of the lode and 300 feet in width on each side of the outcrop. The end lines of the claim must be parallel, but the side lines need not be so. State and local laws sometimes diminish the width to 100 or even 50 feet. If the mineral discovery is made on surveyed land, one corner of the claim must be connected with the public survey. If the land has not been surveyed, the location survey must be connected with some prominent natural object. If there is no section corner of the public survey within two miles, the Deputy Surveyor must establish a permanent mineral location monument. The miner must confine his workings within vertical planes passing through his end lines, but he has the right to follow the vein along its dip beyond his side lines.

A placer claim may contain as much as 20 acres, and if made on unsurveyed land may have any shape whatever, but if the land has been surveyed by the Government, the claim must conform to the smallest legal subdivisions. A placer claim patent conveys no extra-lateral right.

In order to hold a location at least $100 per year must be expended upon it, and $500 must be spent upon it before a patent will be issued.

The applicant must pay the Deputy Surveyor for his services, and he may select any Deputy whom he may prefer. The procedure in obtaining a patent is as follows: The claimant secures from the Surveyor-General an application blank which he fills out, giving the particulars required, and with this he forwards a copy of the record of location of the claim certified by the recorder of the county in which the claim is situated. A deposit of $35 is then made for the office charges, a receipt is issued, and the survey for patent authorized. If the returns made by the Deputy Surveyor are satisfactory, the Surveyor-General sends to the applicant two copies of field-notes and official plats. The applicant must

then have published in the nearest local paper for 60 days a notice of his intention to apply for patent, giving a full description of ground, and also post conspicuously upon his claim one copy of map and notes. Then if no adverse claim is filed the applicant is ready to proceed with final proof before the U. S. Land Office for district in which claim is situated. In making final proof and application to purchase, all the papers in the case must be filed and proof submitted as to citizenship, posting of notices, publication, money expended on claim, and of the fact that there is no adverse claim. The Government price for lode locations is $5 per acre, $2.50 per acre for placer ground, and $20 per acre for coal lands.

The following is a specimen of the field-notes for a mining claim:*

FIELD-NOTES

Of the Survey of the Claim of the "Argentum Mining Co." upon the Gold Queen Lode in Alpine Mining District, Lake County, Colorado. Surveyed by G. Lightfoot, Apr. 22 to 24, 1886.

Gold Queen Lode.—Beginning at corner No. 5, a pine post, 5 feet long, 5 inches square, set 2 feet in the ground, with mound of earth and stone, marked (5) 4225 A, whence corner No. 1 of this survey bears S. 14° 54′ E., 370·16 feet. A pine, 18 inches in diameter, bears S. 33° 15′ W., 51 feet, and a silver spruce, 13 inches in diameter, bears N. 66° W., 23 feet. Both are blazed and marked B. T. (5) 4225 A.

Thence S. 24° 30′ W. (variation 15° 14′ E.), 285 feet intersect line 4–1 of this survey. 315 feet intersect line 4–1, Gottenburg lode, at N. 25° 56′ W., 237·78 feet from corner No. 1. 688·3 feet intersect line 1–2, Gottenburg lode, at N. 64° 04′ E., 12·23 feet from corner No. 2. 1438 feet to trail, coursing N.-W. and S.-E. 1500 feet to corner No. 6, a granite stone, 34 by 14 by 6 inches, set 1 foot in the ground to bed-rock, with mound of stone, chiselled (6) 4225 A, whence a cross, chiselled on ledge of rock, marked B. R. (6) 4225 A, bears due north 12 feet.

Thence N. 65° 30′ W.(variation 15° 20′ E.), 70·3 feet intersect line 3–4 of this survey. 223·37 feet intersect line 4–1, survey No. 2560, at N. 38° 52′ W., 567·28 feet from corner No. 1. 300 feet to corner No. 7, a cross at corner point, and (7) 4225 A, chiselled on a granite boulder, 12 by 6 by 3 feet above ground, whence a cross chiselled on vertical face of cliff, marked B. R. (7) 4225 A, bears N. 72° W., 56·2 feet. A pine, 14 inches in diameter, blazed and marked B. T. (7) 4225 A, bears N. 10° E., 39 feet.

Thence N. 24° 30′ E. (variation not determined on account of local attrac-

* By permission from Bennett H. Brough's "Treatise on Mine Surveying."

tion), 38·43 feet intersect line 4–1, survey No. 2560, at N. 38° 52′ W., 653 feet from corner No. 1. 165 feet to trail, coursing N.-W. and S.-E. 1043·73 feet intersect line 2–3, Gottenburg lode, at N. 25° 56′ W., 379·06 feet from corner No. 2. 1432·90 feet intersect line 4–1, Gottenburg lode, at N. 25° 56′ W., 626·94 feet from corner No. 1. 1500 feet to corner No. 8, a spruce post 6 feet long, 5 inches square, set 2·5 feet in the ground, with mound of stone, marked (8) 4225 A, whence a cross chiselled on rock in place, marked B. R. (8) 4225 A,

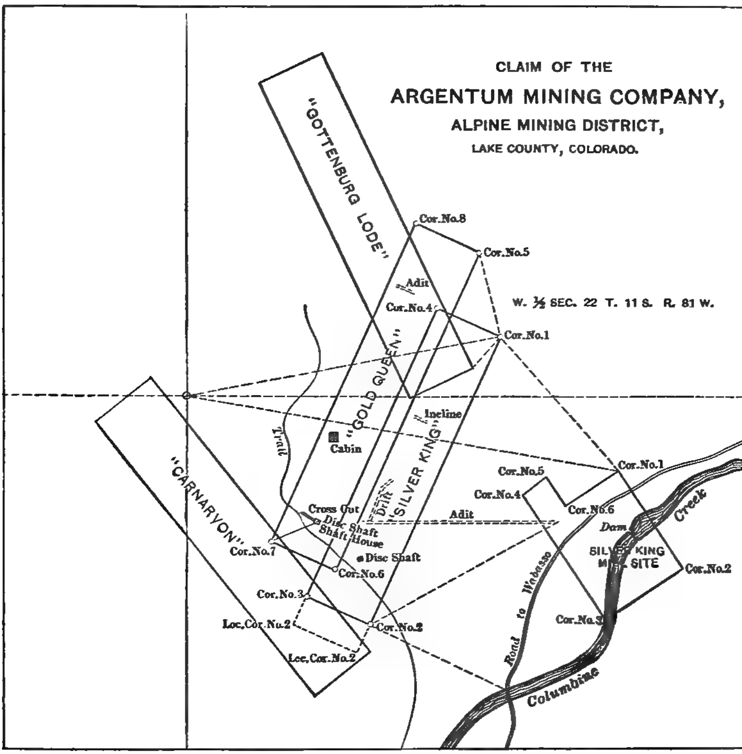


FIG. 270.

bears S. 9° 12′ E., 15·8 feet. A pine, 20 inches in diameter, blazed and marked B. T. (8) 4225 A, bears N. 83° E., 28.5 feet.

Thence S. 65° 30′ E. (variation 15° 16 E.), 300 feet to corner No. 5, the place of beginning.

Area.—Total area of Gold Queen lode. 10.331 acres. Area in conflict with other surveys, 4·022 acres, thus, with survey No. 2560. 0.034 acre, with Gottenburg lode, 2·679 acres, with Silver King lode (exclusive of conflict of said Silver King lode with the Gottenburg lode), 1·309 acres. Net area of Gold Queen lode, 6·309 acres.

CHAPTER XI.

THE SOLAR INSTRUMENT.

189. Description.—The solar instrument is an apparatus for the determination of bearings, latitude, and time by means of observations on the sun. Its primary use in surveying is for the determination of true bearings.

The essential elements are a "vertical axis" as in other apparatus, around which the entire remaining part of the instrument revolves, a "polar axis" lying in the plane of the vertical axis but capable of revolution around it and around the intersection of the two axes in the vertical plane determined by them; a vertical or latitude circle by which this last revolution is measured, a "declination arc" lying in the plane of, and capable of revolution around, the polar axis; a solar line of sight (L_1S_1) lying always in the plane of the declination arc but revolving both around an axis perpendicular to that plane, the amount of revolution being read on the arc, and around the polar axis; and a terrestrial or azimuth line of sight (L_2S_2) lying in and generating the vertical plane of the polar and the vertical axes. As noted above, the first line of sight (L_1S_1) revolves around an axis perpendicular to the plane of the declination arc and therefore perpendicular also to the polar axis. When the vernier of the declination arc reads zero, L_1S_1 is perpendicular to the polar axis. In general, the reading of the declination arc equals the angle made by L_1S_1 with a perpendicular to the polar axis.

190. The Elementary Instrument.—The elementary instrument is shown in Fig. 271, all the elements being revolved parallel to the vertical plane of projection.

The vertical axis is represented at V_AA_V. The polar axis PA_P

revolves in a vertical plane around O, its intersection with the vertical axis. It may also be revolved around the vertical axis so that, in its most general motion, it generates a cone whose axis is the vertical axis of the instrument and whose angle may have any value. This angle is read on the vertical or latitude circle $V_C C_V$. The line of sight L_1S_1 intersects the polar axis at the point O and revolves around that point in the plane of the

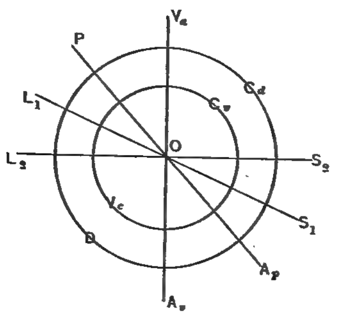


FIG. 271.

polar axis. The angle it makes with the polar axis can be determined by the declination arc or circle DC_D. The general movement of L_1S_1 with regard to PA_P is the same as that of PA_P with regard to V_AA_V. It generates a cone whose axis is PA_P and whose angle may have any value.

191. Astronomical Definitions.—The declination of any heavenly body, in this connection especially the sun, is its angular distance (measured from the center of the earth) north or south of the celestial equator (Art. 50). The declination of the sun changes continually, varying from 23° 27′ north on the twenty-first of June to the same angular distance south on the twenty-first of December. The sun itself being a fixed star, this change of declination is due to the fact that the plane of the earth's orbit is inclined to the equator at an angle of 23° 27′.

In astronomy an "hour circle" is any great circle of the celestial sphere passing through the poles. Suppose a heavenly body, as the sun, situated on any hour circle. Then for any point on the surface of the earth at any particular time, the angle made by the meridian plane of the point with the plane

of the hour circle on which is situated the sun, is known as the "hour angle" of the sun.

A "solar day," or an "apparent solar day," is the time included between two successive meridian passages of the sun's center. If the celestial sphere is divided into twenty-four equal parts by hour circles, one lying in the meridian plane, a solar hour is the time of the sun's passage between adjacent circles, and time reckoned in this manner is known as "apparent solar time."

192. Principle of the Instrument.—Suppose the point O, Fig. 271, located at the center of the earth, that is at the center of the celestial sphere, with the vertical axis directed to a point on the earth's surface whose latitude is ϕ. The prolongation of the vertical axis would be a vertical line at the point in question. Further, suppose the polar axis set at an angle with the vertical axis equal to the co-latitude of the point, but in this position left free to revolve around the vertical axis. It is evident that at one point of the revolution the polar axis will coincide with the axis of the earth. Assume the polar axis revolved into this position, and clamped there. Now, if the line of sight L_1S_1, by making the vernier of the declination arc read zero, is made perpendicular to the polar axis and revolved around it, it will cut the celestial sphere in the line of the celestial equator. But if the declination arc is set at the *declination of the sun at the given time*, in its revolution around the polar axis, L_1S_1 will cut the celestial sphere in the parallel of latitude on which the sun lies and at one point of the revolution will be directed towards the sun. But if PA_P be revolved slightly around the vertical axis so as to shift it somewhat from its position of coincidence with the axis of the earth, and L_1S_1 then is set at the declination and revolved around PA_P, at no point of the revolution will L_1S_1 be directed towards the sun.

In other words, if the instrument is set up at the center of the earth with its vertical axis directed to a point on the surface whose latitude is ϕ; if the polar and vertical axes are set at an angle $(90^\circ - \phi)$ and the line of sight L_1S_1 is set at an angle with a perpendicular to the polarax is equal to the declination of the sun—if then by a combination of the move-

ments of the polar axis around the vertical axis and the line of sight around the polar axis the line of sight is directed to the sun, the polar axis coincides with the axis of the earth.*

193. **Application.**—In practice the instrument is set up at some point on the earth's surface, but the radius of the earth is so small compared with the distance from the sun that the kinematic relations remain the same as in the last article. The instrument is set up with the vertical axis truly vertical, the polar axis is set at angle with the vertical equal to the co-latitude of the place, the line of sight L_1S_1 is set at an angle with the polar axis equal to 90° minus the declination of the sun. Revolution of the polar axis around the vertical axis and of the line of sight L_1S_1 around the polar axis is then had until L_1S_1 is directed to the sun. The polar axis will then be found parallel to the axis of the earth and the line of sight L_2S_2 will lie in the meridian plane.

194. **The Instrument as Practically Made.**—The apparatus is made in two forms, the solar transit, Figs. 272, 273, and 274, and the solar compass, Fig. 275. The solar transit consists of an ordinary transit with the solar apparatus attached to the top of the telescope. Fig. 272 illustrates admirably the working of one form.

The figure is self-explanatory. The line of sight L_1S_1 is peculiar to this style of instrument. It consists of a bar B, Fig. 276, having fastened perpendicular to its face at each end a block (R_1 and R_2). In each block is set a convex lens (C_1 and C_2) having its principal focus on the face of the opposite block at the center of a small square defined by two vertical and two horizontal lines. When either one of the two lines of sight thus formed is directed to the sun the image formed by the lens appears centered on the square at the opposite block. One lens is turned towards the sun in the case of north declination and the other in the case of south. Note that the pivot around which L_1S_1 rotates in the plane of the polar axis is not found at the intersection of the polar and the vertical axis, as in Fig. 271. This

* Strictly speaking there are two positions of the polar axis which will fulfill the desired condition. Ambiguity on this point may be avoided by first orienting the instrument in the meridian by means of the needle.

does not affect the working of the instrument. The line of sight L_2S_2, the latitude circle, and the vertical axis are seen to be the telescopic line of sight, the vertical circle, and the vertical axis

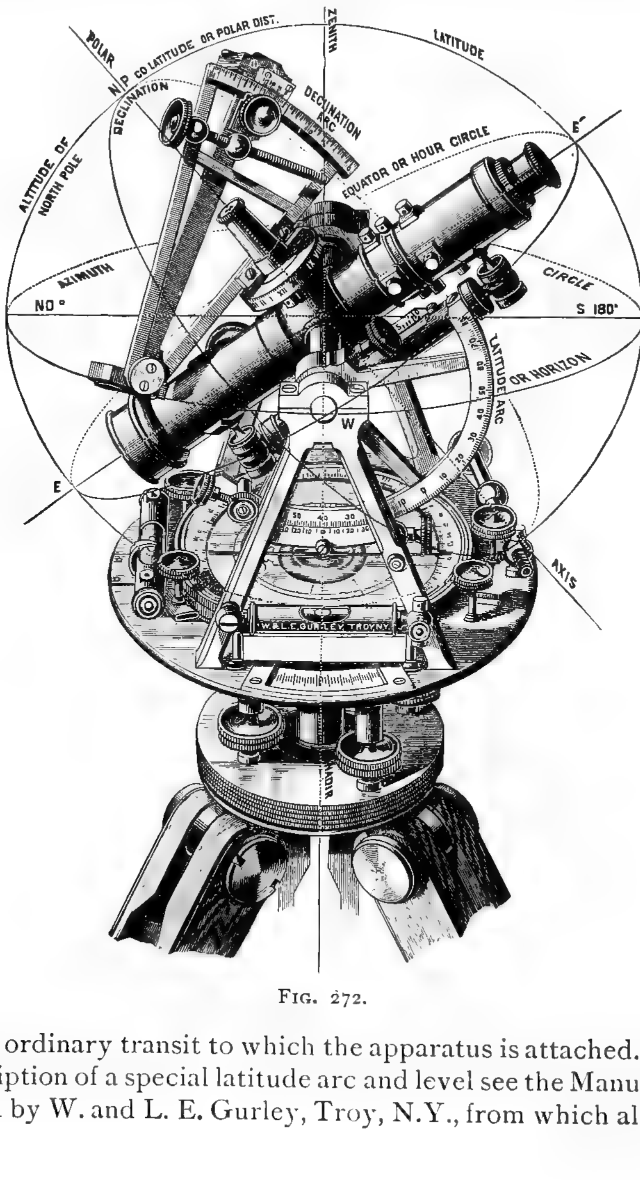


FIG. 272.

of the ordinary transit to which the apparatus is attached. For a description of a special latitude arc and level see the Manual published by W. and L. E. Gurley, Troy, N.Y., from which also Fig.

272 is taken. The solar transits shown in Figs. 273 and 274 differ from the one in Fig. 272 in having the line of sight L_1S_1 telescopic, and dispensing with the declination arc, using in its place a combination of the ordinary vertical circle and an added level or levels, used in connection with the solar apparatus. In Fig. 274 the tube AD is merely an eye-piece placed in this position to facilitate the observations. A cut of the solar compass is given in Fig. 275. It is seen to be entirely similar to the

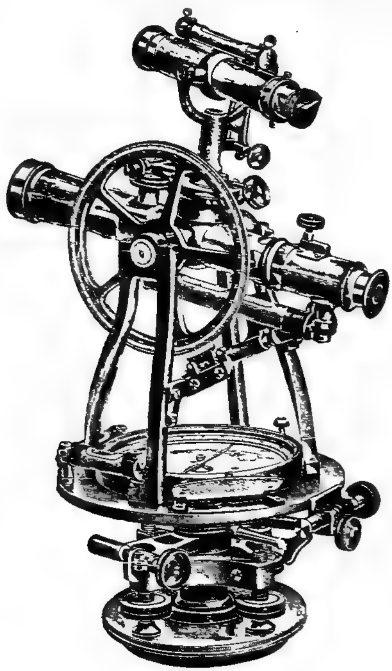


FIG. 273.

apparatus in Fig. 272. This indeed is the original apparatus invented by Burt in 1835. With the aid of the figure and the descriptions above given the construction and use of the solar compass will be readily understood. It is furnished with a horizontal circle reading to single minutes so that, the meridian having been determined by the solar apparatus proper, bearings can be accurately set off. Another adjunct is a trough compass (i.e., a needle held in a narrow box instead of a circular case)

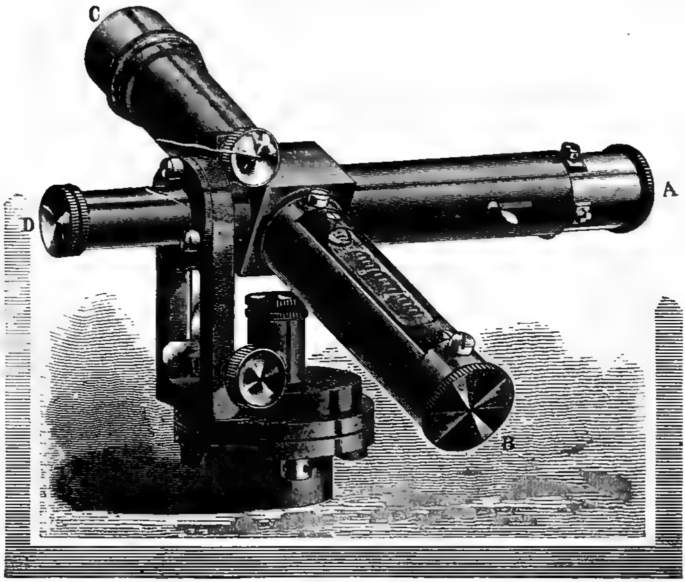


FIG. 274.

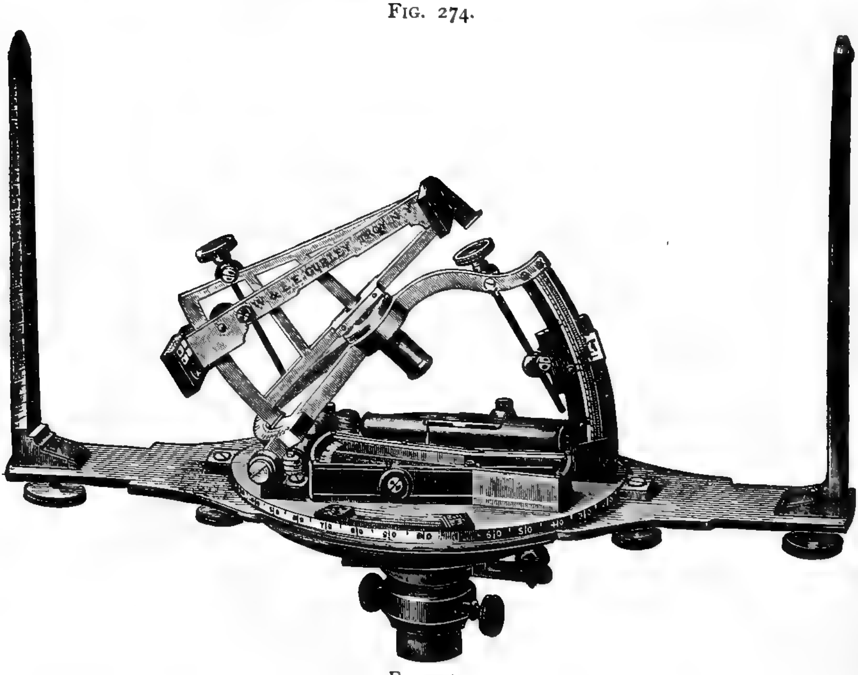


FIG. 275.

by means of which, on comparison with the true meridian, the magnetic declination may be known.

195. Time for Meridian Observations.—An observation for meridian should not be taken within an hour of noon. At this time, the motion of the polar axis around the vertical axis causes the intersection, with the celestial sphere, of the solar line of sight, to move in a direction perpendicular to the meridian plane. Consequently, if the observation is taken

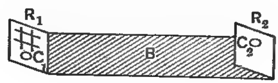


FIG. 276.

very near noon a slight motion in azimuth of the vertical plane containing the polar axis will not throw the sun out of the intersection of the celestial sphere and the solar cone of sight (cone generated by the line of sight L_1S_1 in revolving around the polar axis) and hence an erroneous meridian may be determined. When the sun is near the horizon the atmospheric refraction becomes excessive. For these reasons observations for meridian should be taken between the hours say of 8 and 11 A.M., and 1 and 5 P.M., and preferably about midway between these times.

196. Declination.—For a meridian observation to be made it is necessary to know the latitude of the place and the declination of the sun at the given time. The latitude of the place may be obtained from a good map, from a transit observation on a star (Art. 50) or with the solar instrument itself by the method of Art. 197. The declination of the sun, however, must be taken from a table calculated astronomically. Such a table is published by the Navy Department at Washington in the "Nautical Almanac." It is also issued by G. N. Saegmuller, Washington, D. C., to be used in connection with his solar attachment similar to that shown in Fig. 273. The declination must be corrected for atmospheric refraction, the correction being plus or minus as the declination is north or south. The tables issued by G. N. Saegmuller contain full instructions for taking out declinations and refraction corrections, and converting the "apparent solar" to "mean local" time.

197. Time and Latitude.—Some forms of the solar apparatus are furnished with an "hour circle" around the base of, and perpendicular to, the polar axis. Note that, when the polar axis is parallel with the axis of the earth, the plane of *this* hour circle is perpendicular to the planes of the "hour circles" described in Art. 191. By means of this circle, the revolution of the declination arc and polar line of sight L_1S_1 around the polar axis is read. The hour circle being divided into twenty-four equal parts, subdivided to twelfths, with the zero lying in the meridian plane, when the solar apparatus is oriented in the meridian, its reading equals the hour angle of the sun, expressed in hours and five-minute intervals, or the *apparent solar time*. Now, if the apparent solar time is known (and it can be determined from the tables mentioned in the last article), and if the declination arc is set at the proper reading, then, finally, if by motion of the polar axis in altitude and the whole apparatus in azimuth, the solar line of sight is directed to the sun, the reading of the latitude arc will give the co-latitude of the place, the explanation of which will be clear to the student having carefully considered all that has gone before. The best time to observe for latitude is at apparent solar noon. The objections which hold against observing for meridian at this time do not apply in the present case, as it is not concerned with the *exact* location of the meridian By setting the hour circle at zero and going out a short time before apparent noon (i.e. the sun's culmination), by means of the motions above referred to, the solar line of sight can be kept directed to the sun as it rises in altitude. When it appears to stop for a brief space and then descend will be the time of apparent noon, and the reading of the latitude arc will give the information desired.

If there is no hour circle attached, the instrument can still be set for zero time merely by bringing the solar line of sight into the vertical plane of the terrestrial line of sight and clamping it in this relative position.

198. Solar Observations with the Ordinary Transit.—The meridian may be determined in this manner. The work will depend on the solution of a spherical triangle. Note that the solar apparatus when employed solves this triangle *mechanic-*

ally. The method can be explained with the aid of Fig. 71. With the transit set up at O (a point on the earth's surface in latitude ZOZ'), the sun is supposed to be at S. If the line of sight is directed to S, through what azimuth angle $\theta = PZS = YOX$ must it be turned to lie in the meridian plane ZPO? Measure the altitude XOS of the sun. In the triangle PZS, PS = co-declination of the sun, or, if the declination is south $= 90°$ plus the declination $= a$.

PZ = co-latitude of the place $= b$.

SZ = co-altitude of the sun $= c$.

θ = azimuth angle PZS.

From the principles of spherical trigonometry *

$$\sin^2 \tfrac{1}{2}\theta = \frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}, \quad . \quad . \quad . \quad (112)$$

where

$$s = \frac{a+b+c}{2}.$$

An observation of this sort is facilitated by the use of the Davis Patent Solar Attachment illustrated in Fig. 277. With this device the images of the sun and the cross hairs are received on a ground glass screen attached back of the telescope eye-piece. When the line of sight is directed to the sun, its image on the screen is quartered by that of the cross hairs. If direct vision of the sun is to be had, a piece of colored glass must be inserted between the eye of the observer and the eye-piece. The manufacturers of the Davis attachment † also furnish an improved prism or diagonal eye-piece and colored glass attachment. The Davis attachment is particularly useful when the altitude of the sun is great, but the diagonal eye-piece alone will enable a high altitude observation to be made. A system of four extra cross hairs forming a square with diagonals, vertical and horizontal, enables the sun's image to be centered more accurately than can be done with the ordinary arrangement. An observation on any heavenly body,

* Crockett's Trigonometry, Art. 142.

† C. L. Berger & Sons, Boston.

except when situated directly at the zenith, will have to be corrected for refraction, as noted in Art. 196.

199. Adjustments of the Solar Transits in Figs. 273 and 274.—The ordinary transit with which the solar is used must

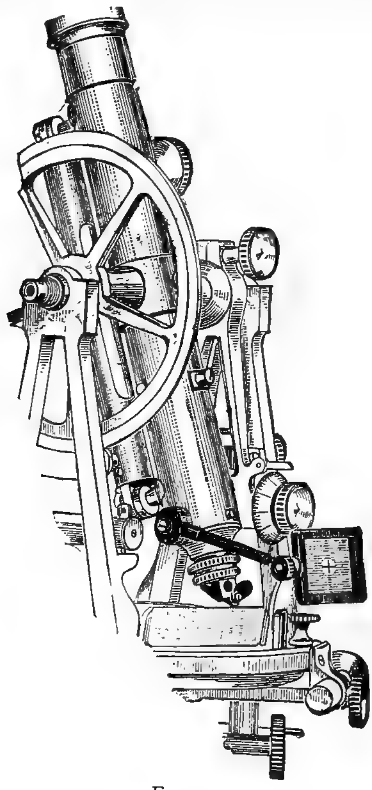


FIG. 277.

of course satisfy the tests of Art. 111. The solar adjustments proper are three in number.

(1) The adjustment to make the polar axis perpendicular to the line of sight (adjusted) and to the horizontal axis of the ordinary transit. The vertical axis of the ordinary transit being made truly vertical, level its telescope and also the solar telescope, the latter being in the plane of one pair of adjusting screws at the base of the solar apparatus. Revolve the solar

180° around the polar axis and note at the end of the revolution whether the bubble is still found in the center. If not, correct one half the error by means of the pair of adjusting screws above mentioned. Test and adjust until the test is satisfied. Then proceed in a similar manner with the solar telescope over the other pair of adjusting screws.

(2) The adjustment to make the line of sight parallel to the tangent of its attached level.

Having brought both the transit and the solar telescope into about the same vertical plane, level both and note the points to which both lines of sight are directed on a vertical surface some distance away. Measure the distance between these two points and note whether it is equal to the distance between the horizontal axes of the two telescopes. If so, the adjustment is correct. If not, the distance must be made equal by moving the intersection of the cross hairs in the solar telescope. In practice, first measure the distance between the axes and rule two parallel lines this distance apart on a piece of paper. The telescopes both being leveled, the paper can be put up with the lower line in the line of sight of the main telescope and the remainder of the operation performed as above.

(3) The adjustment to make the hour arc read apparent time.

If the instrument is provided with an adjustable hour circle, note whether, at the time of a meridian observation, it reads the apparent solar time as taken from the tables. If not, loosen the screws which hold it in place, turn it to the proper reading, and clamp it in this position.

200. Adjustments of the Transit Shown in Fig. 272.—(1) The adjustment to make parallel the two solar lines of sight.

The declination arm is first removed from the declination arc, and the "adjuster," a small bar furnished for the purpose, screwed in its place. Next the declination arm is placed on the adjuster and one of the lines of sight directed to the sun, its image falling within the square on the block. The declination arm is then turned upside down (the same lens being towards the sun) and the position of the image noted on the

block. If it again falls within the square, the line of sight in question is parallel to the planes of the edges of the blocks. If not, one half the error must be corrected by the movement of the silver plate on which the square is drawn, and the adjustment tested until perfect. The other line of sight is adjusted in the same manner. Both being made parallel to the planes of the edges of the blocks, they will be parallel to each other.

(2) The adjustment to make the declination vernier read zero when the lines of sight are perpendicular to the polar axis.

The vernier of the declination arc being set at zero, direct one of the lines of sight to the sun. Next revolve the solar apparatus around its polar axis, until the other line of sight is directed to the sun as near as may be. If, by the motion of mere revolution around the polar axis, the second line of sight can be directed *exactly* to the sun, so that its image appears centered squarely between the lines on the opposite plate, the adjustment is correct. If not, note the angle (on the declination arc) through which the solar line of sight must be turned in order to make the centering perfect, loosen the screws which hold the vernier in place, and by moving it through half the observed angle, correct the error. In this position, turn the screws to a firm bearing. Test and adjust again, if necessary, until the condition is satisfied.

(3) The adjustment to make the polar axis perpendicular to the line of sight (adjusted) and to the horizontal axis of the ordinary transit.

This adjustment is entirely similar to the first adjustment of the last article, except that an auxiliary level has to be employed. This auxiliary level is first placed on the declination arm and its bubble brought to the center by means of the movement of the arm along the declination arc. The remainder of the adjustment is performed exactly as in Art. 199.

(4) The adjustment to make the hour circle read apparent solar time. This is the same as the third adjustment of Art. 199.

CHAPTER XII.

THE U. S. PUBLIC LANDS.—RESURVEYS.

SECTION I.

The U. S. Public Lands.

201. Principal Meridian and Base-line—Guide Meridians and Standard Parallels.—The public lands of the United States Government are surveyed and subdivided by the government surveyors according to the general plan shown in Figs. 278–280. A true meridian and parallel N. S. and W. E., Fig. 278, intersecting at some convenient point O, are first carefully run out. This meridian is known as a "principal meridian," while the parallel is styled a "base-line," and the point O the "initial point." As the principal meridian and the base-line are being run, starting from their intersection, corners are established at every mile and half mile. Next two series of "standard parallels" or "correction lines" and of "guide meridians" are surveyed. The standard parallels are true east and west lines extending to both sides of the principal meridian at intervals of twenty-four miles north and south of the base-line. In Fig. 278 they are shown by the lines $W'_n\ E'_n$, $W''_n\ E''_n$, $W'_s\ E'_s$, etc. The guide meridians are run north from the base-line and standard parallels, at intervals of twenty-four miles east and west of the principal meridian. They are twenty-four miles in length, each guide meridian extending from its beginning point north to its intersection with the next standard parallel or possibly the base-

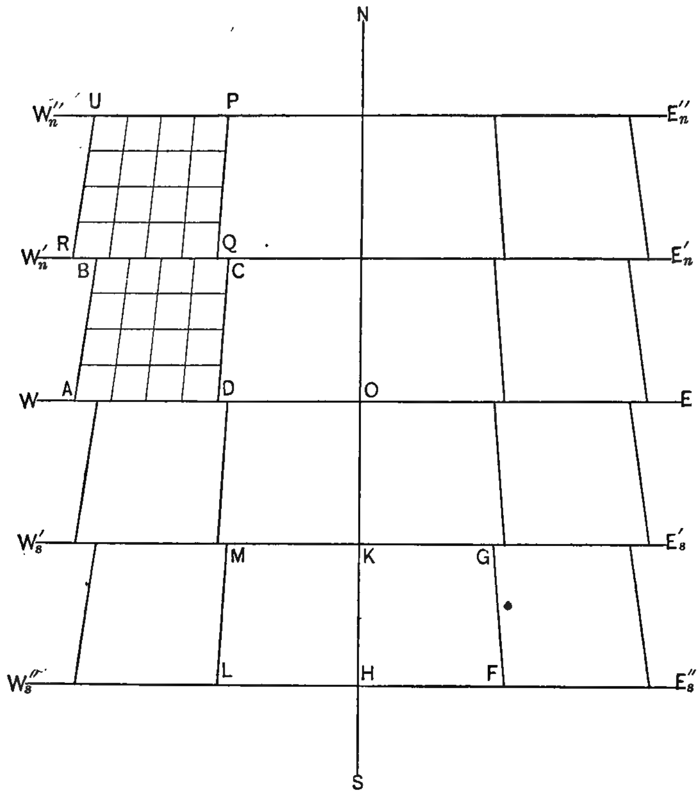


FIG. 278.

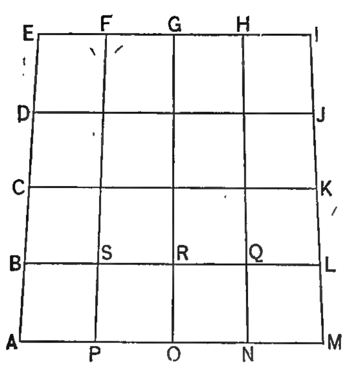


FIG. 279.

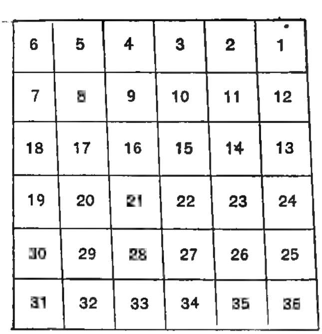


FIG. 280.

line. Evidently, while the distance east and west between the southern ends of any two adjacent guide meridians is twenty-four miles, owing to the convergence of the meridians, the corresponding distance between their northern ends will be somewhat less. In Fig. 278, *AB*, *DC*, *LM*, *FG*, etc., are guide meridians. As the guide meridians and standard parallels are surveyed, corners are established as on the principal meridian and the base-line.

202. Subdivision of Twenty-four Mile "Squares" into Townships.—The area now having been divided into tracts approximately twenty-four miles square, each of these is further subdivided, as shown in Figs. 278 and 279, into "townships" approximately six miles square. In Fig. 279, *AM* and *EI* represent two standard parallels, or the base-line and a standard parallel, either being the base-line. *AE* and *MI* are guide meridians, or one of them may be the principal meridian. Lines *AE*, *IM*, and *MA* are each twenty-four miles long, while *EI* is shorter than twenty-four miles by the convergence of the meridians *AE* and *MI*. The twenty-four mile "square" *AEIM* is divided into six-mile square townships first by running the true meridians *PF*, *OG*, and *NH*. These are known as "range lines" or "township exteriors." Corners are established at every mile and half mile. From the points *J*, *K*, and *L* the "township lines" or also "township exteriors," *JD*, *KC*, and *LB* are then run. These are theoretically true east and west lines, but the practice is, starting from a township corner as *L* on the east line of a twenty-four-mile square, to run a random line in the direction of *Q*, the corner previously established, six miles north of *N* on the range line *NH*. As the line *LQ* is being run, corners are established at each full mile and half mile. This throws the last half mile short by the convergence of the meridians *ML* and *NQ*. The random line not striking the point *Q*, the offset is noted, and the mile and half mile corners just set on the random line are moved south or north the proper proportional distances. Random lines are then run from *Q* to *R*, *R* to *S*, and *S* to *B* in the same manner. The lines *JD* and *KC* being surveyed and marked in the same way, the approximate twenty-four-mile square will now be found subdivided

into approximate six-mile-square townships, a typical one of which is shown in Fig. 280. A line of townships running *north and south* is known as a "range," while a similar line running *east and west* is styled a tier. The ranges and tiers are numbered respectively with reference to the principal meridian and the base-line. Thus all townships lying just east of the principal meridian would be situated in "range one east," a township in the fourth east and west row south of the base-line would lie in "tier four south," etc. In this way the position of any township may easily be described.

203. Subdivision of Townships into Sections.—To return to Fig. 280, it is to be noted that the typical township there shown may have its south boundary less than six miles in length. This would be true except in the case where such south boundary happened to be a part of either the base-line or a standard parallel. Each township is further subdivided into thirty-six sections each a mile square as nearly as may be. The sections are numbered, as shown in Fig. 280. The terms "tier" and "range" have been defined in Art. 202, and relate to east and west and north and south rows of *townships*. In the present article and in the next the expressions "tier" of *sections* and "range" of *sections* will be used, meaning, in each case, an east and west and a north and south row. It must be understood, however, that this is done only for convenience of expression and is not the official usage.

The method and order of township subdivision will be to start at the previously established common S. E. corner of Sec. 35 and S. W. corner of Sec. 36 (Fig. 280), and run thence not truly north, but *parallel* with the east boundary of the township, establishing, at the end of a half mile the common quarter section corner of sections 35 and 36, and at the end of a mile the common section corner of sections 35, 36, 25, and 26. From the corner thus established a random line will be run eastward towards the previously established N. E. corner of Sec. 36. Arriving opposite this corner, the offset N or S is noted, and the line just run corrected to run straight between the established corner. This correction will consist merely of marking the true line by blazed trees and establishing at *its*

middle point the common quarter section corner of sections 36 and 25. For convenience in doing this last, a temporary station will have been left on the random line a half mile east of its starting point, i.e. the N. W. corner of Sec. 36. Starting next at this N. W. corner of Sec. 36, Sec. 25 is surveyed in a precisely similar manner, and so on with sections 24, 13, and 12. When Sec. 1 is reached, its west line, instead of being run parallel to the east line of the township, is run as a straight line connecting the N. W. corner of Sec. 12 with the previously established N. W. corner of 1 and N. E. corner of 2. The quarter section corner of the middle point of the N. line of Sec. 1 will also have previously been established. The quarter section corner common to sections 1 and 2 will be placed on the west line of Sec. 1 just a half mile north of its S. W. corner. If, however, the north line of Sec. 1 happens to be part of a base-line or a standard parallel, i.e. if the township lies in the most northern tier of a twenty-four-mile square, the above proceeding is somewhat varied. In such a case as this the west line of Sec. 1 is run parallel with the east line of the township and the N. W. corner of Sec. 1 established at the intersection of its west line with its previously located north line. Proceeding westward the ranges of sections are surveyed in the same way until the sixth range is reached. The west line of this range will have been surveyed already, and its east line is the just completed west line of the fifth range. It remains only to join by straight lines corresponding points on these two boundaries. The quarter section corners, however, are established at even half miles from the east line of the range. It will be noted that this method makes a full mile square all the sections in a township except those in its most northern tier and western range, and that in these the error is thrown into the northern and the western halves.

Note also that the quarter section corners on the north lines of sections 1 to 6, where such sections lie immediately south of a *base-line* or *standard parallel*, are *not* established by the government surveyors. If it is desired to locate these corners in future work, they should be placed midway between the N. E. and N. W. corners of their sections, except in the case of

section 6. With section 6, the quarter corner on the north line is set just a half mile west of the N. E. corner.

204. Subdivision of Sections.—The sections are not further subdivided by the U. S. surveyors. If, however, it is desired to divide a section into *quarter sections*, and the quarter section corners have been set on the four sides of the section during the original survey, the method will be to join the opposite quarter section corners by straight lines, their intersection giving the corner common to the four quarters. The same rule holds in the case of the sections lying just south of the base-line or a standard parallel. As stated in the last article, the Government surveyors do not set the quarter corners on the north lines of such sections. To quarter one, the subsequent surveyor will first set the quarter corner on the north line as directed in Art. 203. Then lines joining opposite quarter corners will give the correct subdivision. In the case of a section, as a fractional section, "where opposite corresponding corners have not been or can not be fixed, the subdivision lines should be ascertained by running from the established corners due north, south, east, or west lines, as the case may be, to the water course, Indian boundary line, or other boundary of such fractional section.

(*a*) The law presumes the section lines surveyed and marked in the field by the United States Deputy Surveyors to be due north and south or east and west lines, but in actual experience this is not always the case. Hence, in order to carry out the spirit of the law, it will be necessary in running the subdivisional lines through fractional sections to adopt mean courses where the section lines are not due lines, or to run the subdivision line parallel to the east, south, west, or north boundary of the section, as conditions may require, where there is no opposite section line."*

205. Instruments Used.—It is specified that all lines of the Government land surveys shall be run with instruments capable of determining directions independently of needle bearings. This restricts the instruments used to transits and solar instru-

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* Circular on Restoration of Corners, etc., issued by the General Land Office, Washington, D. C.

ments, either compasses or transits. If the transit without solar attachment is used, it must be done in connection with careful Polaris observations.

206. The Manual of Instructions.—A "Manual of Instructions" has been issued from time to time by the Government General Land Office at Washington. The edition of 1894 comprises 236 pages and contains detailed information and instructions concerning the survey of the public lands. A surveyor undertaking work in connection with government surveys should have a copy of the manual, as also a copy of the instructions in force at the time of the original survey. Clevenger's "Government Surveying" is a work of reference in this connection. See also the circular issued by the Land Office on the Restoration of Lost or Obliterated Corners and Subdivision of Sections. Nearly all the work now to be done on the Government lands is in the line of resurveys. Concerning these the next section of the present chapter treats more particularly.

207. Convergence of Meridians.—While the method employed in the survey of the public lands falls within the sphere of plane surveying, still, as indicated in the preceding articles, to the extent of allowing for the convergence of meridians, it takes into account the curvature of the earth. Also as shown in Art. 28, a due east and west line, if prolonged by backsight and foresight, will not, except at the equator, continue due east and west. In some cases, therefore, it becomes necessary to compute the bearing required to hold a northerly or southerly line parallel with a meridian some distance east or west (Art. 203) of the line in question, to determine the difference in length of two east and west lines lying between and limited by two meridians or to calculate the corrections necessary to make a line conform to a true parallel. The following discussion will indicate the method of performing the computation in the first two cases, while in article 209 will be explained the method of running due east and west lines.

At points on the same parallel of latitude, the angle between any two meridians is the angle between their tangents. At the equator this angle is zero, while at the poles it equals the difference in longitude of the meridians. In Fig. 281 let S_1N_1 and

S_2N_2 represent two meridians whose difference in longitude is θ, the length of each being equal to l. The earth is regarded

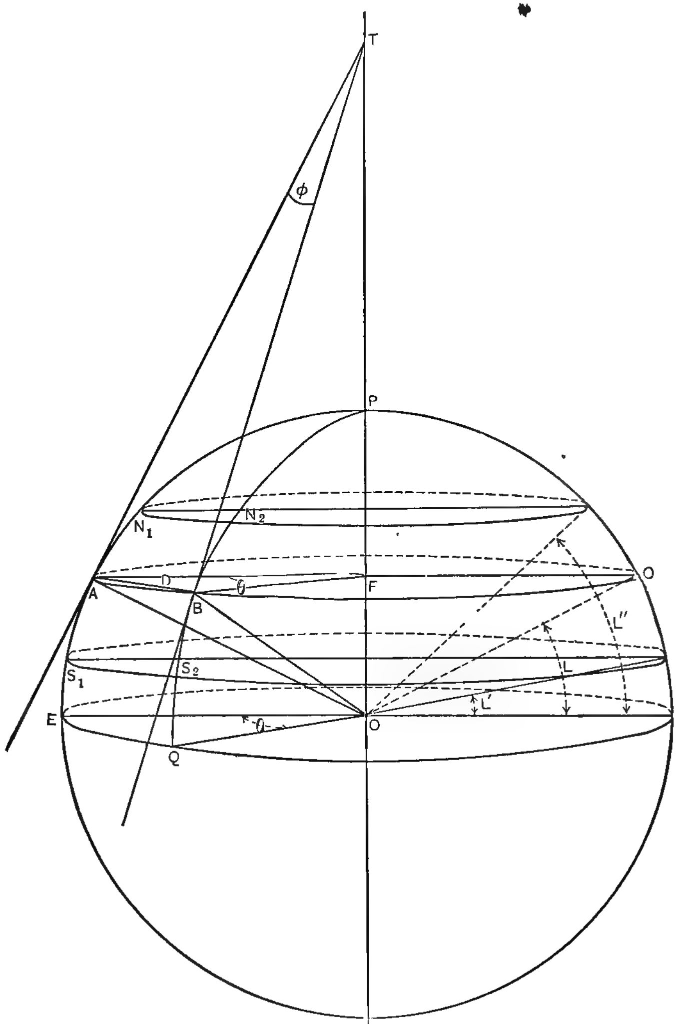


FIG. 281.

as a sphere having its center at O. EQ represents the terrestrial equator and A and B are two points on the meridians lying

on the same parallel of latitude, ABC. The tangents AT and BT to the meridians at A and B will intersect the axis of the earth prolonged at some point T. The object of the discussion is, first, the deduction of a formula giving the value of the angle ϕ between the two meridians at the points A and B, and, second, the determination of the difference in length of two east and west lines S_1S_2 and N_1N_2, lying between two meridians of equal length l. The angle L is the common latitude of the points A and B. Let $R = OE = OQ = OA = OB$ represent the mean radius of the earth.

$$\phi = 2 \sin^{-1} \frac{ADB}{2AT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

$$AT = R \cot L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

$$ADB = 2AF \sin \frac{\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

$$AF = R \cos L. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

Substituting (116) in (115)

$$ADB = 2R \cos L \sin \frac{\theta}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

Substituting (117) and (114) in (113)

$$\phi = 2 \sin^{-1} \frac{2R \cos L \sin \frac{\theta}{2}}{2R \cot L} = 2 \sin^{-1} \sin L \sin \frac{\theta}{2}. \quad (118)$$

With the U. S. land surveying system, in the cases which require an application of equation (118), the values of both ϕ and θ are small, and we may therefore replace the arcs by their sines. With these substitutions equation (118) becomes

$$\phi = \theta \sin L. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

Let L' represent the latitude of S_1 and S_2, and L'' that of N_1 and N_2.

The arc $S_1S_2 = R \cos L' \times \theta$.

" " $N_1N_2 = R \cos L'' \times \theta$.

$$\begin{aligned} S_1S_2 - N_1N_2 = c &= R\theta (\cos L' - \cos L'') \\ &= R\theta \left[- 2 \sin \tfrac{1}{2}(L' + L'') \sin \tfrac{1}{2}(L' - L'')\right] \\ &= - 2R \sin \tfrac{1}{2}(L' - L'') \times \theta \sin \tfrac{1}{2}(L' + L'') \quad . \quad (120) \end{aligned}$$

If A and B are respectively midway between S_1 and N_1, and S_2 and N_2, their latitude L is the mean latitude of the lines S_1N_1 and S_2N_2, and equals $\frac{1}{2}(L' + L'')$. Also in the Government surveys, the extreme length of north and south lines (guide meridians and township exteriors) run without lateral correction is 24 miles, and within this limit, the expression

$$- 2R \sin \tfrac{1}{2}(L' - L'') = 2R \sin \tfrac{1}{2}(L'' - L')$$

of equation (120) is practically equal to l. Making these substitutions in equation (120) we may therefore write

$$c = l\,\theta \sin L. \qquad (121)$$

From (119), $\theta \sin L = \phi =$, for the small values of ϕ occurring in practice, $\sin \phi$, with which (121) becomes

$$c = l \sin \phi, \qquad (122)$$

or *the difference in length between two parallels included between two meridians is equal to the common length of the meridians multiplied into the sine of their mean angle of convergence.*

208. Applications of Equations (119) and (122).—Consider the subdivision of a township into sections, Art. 203. The section lines are to be run *parallel* with the east range lines. The question arises, what bearings must these section lines have? The answer is found by a solution of equation (119) $\phi = \theta \sin L$. L can be found from any map and θ taken from Table X or XI*a*. With these data ϕ can be found at once and the bearing corrected accordingly.

Equation (122) is used in checking the work of running range lines. It will be recollected that these are true meridians and that their extremities lie in parallels previously divided in six mile lengths. By means of equation (122) there can be computed the distance between the end of one of these lines and the nearest previously set corner on the parallel in which it lies. The agreement of the computed and the measured distances checks the work. Table XI gives the length of a degree of latitude, in chains, from latitude 29° to 48°. To convert distance on the meridian to latitude, find from the table the length a degree of latitude in the latitude of one extremity of the line, and also in

the latitude of the other, allowing for this purpose 52″ arc per meridional mile. Take half the sum of these two lengths for the mean length of a degree and convert the line length into arc by simple proportion.* Or the distance may be changed to arc with the aid of Table XI*a*.

Table XII has been computed from eqs. (119) and (122).

209. East and West Lines.—As indicated in Art. 28, Fig. 28, a true east and west line cannot be prolonged indefinitely by means of fore sights and back sights. Neither is it convenient to perform the operation by aid of a compass, either magnetic or solar. It accordingly becomes necessary to devise a practical method of locating a true parallel. Suppose that the transit is set up at a point as A, Fig. 282, and that it is desired to locate a point as B on the same parallel with A, but x miles further west. If a right angle is turned off from the meridian AP at A and the line thus determined prolonged, the great circle GAE will be run out. Let EP be a meridian midway between AP and BP. Then, in the right spherical triangle PAD,

$$\cos AP = \cot PAD \cot APD$$

and

$$\cot PAD = \frac{\cos AP}{\cot APD}, \quad \ldots \ldots \ldots \quad (123)$$

$$PAD = \cot^{-1} \frac{\cos AP}{\cot APD}, \quad \ldots \ldots \quad (124)$$

or, with the notation of Fig. 280,

$$PAD = \cot^{-1} \frac{\sin L}{\cot \frac{\theta}{2}}. \quad \ldots \ldots \quad (125)$$

Also

$$DAE = 90^\circ - PAD.$$

Therefore

$$DAE = \tan^{-1} \frac{\sin L}{\cot \frac{\theta}{2}} = \tan^{-1} \tan \frac{\theta}{2} \sin L. \qquad (126)$$

In practice DAE and θ are both small. We may, therefore, write

$$DAE = \tfrac{1}{2}\theta \sin L = \tfrac{1}{2} ATB \quad \ldots \ldots \quad (127)$$

* Manual of Instructions of 1894, p. 129.

from eq. (119). If, therefore, we turn off from the meridian at A an angle equal to 90° minus one half the angle of convergence of the meridians at A and B, the line thus determined, if prolonged, will pass through the point B. Intermediate points will lie not on the parallel ACB, but on the great circle ADB. Offsets, however, can be calculated and laid off, thus locating

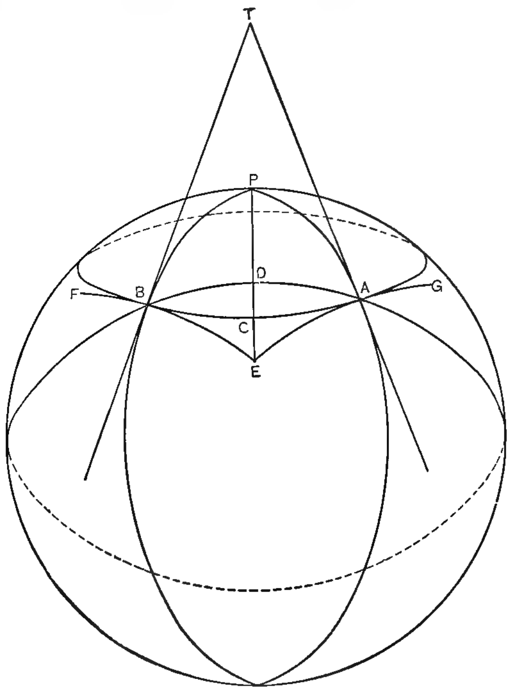


FIG. 282.

points on the parallel as ADB is being run. It is the practice in the government surveys to locate the points A and B six miles apart—the length of a township line. At B a new meridian is established and the work repeated. Points are located on the parallel at every half mile. The Manual recommends the "secant" method. This consists in running, instead of the line ADB, an intermediate or secant line which passes south of A and B and intersects the parallel at two points one mile west of A and one mile east of B. Tables are given containing the

azimuths of the secant (i.e. angles made with the meridian) and offsets to the parallel at each half mile. In latitude 50—the highest given in the table—the secant would start from a point 3.96 feet south of A and the offset DC would be 3.17 feet. The offsets may be taken perpendicular to ADB.

SECTION II.

Resurveys.

210. Nature of the Work.—The object of a resurvey is the location of the boundaries of a tract *as they were determined by the surveyor who originally laid them out.* This definition can be best illustrated by an example. Suppose that A, the original owner of a large tract, has at some past time sold a portion of it to B, and that in the deed from A to B the portion sold is described as being, say, one mile square, with boundaries due north and south and east and west. B sells the property to C and C to D. D employs a surveyor to locate the tract. It is the business of D's surveyor to locate the boundaries *as they were originally located* by A's surveyor at the time of the sale to B. In doing this the surveyor may find the original notes woefully in error. The actual tract may have no side a mile in length, parallel to any other side, or even a straight line. This makes no difference. The surveyor must find *the original lines.* As may be supposed, this leads to a great deal of difficulty in many cases. The author has one instance in mind where two surveyors, in the endeavor to find an original line, located two separate and distinct lines a mile apart. The trouble arises mainly from the fact that the original surveys were not made with due care and skill, especially in the matter of recording the notes. A line would be recorded as running to the bank of a certain stream, when as a matter of fact it stopped at the bank of some branch of the stream, etc. Erroneous determination of the magnetic declination has been a fruitful source of trouble. Bearings have been stated as *true* which were really *magnetic*, and so on. Again, many original surveys have been made on paper simply, though the notes call for actual

field work. In such a case as this discrepancies are very apt to be discovered.

211. General Rules.—It is impossible to lay down comprehensive rules for the guidance of the surveyor in relocations. The intelligence of each individual will have to be exercised in his own practice.

Generally, it may be stated, first, that marked boundaries and corners take precedence over all other evidence in questions of location. Thus in country work the original lines were generally marked, while being run, by blazing trees to either side. If the notes of such a line call for a bearing of N. 10°00′ E. and no marked line is found running in that direction, while an old marked line is discovered with a bearing of N. 5°00′ W., the latter would probably be accepted as the true line. Second, "distances govern courses." This proceeds on the supposition that an error is more apt to be made in connection with bearings than with line lengths. If a location can be made to fit the original field-notes with the exception of one length or one bearing, the latter would yield to the former. Third, the area of the tract is, generally speaking, the least important consideration.

212. True Position of the Surveyor.—The surveyor should remember, however, that it is no part of his duty to assume judicial functions. All that can be expected of him is to inform his clients as to the *actual state of affairs on the ground.* Thus, in the case cited above, suppose the surveyor to find marked lines and other evidence that the tract surveyed in connection with the sale from A to B was entirely different from the one *described* in the deed between those parties. He should simply report his findings to D, joined with his opinion as to the position of the lines run out by the original surveyor, but assuming no authority to fix them in any given location.

After a careful examination of the original field-notes and a searching investigation of the facts on the ground, coupled perhaps with the testimony of parties long familiar with the property in question, the experienced surveyor will generally feel morally certain as to the location of the original lines. But even with this he should not assume the responsibility for

finally fixing their positions. Many clients will experience difficulty in appreciating these facts, and will expect the surveyor to state *positively* "here is the N. E. corner of the original tract sold to B by A." This is something that no man on earth can do unless he saw the original corner placed.

213. City Property.—In many cases street lines are not marked by any permanent monuments. Even when so marked the blocks may prove to be larger or smaller than called for in the original plot. Suppose the case of a vacant block which on the original plot is shown with square corners and equal parallel sides, but which on resurvey is found to have no two sides equal or parallel. If the block is shown subdivided into lots which have been sold by number to various parties, the only practical location of the lines, unless of course the original corner monuments can be found, is by agreement among all the owners. Here it may be noted that in the case of the location of a property line between adjacent owners, it is always wisest to have an agreement between the parties. In the case of a vacant block divided into lots whose street frontage is a little longer or shorter than called for in the original plot the surplus or deficiency will generally be equally divided among all the lots. The case where some of the lots have been sold and perhaps improved and built upon presents more difficulties. The surveyor finds his only safeguard in stating simply the facts as he finds them, coupled with his opinion as to the correct locations. He should be especially careful in regard to the acceptance of such monuments as he may find. Unless they can be shown to be the *original monuments* they are only valuable as giving the results reached by some previous resurvey. However, monuments of any sort become valuable as evidence of true location in proportion to the time during which they have stood and generally been regarded as correct. In locating a lot, it is always safer to test the exterior lines and angles of the whole block. Probably the majority of surveyors do not go to this trouble.

214. Country Property.—Generally speaking, this presents less difficulty. In wooded regions the original lines have been marked by blazing trees. The method of marking these line

trees varies somewhat in different localities. The commonest mark is three hacks. "Fore and aft" trees, i.e., those standing directly on the line, are generally marked on the sides where the line enters and leaves. For instance, in the case of a north and south line, or the north and south. The ordinary line trees are hacked on the sides facing the line.

Bearing trees have generally been established near the corners, and will be described in the field-notes about as follows: "The S. E. corner of Sec. 13 from which a pin oak, 15 inches diameter, marked *X*, bears S. 13° W., 18 feet," etc. Many of the old bearing and line trees have been destroyed, but often the stumps afford sufficient evidence of their locations. In the case of a prairie region with practically no woods the work becomes more difficult. The author has seen a section shifted over the country in a most surprising manner. At the second survey it was moved from under the house of the owner, who then placed the building directly in the middle of the second location to allow ample freedom for future motion. Hodgman's "Land Surveying" and Dorr's "Surveyors' Guide" are valuable works in this connection, especially with reference to the U. S. lands. See also the circular of the General Land Office alluded to in Art. 206.

215. Descriptions.—The original field-notes may generally be found in the office of the county clerk, parish clerk, or corresponding official. The surveyor cannot exercise too much care in obtaining a full description. To illustrate, the author was employed to locate a city lot. The owner on being asked for a description of the property produced a copy from the deed, by metes and bounds. On being further asked if this was the *entire* description in the deed, he replied that it was, and the property was located accordingly. Later the author obtained the deed. On examination, the description was found to continue, stating that the property in question was lot No. ——, block ——, in the —— addition to the city of ——, all as per a map of record in the office of the county clerk. As a matter of fact, the most important part of the description had not been furnished to the surveyor.

In conclusion, too great care cannot be exercised by the surveyor both in obtaining the full field-notes and in weighing the local evidence. But even with this he should merely report and advise, in no uncertain tone, perhaps, but without assuming the responsibility of final location.

APPENDIX A.

PROBLEMS.

ARTICLE 4.

1. Suppose a tract surveyed with a half chain and described in the notes as having been surveyed with a "chain," what would be the error of the calculated area?

2. What would have to be the length of a chain such that, if the area of a tract be given in square "chains," dividing by 2.5 and moving the decimal point one place to the right will reduce it to square rods?

ARTICLE 9.

3. Suppose a line AB of true length 1460 ft, measured with a surveyor's chain. If the pins are placed 4 inches off line alternately to the right and left, what is resulting error, in links, in the measurement of the line?

ARTICLE 10.

4. A line is recorded as being 1235.3 links in length. If the surveyor's chain with which the measurement has been made is later found to be 2.2 inches short, what is the true length of the line in feet?

5. In the above example if the measuring instrument is an engineer's chain and the true length of the line 1170.5 feet, what is the true length of the chain?

6. A chain whose true length is 75 ft. is used in measuring a line whose true length is 1000 ft. If the result of the measurement is 987 ft., what is the standard length of the chain?

ARTICLE 11.

7. Assume a line 1235 ft. in length measured with a fifty-foot steel tape, the tape being held at an angle of 6 degrees with the horizontal. What is the resulting error in the measurement of the line?

8. A straight line AB is measured through a tunnel at the base of a mountain. Two points A' and B', having the same latitudes and longitudes as A and B, are found higher up on the mountain and the line between them measured. Will the lines AB and $A'B'$ be of equal length? If not, find an expression for their difference in length, supposing the difference in elevation of A and A' 500 and of B and B' 600 ft.

ARTICLE 14.

9. Determine the length of a line by estimation, pacing, and chain measurement, and compare the results.

ARTICLE 16.

10. Prolong a line by means of rods about 1000 ft. Then retrace the line in the same manner and note how far the return line passes from the starting point.

ARTICLE 20.

11. Draw the notes for locating an irregular 5-sided building none of whose sides are parallel with the line being run.

12. A point is described in the notes as being respectively 5 and 7 ft. from two marked points. Can the point be accurately located?

ARTICLE 23.

Find area of following triangles in acres, the sides being denoted by a, b, and c.

13. $a = 535$ ft., $b = 720$ ft., $c = 618$ ft.

14. $a = 210$ varas, $b = 178$ varas, $c = 231$ varas.

15. $a = 25$ rods, $b = 35$ rods, $c = 33$ rods.

16. $a = 80$ yds., $b = 63$ yds., $c = 58$ yds.

ARTICLE 25.

17. A field is surveyed and the area determined as 157.3 Ac. The chain used is 73 ft. long and the true area is 163 Ac. What is the standard length of the chain?

18. A tract of land when surveyed is found to contain 105 Ac. The surveyor's chain used is then found to be $\frac{5}{10}$ of a ft. too long; what is the true area of the tract?

19. A field whose true area is 305 Ac. is found by survey to contain 300 Ac.; what is the true length of the surveyor's chain used?

20. A field whose true area is 213.4 Ac. is surveyed with a 50-vara chain which is 2 inches short; what is the area found?

ARTICLE 29.

21. A line is run out with the magnetic bearing N 13° 15′ E. At the same time the declination is noted as being 0° 55′ W; what will be the magnetic bearing of the line at a later date when the declination is 1° 05′ E?

ARTICLE 31.

Find the correct bearing of the line *AB* in each of the following traverses:

		Foresight.	Backsight.
22.	*AB*	N 35° 14′ W	S 33° 55′ E
	BC	N 15° 15′ W	S 14° 13′ E
	CD	N 17° 13′ W	S 17° 13′ E
23.	*AB*	N 2° 13′ E	S 0° 10′ E
	BC	S 14° 05′ W	N 3° 15′ W
	CD	N 18° 16′ E	S 17° 50′ W
	DE	S 4° 10′ W	N 5° 00′ E
	EF	N 18° 15′ E	S 18° 15′ W
24.	*AB*	N 15° 00′ E	S 15° 00′ E
	BC	S 14° 35′ W	N 14° 30′ E
	CD	N 25° 10′ E	S 23° 55′ W
	DE	S 85° 05′ W	N 85° 05′ E

Is the above problem possible? If so, under what circumstances?

25. In the above examples 22–24 from consideration of the fact that the *angle* of the traverse determined by the compass at each station is correct, devise another method of determining the bearing of *AB*.

ARTICLE 37.

Compute the error of closure, its bearing, and the ratio of closure in each of the following closed traverses:

	Line.	Wt.	Bearing.	Length.
26.	*AB*		S 82° 00′ E	800.0 ft.
	BC		S 13° 50′ W	572.4 "
	CD		S 74° 55′ W	490.3 "
	DA		N 10° 17′ W	625.0 "
27.	*AB*		S 34° 30′ E	204.3 varas.
	BC		S 85° 49′ W	506.1 "
	CD		N 13° 55′ E	212.0 "
	DA		S 89° 00′ E	325.4 "
28.	*AB*	1	N 53° 25′ E	344.1 yds.
	BC	2	S 28° 00′ E	252.0 "
	CD	4	S 37° 53′ W	320.5 "
	DA	2	N 36° 50′ W	350.0 "
29.	*AB*		N 84° 50′ E	6.88 ch.
	BC		S 29° 48′ E	4.90 "
	CD		S 84° 50′ W	8.75 "
	DA		N 6° 55′ W	4.70 "
30.	*AB*	3	S 75° 00′ E	344.1 ft.
	BC	1	S	521.0 "
	CD	2	N 84° 10′ W	437.2 "
	DE	2.5	N 34° 15′ E	460.3 "
	EA	1.5	N 38° 31′ W	250.4 "
31.	*AB*		N 35° 15′ E	1517.5 "
	BC		S 54° 45′ E	1110.0 "
	CD		S 35° 15′ W	1508.2 "
	DA		N 54° 45′ W	1115.3 "

Article 40.

32. Suppose a closed traverse surveyed with a compass with the backsight agreeing with the foresight on each line. Within what limits could Eq. (10) be expected to be fulfilled?

Article 42.

33–38. Compute the area in Ac. in each example under Art. 37.

39. Suppose a straight line 100 yds. in length with offsets measured to the bank of a stream at every 50 ft., the length of the offsets, in feet, being as follows: 5.2, 20.3, 22.0, 15.4, 2.0, 17.3, 3.5; compute the area between the line and the stream.

Article 45.

40. Find the length of the line *AC* in the 1st ex. under Art. 37.

41. Find the bearing of the line *BD* in the 3d ex. under Art. 37.

Supply the missing data in the following closed traverses:

	Line.	Bearing.	Length.
42.	*AB*	N 13° 05′ W	417.3 ft.
	BC		503.0 "
	CD	S 45° 00′ E	400.0 "
	DA		317.5 "
43.	*AB*	N 18° 15′ W	1000.0 "
	BC	N 10° 14′ W	700.5 "
	CD	S 45° 15′ E	
	DA	S 25° 00′ W	
44.	*AB*	N 25° 13′ E	915.3 "
	BC		450.0 "
	CD	S 37° 00′ W	375.0 "
	DA	N 43° 10′ E	

45. Suppose the bearing of the line *BC* omitted from the 2d ex. under Art. 37 and compute it from the remaining data.

46. Suppose the length of the line *CD* omitted from the 4th ex. under Art. 37 and compute as above.

ARTICLE 46.

47. In the field of Ex. 33 make the necessary calculations for cutting off an area of 3.4 Ac. by a line starting from Station A.

48. Make the necessary calculations for dividing in two equal parts, by a line parallel with *CD*, the field of Ex. 36.

49. In Ex. 35, suppose a spring situated S 13° 15′ E 150 yards from Station B. Divide the tract into two equal parts by a line running from the spring.

50. In the field of Ex. 37 suppose the land in the triangle *ABE* worth $100 per acre, that in *EBD*, $75, and that in *DBC*, $50. It is desired to partition the entire tract between X and Y so that the share of X shall be one fourth that of Y. Explain the method and write the field-notes of the division line.

51. Two boundaries of a tract intersect at an angle of 76 degrees. Find the corners of a ten-acre field having an equal frontage on each boundary.

ARTICLE 47.

52. Determine a general method of obtaining the length of an inaccessible line.

53. State the method to be employed in the above problem when there is only one point from which both ends of the line are visible.

54. State the method when there is no point from which both ends of the line are visible.

ARTICLE 49.

55. Design a vernier to read to hundredths of a foot the scale divisions being each $\frac{6}{10}$ of an inch.

56. The scale divisions being each $\frac{6}{10}$ of a foot, design a vernier to read to thousandths of a foot.

57. There being 25 divisions on a vernier and scale divisions, equal each to one tenth of a foot, find the least count.

ARTICLE 77.

58. A lookout on a ship is 16 feet above the water and the light of a lighthouse 100 feet high is just visible on the horizon. Required the distance from the ship to the lighthouse.

ARTICLE 95.

59. From two points on a level area and in the same vertical plane with a mountain top, the angles of elevation to the top are determined as $39° 37'$ and $27° 40'$. The distance between the two points is 3810.4 ft. Required the elevation of the top above the points.

60. The line connecting two points A and B is 500 ft. long and at an inclination of 30° to the horizontal. The angles of elevation to a point in the same vertical plane with A and B are 60° and $49° 19'$. Find the elevation of the point above A.

61. Three points x, y, and z are in the same horizontal line, the distances xy and yz being respectively equal to m and n. The angles of elevation to the top of a tower are X, Y, and Z. Find the elevation of the top of the tower above the points.

ARTICLE 102.

62. Describe a method of locating a circular curve by means of offsets from the tangent at the P. C.

63. Find a method as above by offsets from the "long chord," i.e. the chord joining the P. C. and P. T.

ARTICLE 106.

64. In Fig. 174 the angles on the lower side of the stream from 1 to 7 are as follows: $62° 15'$, $48° 25'$, $49° 30'$, $55° 14'$, $59° 00'$, $46° 24'$, $68° 17'$, $46° 05'$, $65° 18'$. If the bearing of 1–3 is S $18° 30'$ E, find the bearing of 7–8.

ARTICLE 127.

Determine the best methods for evaluating the following expressions;

65. $\dfrac{a \times b \times c \times d \times e}{f \times g \times h} = ?$

66. $\dfrac{h(a^2 + b^2 + c + de + f^3)}{g} = ?$

ARTICLE 135.

67. Can stadia wires be inserted in the ordinary field-glass?

ARTICLE 144.

68. Make a sketch showing the error consequent on the line of sight and the edge of the alidade ruler not being in the same plane.

ARTICLE 185.

69. The dip of a vein being 60°, it is required to determine the depth of a shaft from a point on the surface 75 ft. lower than the outcrop, and distant from it 560 ft. in a direction perpendicular to the strike.

70. A tunnel is to be run to connect the surface with a vein whose strike is N 46°00′ E and dip 50°00′ to the S. E. The mouth of the tunnel is 50 ft. lower than, and bears S 85°25′ E from, a point on the outcrop, distance 415 ft.; what is the mimimum length of the tunnel?

71. In the above example, suppose the tunnel to bear N 35°15′ W on a descending two per cent grade; what will be its length?

72. A tunnel runs from its mouth, azimuth 40°15′, 395 ft., vert. angle + 1°13′. Thence, azimuth 49°23′, 220 ft., vert. angle + 0°55′. From the center of a 122-ft. shaft, the mouth of the tunnel bears azimuth 215°00′, distance 800 ft., vert. angle − 15°00′. Find the length, direction, and grade of the passage connecting the tunnel with the bottom of the shaft.

APPENDIX B.

THE CYCLOTOMIC TRANSIT.*

THE PRINCIPLE OF THE INSTRUMENT.

The evolution of this instrument is due to a constant tendency to create a transit with *one spindle*, i.e. having but one central cone turning within the leveling head, that shall, at the same time, sacrifice none of the advantages that the so-called compound center possesses.

It goes without saying that the principal advantage of the double spindle lies in the fact that, no matter in what direction the telescope may be pointed, the operator is enabled to make any azimuth of his graduated plate agree therewith. How this may be done without giving the lower plate an independent motion around the vertical axis of the instrument is the problem to be solved.

The lower plate is the important member that carries the graduated azimuth circle, and if it be made a part of the rigid sub-structure—of the leveling head and base-plate—the control of it in reference to known azimuths is apparently lost. If we were enabled, however, to shift the figure series—the nomenclature of the circle—at will, so as to make any one of the graduation lines the zero, the advantage lost by having a rigid lower plate would be regained.

The novelty of the new transit lies in a floating exterior ring, placed around the periphery of the lower plate, upon which the figures from 0 to 360 are engraved. These figures are then no longer a fixed part of the circle, but possess that independent rotation which the lower plate had in

* From the catalogue of the A. Lietz Co., San Francisco.

the case of the double spindle. Instead of turning the whole plate around its vertical axis, we turn a narrow metal band around the stationary plate, which is the same thing.

As this band appears to be sliced from the plate, the name *Cyclotome* has been applied to it, from *κυκλος*, ring or circle, and *τεμνειν*, to cut, that is, a ring cut or severed as from a disk.

Since the object of the ring is merely to designate the graduated lines upon the plate by corresponding figures, absolute concentricity of the cyclotome is not a matter of importance.

THE CONSTRUCTION.

Attention is drawn to the illustrations herewith, Fig. 283 showing a vertical section through the plates, and Figs. 284 and 285 a top and bottom view respectively of the upper plate. In the vertical section the arrangement of the principal parts may be readily understood.

The lower plate and the leveling head become one member, which is mounted upon the base-plate in the ordinary manner. The cyclotome *C* is fitted exteriorly around the plate, its top resting upon the graduation, of which it is a part.

The upper plate revolves upon the lower by means of its long and stout spindle, within the socket of the leveling head. It carries the vernier *V*, visible through an opening in the plate, which also exposes a part of the graduated lower plate and a part of the cyclotome. The horizontal motion of the instrument is arrested by the clamp and collar, and the position adjusted by a tangent screw, as common to all transits.

Compass box and telescope are mounted on the top of the plate, as usual.

The flange forming the top of the lower plate is graduated into 720 even spaces of half-degree divisions. The vernier moves along the inner rim of this graduation, and is held whenever the line of collimation (the telescope) has the desired direction. In order to effect a coincidence between the vernier's zero and the nearest half-degree division line, the entire vernier may be shifted independently to the right or left by means of the screw *A*, shown in the illustrations, and in the manner presently to be explained.

And having thusly determined upon and indicated one of the 720 lines to be the initial or starter, it would be necessary only to bring the zero of the cyclotome—or any other reading for that matter—to match this line.

In the simpler form of the new transit, the exterior ring or cyclotome is revoluble by hand around the periphery of the plate, and the required

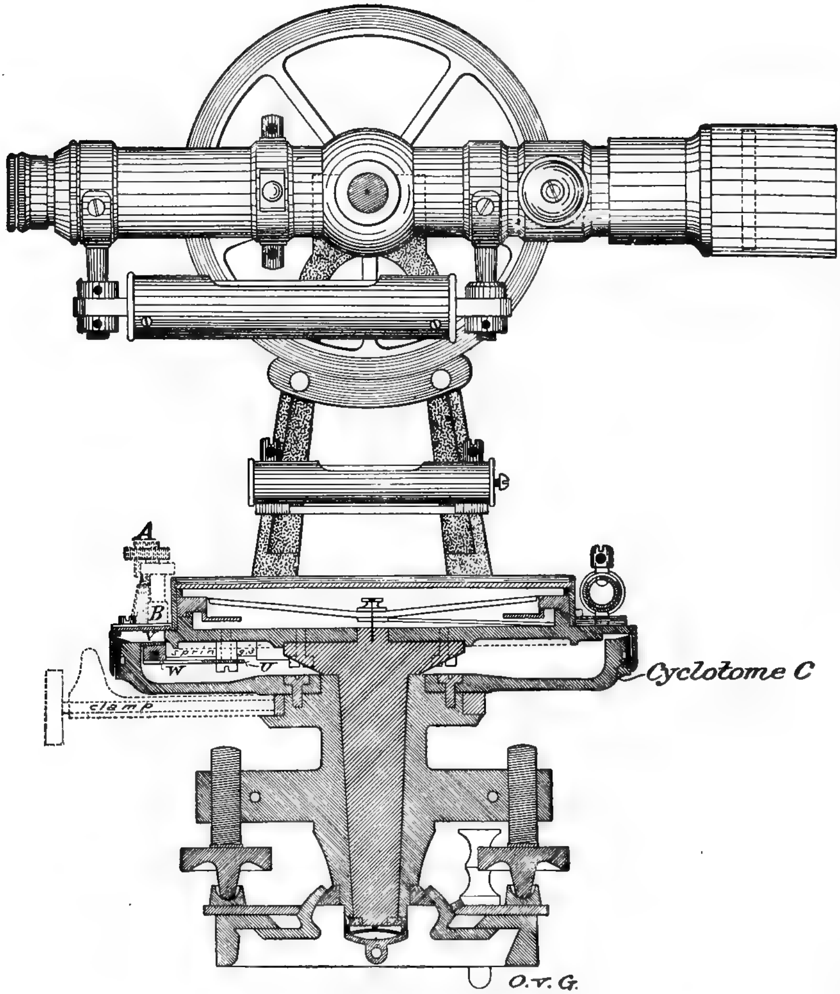


FIG. 283.—Section through Plates on Line X-center-Y, with Side View of Telescope.

azimuth is thus readily set off. In the improved form, as shown by the illustration, the ring is encased, and so arranged that the upper plate in its rotation may or may not carry the cyclotome with it. It is picked up and

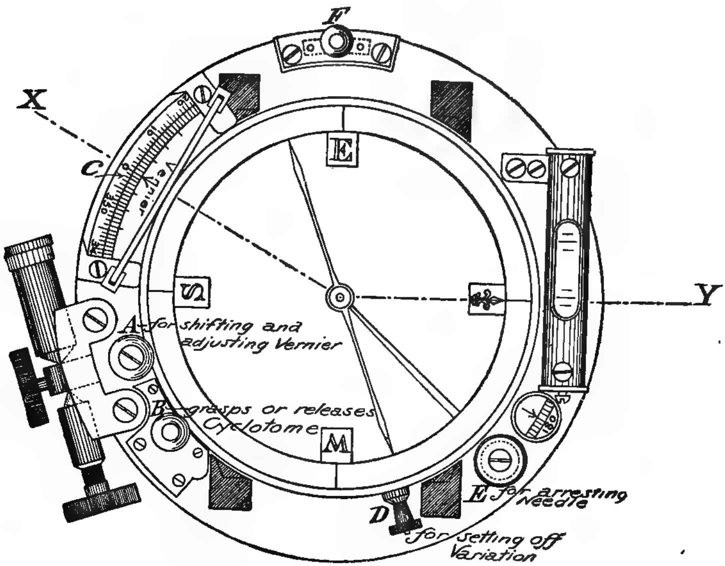


FIG. 284.—Top View of Upper Plate.

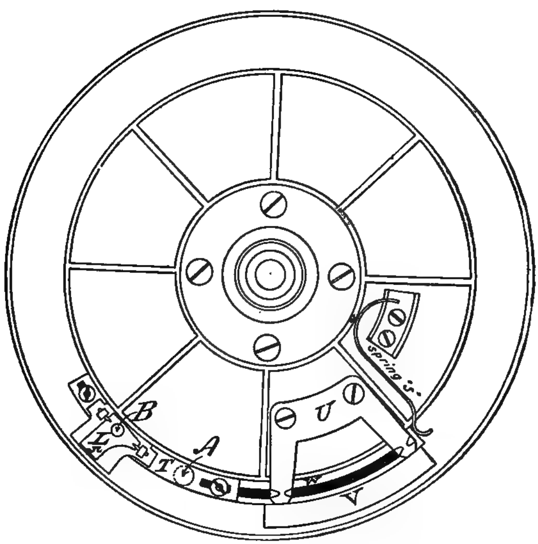


FIG. 285.—Bottom View of Upper Plate.

revolved together with the telescope, or left at rest upon the lower plate in any desired position. It is within the power of the operator to manipulate this at will, and there are two means of doing so, as will be noted further on.

As it is generally required to place the *zero* upon the azimuth from which observations are started, an automatic catch L (see Fig. 285), having a small projecting pin n, is so arranged that whenever it is desired to make the cyclotome travel together with the upper plate, the pin n must be made to drop into a hole provided for it in the cyclotome; the moment this takes place, the two (plate and cyclotome) are connected, and—this is a peculiar feature of the device—in such a position that the zero of the vernier V and the zero of the cyclotome C are brought together, separated, of course, by the intervening graduated flange of the lower plate. If the vernier be now revolved with the upper plate, the figure system will travel with it, their respective zeros coinciding.

The bottom of the upper plate, Fig. 285, illustrates the mechanism with which all this is accomplished. U is a guide, fastened to the plate, for the arc W, carrying the vernier V. A strong spring S presses the arc against the side T, the position of the whole being regulated by the exterior screw A, which allows the adjustment of the vernier already referred to. The catch L is poised in T. The screw B raises or lowers the catch, so that with it we may throw the cyclotome either in or out. A small spring under the catch L admits of this. The mechanism is so simple that it needs no further description.

With this device there is no difficulty in placing the zero of the horizontal circle so as to correspond with any pointing of the telescope.

USE IN THE FIELD.

The field manipulation is reduced to a minimum.

Having set the instrument over a point (1) in the usual manner, it is desired to direct the telescope to another point (2), and to make the zero of the horizontal graduation correspond with this azimuth. The main clamp being loose, the first operation is to turn the screw B so that the catch L is depressed; the upper plate is then turned, until a click indicates that the little pin n has caught the cyclotome and is carrying it along, with the zero in position, as explained. The operation is automatic to this extent, that the manipulator need not watch his plate to set the zeros. He will now direct the telescope to point (2), clamp the plate,

and bisect the object with the tangent screw. His attention is thereupon directed to the vernier, for it is essential that its zero should correspond exactly with a line of the fixed graduation. He turns the screw A to the right, or left, shifting the vernier sufficiently to accomplish this. The cyclotome travels with the vernier, so that he does not need to watch it. The instrument is now oriented, the vernier indicating the starting azimuth, and measurements to other points may begin. Before commencing, however, the screw B is turned so as to release the catch and allow the cyclotome to remain in position. The instrument is now unclamped and ready for operation. Any subsequent reading will indicate directly in degrees and minutes the deflection from the starting point. The whole operation is simple and rapid, and will require less time than the setting of the compound-center instrument.

If it be desired to set any other azimuth to a telescope pointing, recourse is had to the clamp F (see top view of plate, Fig. 284), by which the cyclotome may be connected to the upper plate at any point.

The operation is as follows:

Set up instrument; drop catch L by a turn of screw B; revolve plate on center, click indicates that L has caught cyclotome C; point telescope, clamp plate and bisect object; shift zeros to the nearest graduation line by screw A; release cyclotome by screw B; unclamp instrument and lay off the reading of the required azimuth to the nearest thirty minutes by means of the clamp and tangent screws, and then to the minute with precision by means of the screw A; now turn down the screw F, which catches the cyclotome; unclamp instrument, revolve on center, direct telescope to original object, clamp and bisect. The reading of the vernier will now indicate the azimuth wanted. Release the screw F and the cyclotome will remain in the position into which it has been brought.

The reason why the reading is laid off to the nearest thirty-minute mark only, and then adjusted to precise reading by shifting the vernier, becomes obvious, if we remember that it is always necessary to match the graduation lines of the plate with those of the cyclotome, and that any setting disturbing their coincidence (readings from 1′ to 29′ and 31′ to 59′) will have to be corrected by a vernier displacement.

This operation is rapid, although perhaps a trifle slower than the manipulation with the hand cyclotome, mentioned above, in which case the telescope is directed, plate clamped, object bisected, vernier zero brought to a line, cyclotome turned by hand to read within the nearest

half degree of the line, after which the vernier is adjusted to the exact reading.

The principle remains the same in either method, the only difference being that in the case of the hand cyclotome one is able to set it irrespective of the motions of the upper plate.

After these explanations it becomes very obvious that there are no advantages that the double spindle system can claim over the cyclotomic system in the ready manipulation of the horizontal arc.

ANGULAR REPETITION.

While the reiteration of an angle, resorted to in geodetic measurements, to obtain the value of an arc with its probable error to the fraction of a second, is not possible with the cyclotomic transit, because the main graduation is fixed and cannot be turned in reference to the direction of the objects observed upon, it is perfectly feasible to take the same angle on different parts of the plate. Since there are two verniers, located 180 degrees apart, two readings may also be had of each measurement and the mean taken.

Unless a double spindle transit be of the very best workmanship, that is, a first-class and therefore a high-priced article, all the reiteration and repetition will fail to reach a better result than that attainable with a well-built cyclotomic instrument, which is made to read to half minutes directly, or to twenty seconds in the larger sizes; and anything within the limits of this accuracy is guaranteed by the maker. As the reiteration of an angle is uncalled for in any but the most refined measurements, the cyclotomic transit does not lack completeness for the want of this particular feature.

APPENDIX C.

RESTORATION OF LOST OR OBLITERATED CORNERS AND SUBDIVISION OF SECTIONS.*

DEPARTMENT OF THE INTERIOR,
GENERAL LAND OFFICE,
Washington, D. C., March 14, 1901.

The increasing number of letters from county and local surveyors received at this office making inquiry as to the proper method of restoring to their original position lost or obliterated corners marking the survey of the public lands of the United States, or such as have been willfully or accidentally moved from their original position, have rendered the preparation of the following general rules necessary, particularly as in a very large number of cases the immediate facts necessary to a thorough and intelligent understanding are omitted. Moreover, surveys having been made under the authority of different acts of Congress, different results have been obtained, and no special law has been enacted by that authority covering and regulating the subject of the above-named inquiries. Hence, the general rule here given must be considered merely as an expression of the opinion of this office on the subject, based, however, upon the spirit of the several acts of Congress authorizing the surveys, as construed by this office, and by United States court decisions. When cases arise which are not covered by these rules, and the advice of this office is desired, the letter of inquiry should always contain a description of the particular corner, with reference to the township, range, and section of the public surveys, to enable this office to consult the record.

* Issued by the General Land Office, Washington, D. C.

An *obliterated* corner is one where no visible evidence remains of the work of the original surveyor in establishing it. Its location may, however, have been preserved beyond all question by acts of landowners, and by the memory of those who knew and recollect the true situs of the original monument. In such cases it is not a lost corner.

A *lost* corner is one whose position can not be determined, beyond reasonable doubt, either from original marks or reliable external evidence.

Surveyors sometimes err in their decision whether a corner is to be treated as *lost* or only *obliterated.*

Surveyors who have been United States deputies should bear in mind that in their private capacity they must act under somewhat different rules of law from those governing original surveys, and should carefully distinguish between the provisions of the statute which guide a Government deputy and those which apply to retracement of lines once surveyed. The failure to observe this distinction has been prolific of erroneous work and injustice to landowners.

To restore extinct boundaries of the public lands correctly, the surveyor must have some knowledge of the manner in which townships were subdivided by the several methods authorized by Congress. Without this knowledge he may be greatly embarrassed in the field, and is liable to make mistakes invalidating his work, and leading eventually to serious litigation. It is believed that the following synopsis of the several acts of Congress regulating the surveys of the public lands will be of service to county surveyors and others, and will help to explain many of the difficulties encountered by them in the settlement of such questions.

Compliance with the provisions of Congressional legislation at different periods has resulted in *two* sets of corners being established *on township lines* at one time; at other times *three* sets of corners have been established on *range lines;* while the system now in operation makes but *one* set of corners on *township boundaries*, except on standard lines—i.e., base and correction lines, and in some exceptional cases.

The following brief explanation of the modes which have been practiced will be of service to all who may be called upon to restore obliterated boundaries of the public land surveys:

Where two sets of corners were established on township boundaries, one set was planted at the time the exteriors were run, those on the north boundary belonging to the sections and quarter sections north of said line, and those on the west boundary belonging to the sections and

quarter sections west of that line. The other set of corners was established when the township was subdivided. This method, as stated, resulted in the establishment of two sets of corners on all four sides of the townships.

Where three sets of corners were established on the range lines, the subdivisional surveys were made in the above manner, except that the east and west section lines, instead of being closed upon the corners previously established on the east boundary of the township, were run due east from the last interior section corner, and new corners were erected at the points of intersection with the range line.

The method now in practice requires section lines to be initiated from the corners on the south boundary of the township, and to close on existing corners on the east, north, and west boundaries of the township, except when the north boundary is a base line or standard parallel.

But in some cases, for special reasons, an opposite course of procedure has been followed, and subdivisional work has been begun on the north boundary and has been extended southward and eastward or southward and westward.

In the more recent general instructions, greater care has been exercised to secure rectangular subdivisions by fixing a strict limitation that no new township exteriors or section lines shall depart from a true meridian or east and west line more than twenty-one minutes of arc; and that where a random line is found liable to correction beyond this limit, a true line on a cardinal course must be run, setting a closing corner on the line to which it closes.

This produces, in new surveys closing to irregular old work, a great number of exteriors marked by a double set of corners. All retracing surveyors should proceed under these new conditions with full knowledge of the field notes and exceptional methods of subdivision.

SYNOPSIS OF ACTS OF CONGRESS.

The first enactment in regard to the surveying of the public lands was an ordinance passed by the Congress of the Confederation May 20, 1785, prescribing the mode for the survey of the "Western Territory," and which provided that said territory should be divided into "townships of six miles square, by lines running due north and south, and others crossing them at right angles" as near as might be.

Ordinance of the Congress of the Confederation of May 20, 1785. U.S. Land Laws, p. 349. Edition 1828.

It further provided that the first line running north and south should

begin on the Ohio River, at a point due north from the western terminus of a line run as the south boundary of the State of Pennsylvania, and the first line running east and west should begin at the same point and extend through the whole territory. In these initial surveys only the exterior lines of the townships were surveyed, but the plats were marked by subdivisions into sections 1 mile square, numbered from 1 to 36, commencing with No. 1 in the southeast corner of the township, and running from south to north in each tier to No. 36 in the northwest corner of the township; mile corners were established on the township lines. The region embraced by the surveys under this law forms a part of the present State of Ohio, and is generally known as "the Seven Ranges."

The Federal Congress passed a law, approved May 18, 1796, in regard to surveying the public domain, which applied to "the territory northwest of the River Ohio, and above the mouth of the Kentucky River."

Act of May 18, 1796. U. S. Statutes at Large, vol. 1, p. 465. Section 2395, U. S. Revised Statutes.

Section 2 of said act provided for dividing such lands as had not been already surveyed or disposed of "by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form townships of 6 miles square," etc. It also provided that "one-half of said townships, taking them alternately, should be subdivided into sections containing, as nearly as may be, 640 acres each, by running through the same each way parallel lines at the end of every two miles; and by marking a corner on each of said lines at the end of every mile." The act also provided that "the sections shall be numbered, respectively, beginning with the number one in the northeast section, and proceeding west and east alternately through the township, with progressive numbers till the thirty-sixth be completed." This method of numbering sections is still in use.

An act amendatory of the foregoing, approved May 10, 1800, required the "townships west of the Muskingum, which are directed to be sold in quarter townships, to be subdivided into half sections of 320 acres each, as nearly as may be, by running parallel lines through the same from east to west, and from south to north, at the distance of one mile from each other, and marking corners, at the distance of each half mile on the lines running from east to west, and at the distance of each mile on those running from south to north. And the interior lines of townships intersected by the Muskingum, and of all townships lying east of that river, which have not been heretofore actually subdivided

Act of May 10, 1800. U. S. Statutes at Large, vol. 2, p. 73. Section 2395, U. S. Revised Statutes.

into sections, shall also be run and marked. . . . And in all cases where the exterior lines of the townships thus to be subdivided into sections or half sections, shall exceed or shall not extend six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western or northern ranges of sections or half sections in such townships, according as the error may be in running the lines from east to west or from south to north." Said act also provided that the northern and western tiers of sections should be sold as containing only the quantity expressed on the plats, and all others as containing the complete legal quantity.

The act approved June 1, 1796, "regulating the grants of land appropriated for military services," etc., provided for dividing the "United States Military Tract," in the State of Ohio, into townships 5 miles square, each to be subdivided into quarter townships containing 4000 acres.

Act of June 1, 1796. U. S. Statutes at Large, vol. 1, p. 490.

Section 6 of the act approved March 1, 1800, amendatory of the foregoing act, enacted that the Secretary of the Treasury was authorized to subdivide the quarter townships into lots of 100 acres, bounded as nearly as practicable by parallel lines 160 perches in length by 100 perches in width. These subdivisions into lots, however, were made upon the plats in the office of the Secretary of the Treasury, and the actual survey was only made at a subsequent time when a sufficient number of such lots had been located to warrant the survey. It thus happened, in some instances, that when the survey came to be made the plat and survey could not be made to agree, and that fractional lots on plats were entirely crowded out. A knowledge of this fact may explain some of the difficulties met with in the district thus subdivided.

Act of March 1, 1800. U. S. Statutes at Large, vol. 2, p. 14.

The act of Congress approved February 11, 1805, directs the subdivision of the public lands into quarter sections, and provides that all corners marked in the field shall be established as the proper corners of the sections or quarter sections which they were intended to designate, and that corners of half and quarter sections not marked shall be placed as nearly as possible "equidistant from those two corners which stand on the same line." This act further provides that "the boundary lines actually run and marked" (in the field) "shall be established as the proper boundary lines of the sections, or subdivisions, for which they were intended, and the length of such lines as returned by either of the surveyors aforesaid shall be held and considered as the true

Act of February 11, 1805. U. S. Statutes at Large, vol. 2, p. 313. Section 2396, U. S. Revised Statutes.

length thereof. And the boundary lines which shall not have been actually run and marked as aforesaid shall be ascertained by running straight lines from the established corners to the opposite corresponding corners, but in those portions of the fractional townships where no such opposite or corresponding corners have been or can be fixed, the said boundary lines shall be ascertained by running from the established corners due north and south, or east and west lines, as the case may be, to the water course, Indian boundary line, or other external boundary of such fractional township."

Act of April 24, 1820. U. S. Statutes at Large, vol. 3, p. 566. Section 2397, U. S. Revised Statutes.

The act of Congress approved April 24, 1820, provides for the sale of public lands in half-quarter sections, and requires that "in every case of the division of a quarter section the line for the division thereof shall run north and south," "and fractional sections, containing 160 acres and upwards, shall in like manner, as nearly as practicable, be subdivided into half-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Treasury; but fractional sections containing less than 160 acres shall not be divided."

Act of May 24, 1824. U. S. Statutes at Large, vol. 4, p. 34.

The act of Congress approved May 24, 1824, provides "that whenever, in the opinion of the President of the United States, a departure from the ordinary mode of surveying land on any river, lake, bayou, or water course would promote the public interest, he may direct the surveyor-general in whose district such land is situated, and where the change is intended to be made, under such rules and regulations as the President may prescribe, to cause the lands thus situated to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or water course, and running back the depth of forty acres."

Act of April 5, 1832. U. S. Statutes at Large, vol. 4, p. 503. Section 2397, U. S. Revised Statutes.

The act of Congress approved April 5, 1832, directed the subdivision of the public lands into quarter-quarter sections; that in every case of the division of a half-quarter section the dividing line should run east and west, and that fractional sections should be subdivided, under rules and regulations prescribed by the Secretary of the Treasury. Under the latter provision the Secretary directed that fractional sections containing less than 160 acres, or the residuary portion of a fractional section, after the subdivision into as many quarter-quarter sections as it is susceptible of, may be subdivided into lots, each containing the quantity of a quarter-quarter section as nearly as practicable, by so laying down the line of subdivision that they shall be 20 chains wide, which dis-

tances are to be marked on the plat of subdivision, as are also the areas of the quarter-quarters and residuary fractions.

These two acts last mentioned provided that the corners and contents of half-quarter and quarter-quarter sections should be ascertained as nearly as possible in the manner and on the principles prescribed in the act of Congress approved February 11, 1805.

GENERAL RULES.

From the foregoing synopsis of Congressional legislation it is evident—

1st. That the boundaries of the public lands established and returned by the duly appointed Government surveyors, when approved by the surveyors general and accepted by the Government, *are unchangeable.*

2d. That the original township, section, and quarter-section corners established by the Government surveyors must stand as the true corners which they were intended to represent, whether the corners be in place or not.

3d. That quarter-quarter corners not established by the Government surveyors shall be placed on the straight lines joining the section and quarter-section corners and midway between them, except on the last half mile of section lines closing on the north and west boundaries of the township, or on other lines between fractional sections.

4th. That all subdivisional lines of a section running between corners established in the original survey of a township must be straight lines, running from the proper corner in one section line to its opposite corresponding corner in the opposite section line.

5th. That in a fractional section where no opposite corresponding corner has been or can be established, any required subdivision line of such section must be run from the proper original corner in the boundary line due east and west, or north and south, as the case may be, to the water course, Indian reservation, or other boundary of such section, with due parallelism to section lines.

From the foregoing it will be plain that extinct corners of the Government surveys must be restored to their original locations, whenever it is possible to do so; and hence resort should always be first had to the marks of the survey in the field. The locus of the missing corner should be first identified on the ground by the aid of the mound, pits, line trees, bearing trees, etc., described in the field notes of the original survey.

The identification of mounds, pits, witness tress, or other permanent objects noted in the field notes of survey, affords the best means of relocating the missing corner in its orignal position. If this cannot be done, clear and convincing testimony of citizens as to the locality it originally occupied should be taken, if such can be obtained. In any event, whether the locus of the corner be fixed by the one means or the other, such locus should always be tested and confirmed by measurements to *known* corners. No definite rule can be laid down as to what shall be sufficient evidence in such cases, and much must be left to the skill, fidelity, and good judgment of the surveyor in the performance of his work.

EXCEPTIONAL CASES.

When new measurements are made on a single line to determine the position thereon for a restored lost corner (for example, a quarter-section corner on line between two original section corners), or when new measurements are made between original corners on two lines for the purpose of fixing by their intersection the position of a restored missing corner (for example, a corner common to four sections or four townships), it will almost invariably happen that discrepancies will be developed between the new measurements and the original measurements in the field notes. When these differences occur the surveyor will in all cases establish the missing corner by proportionate measurements on lines conforming to the original field notes, and by the method followed in the original survey. From this rule there can be no departure, since it is the basis upon which the whole operation depends for accuracy and truth.

In cases where the relocated corner cannot be made to harmonize with the field notes in all directions, and unexplained error in the first survey is apparent, it sometimes becomes the task of the surveyor to place it according to the requirements of one line and against the calls of another line. For instance, if the line between sections 30 and 31, reported 78 chains long, would draw the missing corner on range line 1 chain eastward out of range with the other exterior corners, the presumption would be strong that the range line had been run straight and the length of the section line wrongly reported, because experience shows that west random lines are regarded as less important than range lines and more liable to error.

Again, where a corner on a standard parallel has been obliterated, it is proper to assume that it was placed in line with other corners, and

if an anomalous length of line reported between sections 3 and 4 would throw the closing corner into the northern township, a surveyor would properly assume that the older survey of the standard line is to control the length of the later and minor line. The marks or corners found on such a line closing to a standard parallel fix its *location*, but its *length* should be limited by its actual intersection, at which point the lost closing corner may be placed.

The strict rule of the law that "all corners marked in the field shall be established as the corners which they were intended to designate," and the further rule that "the length of lines returned by the surveyors shall be held and considered as the true length thereof," are found in some cases to be impossible of fulfillment in all directions at once, and a surveyor is obliged to choose, in his own discretion, which of two or more lines must yield, in order to permit the rules to be applied at all.

In a case of an erroneous but existing closing corner, which was set some distance out of the true State boundary of Missouri and Kansas, it was held by this office that a surveyor subdividing the fractional section should preserve the boundary as a straight line, and should not regard said closing corner as the proper corner of the adjacent fractional lots. The said corner was considered as fixing the position of the line between two fractional sections, but that its length extended to a new corner to be set on the true boundary line. The surveyor should therefore preserve such an original corner as evidence of the line; but its erroneous position cannot be allowed to cause a crook between mile corners of the original State boundary.

It is only in cases where it is manifestly impossible to carry out the literal terms of the law, that a surveyor can be justified in making such a decision.

The principle of the preponderance of one line over another of less importance has been recognized in the rule for restoring a section corner common to two townships in former editions of this circular. The new corner should be placed *on* the township line; and measurements to check its position by distances to corners within the townships are useful to confirm it if found to agree well, but should not cause it to be placed off the line if found not to agree, if the general condition of the boundary supports the presumption that it was properly alined.

TO RESTORE LOST OR OBLITERATED CORNERS.

1. *To restore corners on base lines and standard parallels.*—Lost or obliterated standard corners will be restored to their original positions on a base line, standard parallel, or correction line, by proportionate measurements on the line, conforming as nearly as practicable to the original field notes and joining the nearest identified original standard corners on opposite sides of the missing corner or corners, as the case may be.

(*a*) The term "standard corners" will be understood to designate standard township, section, quarter-section, and meander corners; and, in addition, closing corners, as follows: Closing corners used in the original survey to determine the position of a standard parallel, or established during the survey of the same, will, with the standard corners, govern the alinement and measurements made to restore lost or obliterated standard corners; but no other closing corners will control in any manner the restoration of standard corners on a base line or standard parallel.

(*b*) A lost or obliterated closing corner from which a standard parallel has been initiated or to which it has been directed will be reestablished in its original place by proportionate measurement from the corners used in the original survey to determine its position. Measurements from corners on the opposite side of the parallel will not control in any manner the relocation of said corner.

(*c*) A missing closing corner originally established during the survey of a standard parallel as a corner from which to project surveys *south* will be restored to its original position by considering it a standard corner and treating it accordingly.

(*d*) Therefore, paying attention to the preceding explanations, we have for the restoration of one or several corners on a standard parallel, and for general application to all other surveyed lines, the following proportion:

As the original field-note distance between the selected known corners *is to* the new measure of said distance, *so is* the original field-note length of any part of the line to the required new measure thereof.

The sum of the computed lengths of the several parts of a line must be equal to the new measure of the whole distance.

(*e*) As has been observed, existing original corners can not be disturbed; consequently, discrepancies between the new and the original field-note measurements of the line joining the selected original corners will not in any manner affect measurements beyond said corners, but

the differences will be distributed proportionately to the several intervals embraced in the line in question.

(*f*) After having checked each new location by measurement to the nearest known corners, new corners will be established permanently and new bearings and measurements taken to prominent objects, which should be of as permanent a character as possible, and the same recorded for future reference.

2. *Restoration of township corners common to four townships.*—Two cases should be clearly recognized: 1st. Where the position of the original township corner has been made to depend upon measurements on two lines at right angles to each other. 2d. Where the original corner has been located by measurements on one line only; for example, on a guide meridian.

(*a*) For restoration of a township corner originally subject to the first condition: A line will first be run connecting the nearest identified original corners on the meridional township lines, north and south of the missing corner, and a temporary corner will be placed at the proper proportionate distance. This will determine the corner in a north and south direction only.

Next, the nearest original corners on the latitudinal township lines will be connected and a point thereon will be determined in a similar manner, independent of the temporary corner on the meridional line. Then through the first temporary corner run a line east (or west) and through the second temporary corner a line north (or south), as relative situations may suggest. The intersection of the two lines last run will define the position of the restored township corner, which may be permanently established.

(*b*) The restoration of a lost or obliterated township corner established under the second conditions, i.e., by measurements, on a single line, will be effected by proportionate measurements on said line, between the nearest identified original corners on opposite sides of the missing township corner, as before described.

3. *Reestablishment of corners common to two townships.*—The two nearest known corners on the township line, the same not being a base or a correction line, will be connected as in case No. 1, by a right line, and the missing corner established by proportionate distance as directed in that case; the location thus found will be checked upon by measurements to nearest known section or quarter-section corners north and south, or east and west, of the township line, as the case may be.

4. *Reestablishment of closing corners.*—Measure from the quarter-section, section, or township corner east or west, as the case may be, to the next preceding or succeeding corner in the order of original establishment, and reestablish the missing closing corner by proportionate measurement. The line upon which the closing corner was originally established should always be remeasured, in order to check upon the correctness of the new location. See pages 406, 407, 412, 413, and 414 for details.

5. *Reestablishment of interior section corners.*—This class of corners should be reestablished in the same manner as corners common to four townships. In such cases, when a number of corners are missing on all sides of the one sought to be reestablished, the entire distance must, of course, be remeasured between the nearest existing recognized corners both north and south, and east and west, in accordance with the rule laid down, and the new corner reestablished by proportionate measurement. The mere measurement in any one of the required directions will not suffice, since the direction of the several section lines running northward through a township, or running east and west, are only in the most exceptional cases true prolongations of the alinement of the section lines initiated on the south boundary of the township; while the east and west lines running through the township, and theoretically supposed to be at right angles with the former, are seldom in that condition, and the alinements of the closing lines on the east and west boundaries of the township, in connection with the interior section lines, are even less often in accord. Moreover, the alinement of the section line itself from corner to corner, in point of fact, also very frequently diverges from a right line, although presumed to be such from the record contained in the field notes and so designated on the plats, and becomes either a broken or a curved line. This fact will be determined, in a timbered country, by the blazes which may be found upon trees on either side of the line, and although such blazed line will not strictly govern as to the absolute direction assumed by such line, it will assist very materially in determining its approximate direction, and should never be neglected in retracements for the reestablishment of lost corners of any description. Sight trees described in the field notes, together with the recorded distances to same, when fully identified, will, it has been held, in one or more States, govern the line itself, even when not in a direct or straight line between established corners, which line is then necessarily a broken line by passing through said sight trees. Such trees, when in existence

and properly identified beyond a question of doubt, will very materially assist in evidencing the correct relocation of a missing corner. It is greatly to be regretted that the earlier field notes of survey are so very meager in the notation of the topography found on the original line, which might in very many instances materially lessen a surveyor's labors in retracement of lines and reestablishment of the required missing corner. In the absence of such sight trees and other evidence regarding the line, as in an open country, or where such evidence has been destroyed by time, the elements, or the progress of improvement, the line connecting the known corners should be run straight from corner to corner.

6. *Reestablishment of quarter-section corners on township boundaries.*—Only one set of quarter-section corners are actually marked in the field on township lines, and they are established at the time when the township exteriors are run. When double section corners are found, the quarter-section corners are considered generally as standing midway between the corners of their respective sections, and when required to be established or reestablished, as the case may be, they should be generally so placed; but great care should be exercised not to mistake the corners belonging to one township for those of another. After determining the proper section corners marking the line upon which the missing quarter-section corner is to be reestablished, and measuring said line, the missing quarter-section corner will be reestablished in accordance with the requirements of the original field notes of survey, by proportionate measurement between the section corners marking the line.

Where there are double sets of section corners on township and range lines, and the quarter-section corners for sections south of the township or east of the range lines are required to be established in the field, the said quarter-section corners should be so placed as to suit the calculation of areas of the quarter sections adjoining the township boundaries as expressed upon the official township plat, adopting proportionate measurements when the present measurement of the north and west boundaries of the section differ from the original measurements.

7. *Reestablishment of quarter-section corners on closing section lines between fractional sections.*—This class of corners must be reestablished according to the original measurement of 40 chains from the last interior section corner. If the measurements do not agree with the original survey, the excess or deficiency must be divided propor-

tionately between the two distances, as expressed in the field notes of original survey. The section corner started from and the corner closed upon should be connected by a right line, unless the retracement should develop the fact that the section line is either a broken or curved line, as is sometimes the case.

8. *Reestablishment of interior quarter-section corners.*—In some of the older surveys these corners are placed at variable distances, in which case the field notes of the original survey must be consulted, and the quarter-section corner reestablished at proportionate distances between the corresponding section corners, in accordance therewith. The later surveys being more uniform and in stricter accordance with law, the missing quarter-section corner must be reestablished equidistant between the section corners marking the line, according to the field notes of the original survey. The remarks made under section 5, in relation to section lines, apply with full force here also; the caution there given not to neglect sight trees is equally applicable, since the proper reestablishment of the quarter-section corner may in some instances very largely depend upon its observance, and avoid one of the many sources of litigation.

9. *Where double corners were originally established, one of which is standing, to reestablish the other.*—It being remembered that the corners established when the exterior township lines were run belong to the sections in the townships north and west of those lines, the surveyor must first determine beyond a doubt to which sections the existing corner belongs. This may be done by testing the courses and distances to witness trees or other objects noted in the original field notes of survey, and by remeasuring distances to known corners. Having determined to which township the existing corner belongs, the missing corner may be reestablished in line north or south of the existing corner, as the case may be, at the distance stated in the field notes of the original survey, by proportionate measurement, and tested by retracement to the opposite corresponding corner of the section to which the missing section corner belongs. These double corners being generally not more than a few chains apart, the distance between them can be more accurately laid off, and it is considered preferable to first establish the missing corner as above, and check upon the corresponding interior corner, than to reverse the proceeding; since the result obtained is every way more accurate and satisfactory.

10. *Where double corners were originally established, and both are missing, to reestablish the one established when the township line was run.*—The surveyor will connect the nearest known corners on the township line by a right line, being careful to distinguish the section from the closing corners, and reestablish the missing corner at the point indicated by the field notes of the original survey by proportionate measurement. The corner thus restored will be common to two sections either north or west of the township boundary, and the section north or west, as the case may be, should be carefully retraced, thus checking upon the reestablished corner, and testing the accuracy of the result. It cannot be too much impressed upon the surveyor that any measurements to objects on line noted in the original survey are means of determining and testing the correctness of the operation.

11. *Where double corners were originally established, and both are missing, to reestablish the one established when the township was subdivided.*—The corner to be reestablished being common to two sections south or east of the township line, the section line closing on the missing section corner should be first retraced to an intersection with the township line in the manner previously indicated, and a temporary corner established at the point of intersection. The township line will of course have been previously carefully retraced in accordance with the requirements of the original field notes of survey, and marked in such a manner as to be readily identified when reaching the same with the retraced section line. The location of the temporary corner planted at the point of intersection will then be carefully tested and verified by remeasurements to objects and known corners on the township line, as noted in the original field notes of survey, and the necessary corrections made in such relocation. A permanent corner will then be erected at the corrected location on the township line, properly marked and witnessed, and recorded for future requirements.

12. *Where triple corners were originally established on range lines, one or two of which have become obliterated, to reestablish either of them.*—It will be borne in mind that only two corners were established as actual corners of sections, those established on the range line not corresponding with the subdivisional survey east or west of said range line. The surveyor will, therefore, first proceed to identify the existing corner or corners, as the case may be, and then reestablish the missing corner or corners in line north or south, according to the distances stated in the original field notes of survey in the manner indicated for the reestablishment of double corners, testing the accuracy of the result

obtained, as hereinbefore directed in other cases. If, however, the distances between the triple corners are not stated in the original field notes of survey, as is frequently the case in the returns of older surveys, the range line should be first carefully retraced, and marked in a manner sufficiently clear to admit of easy identification upon reaching same during the subsequent proceedings. The section lines closing upon the missing corners must then be retraced in accordance with the original field notes of survey, in the manner previously indicated and directed, and the corners reestablished in the manner directed in the case of double corners. The surveyor cannot be too careful, in the matter of retracement, in following closely all the recorded indications of the original line, and nothing, however slight, should be neglected to insure the correctness of the retracement of the original line; since there is no other check upon the accuracy of the reestablishment of the missing corners, unless the entire corresponding section lines are remeasured by proportional measurement and the result checked by a recalculation of the areas as originally returned, which, at best, is but a very poor check, because the areas expressed upon the margin of many plats of the older surveys are erroneously stated on the face of the plats, or have been carelessly calculated.

13. *Where triple corners were originally established on range lines, all of which are missing, to reestablish same.*—These corners should be reëstablished in accordance with the foregoing directions, commencing with the corner originally established when the range line was run, establishing the same in accordance with previously given directions for restoring section and quarter-section corners; that is to say, by remeasuring between the nearest known corners on said township line, and reestablishing the same by proportionate measurement. The two remaining will then be reestablished in conformity with the general rules for reestablishment of double corners.

14. *Reestablishment of meander corners.*—Before proceeding with the reestablishment of missing meander corners, the surveyor should have carefully rechained at least three of the section lines between known corners of the township within which the lost corner is to be relocated, in order to establish the proportionate measurement to be used. This requirement of preliminary remeasurement of section lines must in no case be omitted; since it gives the only data upon which the fractional section line can be remeasured proportionately, the corner marking the terminus, or the meander corner, being missing, which it is intended to reestablish. The missing meander corner will be re-

established on the section or township line retraced in its original location, by the proportionate measurement found by the preceding operations, from the nearest known corner on such township or section line, in accordance with the requirements of the original field notes of survey.

Meander corners hold the peculiar position of denoting a point on line between landowners, without usually being the legal terminus or corner of the lands owned. Leading judicial decisions have affirmed that meander lines are not strictly boundaries, and do not limit the ownership to the exact areas placed on the tracts, but that said title extends to the water, which, by the plat, appears to bound the land.

As such water boundaries are, therefore, subject to change by the encroachment or recession of the stream or lake, the precise location of old meanders is seldom important, unless in States whose laws prescribe that dried lake beds are the property of the State.

Where the United States has disposed of the fractional lots adjacent to shores, it claims no marginal lands left by recession or found by reason of erroneous survey. The lines between landowners are therefore regarded as extended beyond the original meander line of the shore, but the preservation or relocation of the meander corner is important, as evidence of the *position* of the section line.

The different rules by which division lines should be run between private owners of riparian accretions are a matter of State legislation, and not subject to a general rule of this office.

15. *Fractional section lines.*—County and local surveyors being sometimes called upon to restore fractional section lines closing upon Indian, military, or other reservations, private grants, etc., such lines should be restored upon the same principles as directed in the foregoing pages, and checked whenever possible upon such corners or monuments as have been placed to mark such boundary lines.

In some instances corners have been moved from their original position, either by accident or design, and county surveyors are called upon to restore such corners to their original positions, but, owing to the absence of any and all means of identification of such location, are unable to make the result of their work acceptable to the owners of the lands affected by such corner. In such cases the advice of this office has invariably been to the effect that the relocation of such corner must be made in accordance with the orders of a court of competent jurisdiction, the United States having no longer any authority

to order any changes where the lands affected by such corner have been disposed of.

RECORDS.

The original evidences of the public-land surveys in the following States have been transferred, under the provisions of sections 2218, 2219, and 2220, United States Revised Statutes, to the State authorities, to whom application should be made for such copies of the original plats and field notes as may be desired, viz.:

Alabama: Secretary of State, Montgomery.
Arkansas: Commisssoner of State Lands, Little Rock.
Illinois: Auditor of State, Springfield.
Indiana: Auditor of State, Indianapolis.
Iowa: Secretary of State, Des Moines.
Kansas: Auditor of State and Register of State Lands, Topeka.
Michigan: Commissioner of State Land Office, Lansing.
Mississippi: Commissioner of State Lands, Jackson.
Missouri: Secretary of State, Jefferson City.
Nebraska: Commissioner of Public Lands and Buildings, Lincoln.
Ohio: Auditor of State, Columbus.
Wisconsin: Commissioners of Public Lands, Madison.

In other public-land States the original field notes and plats are retained in the offices of the United States surveyors general.

SUBDIVISION OF SECTIONS.

This office being in receipt of many letters making inquiry in regard to the proper method of subdividing sections of the public lands, the following general rules have been prepared as a reply to such inquiries. The rules for subdivision are based upon the laws governing the survey of the public lands. When cases arise which are not covered by these rules, and the advice of this office in the matter is desired, the letter of inquiry should, in every instance, contain a description of the particular tract or corner, with reference to township, range, and section of the public surveys, to enable the office to consult the record; also a diagram showing conditions found.

1. *Subdivision of sections into quarter sections.*—Under the provisions of the act of Congress approved February 11, 1805, the course to be pursued in the subdivision of sections into quarter sections is to run straight lines from the established quarter-section corners, United States surveys, to the opposite corresponding corners. The point of

intersection of the lines thus run will be the corner common to the several quarter sections, or, in other words, the legal center of the section.

(*a*) Upon the lines closing on the north and west boundaries of a township, the quarter-section corners are established by the United States deputy surveyors at 40 chains to the north or west of the last interior section corners, and the excess or deficiency in the measurement is thrown into the half mile next to the township or range line, as the case may be.

(*b*) Where there are double sets of section corners on township and range lines, the quarter corners for the sections south of the township lines and east of the range lines are not established in the field by the United States deputy surveyors, but in subdividing such sections said quarter corners should be so placed as to suit the calculations of the areas of the quarter sections adjoining the township boundaries as expressed upon the official plat, adopting proportionate measurements where the new measurements of the north or west boundaries of the section differ from the original measurements.

2. *Subdivision of fractional sections.*—Where opposite corresponding corners have not been or can not be fixed, the subdivision lines should be ascertained by running from the established corners due north, south, east, or west lines, as the case may be, to the water course, Indian boundary line, or other boundary of such fractional section.

(*a*) The law presumes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north and south or east and west lines, but in actual experience this is not always the case. Hence, in order to carry out the spirit of the law, it will be necessary in running the subdivisional lines through fractional sections to adopt mean courses where the section lines are not due lines, or to run the subdivision line parallel to the east, south, west, or north boundary of the section, as conditions may require, where there is no opposite section line.

3. *Subdivision of quarter sections into quarter quarters.*—Preliminary to the subdivision of quarter sections, the quarter-quarter corners will be established at points midway between the section and quarter-section corners, and between quarter corners and the center of the section, except on the last half mile of the lines closing on the north or west boundaries of a township, where they should be placed at 20 chains, proportionate measurement, to the north or west of the quarter-section corner.

(*a*) The quarter-quarter section corners having been established as

directed above, the subdivision lines of the quarter section will be run straight between opposite corresponding quarter-quarter section corners on the quarter-section boundaries. The intersection of the lines thus run will determine the place for the corner common to the four quarter-quarter sections.

4. *Subdivision of fractional quarter sections.*—The subdivision lines of fractional quarter sections will be run from properly established quarter-quarter section corners (paragraph 3) due north, south, east, or west, to the lake, water course, or reservation which renders such tracts fractional, or parallel to the east, south, west, or north boundary of the quarter section, as conditions may require. (See paragraph 2 (*a*).)

5. *Proportionate measurement.*—By "proportionate measurement," as used in this circular, is meant a measurement having the same ratio to that recorded in the original field notes as the *length of chain* used in the new measurement has to the *length of chain* used in the original survey, assuming that the original and new measurements have been correctly made.

For example: The length of the line from the quarter-section corner on the west side of sec. 2, T. 24 N., R. 14 E., Wisconsin, to the north line of the township, by the United States deputy surveyor's chain, was reported as 45.40 chains, and by the county surveyor's measure is reported as 42.90 chains; then the distance which the quarter-quarter section corner should be located north of the quarter-section corner would be determined as follows:

As 45.40 chains, the Government measure of the whole distance, is to 42.90 chains, the county surveyor's measure of the same distance, so is 20.00 chains, original measurement, to 18.90 chains by the county surveyor's measure, showing that by proportionate measurement in this case the quarter-quarter section corner should be set at 18.90 chains north of the quarter-section corner, instead of 20.00 chains north of such corner, as represented on the official plat. In this manner the discrepancies between original and new measurements are equitably distributed.

BINGER HERMANN,
Commissioner.

DEPARTMENT OF THE INTERIOR,
March 14, 1901.

Approved:
E. A. HITCHCOCK,
Secretary.

APPENDIX D.*

PHOTOTOPOGRAPHIC METHODS AND INSTRUMENTS.

INTRODUCTION.

TOPOGRAPHY is that branch of surveying which pictures the shape of the outer visible surface of the earth, in reduced scale, as a horizontal projection, yet showing the relative positions of points of the terrene also in the vertical sense. It is, therefore, supplementary to geodesy in representing areas of the earth's surface, including all the necessary details and changes in the terrene, by means of instrumental measurements made in the field.

The work of filling in the details—topographic surveying in the closer sense—may be accomplished by various methods, differing in the matter of costs, time, and attainable accuracy; one may be advantageously employed for one class of work, while another may be preferable for another class or locality, under different conditions, and the method best adapted for any particular region should be employed to obtain the best results. Minute and detailed methods, with ensuing accurate results, should be applied to cities and all closely settled regions, to the coastal belts, larger river valleys and lakes, particularly when navigable, and this work should be platted on a large scale.

Arid, barren, and mountainous regions, as well as prairies and swamp lands, when sparsely settled, should be more generalized in their cartographic representation and platted on a small scale.

* From Appendix 10, Report for 1897, U. S. Coast and Geodetic Survey, by J. A. Flemer.

Topographic surveys may be accomplished in various ways, of which the following are the methods and instrumental outfits more frequently in use:

I. The direct platting to scale in the field of all features to be represented on the finished chart:

(*a*) With a plane-table and steel tape measure.

(*b*) With a plane-table and telemeter or stadia rods.

(*c*) With a tachygraphometer and telemeter or stadia rods.

(*d*) With either outfit mentioned under *a* and *b*, but with a leveling instrument in addition for a more precise location of the horizontal contours.

(*e*) Using a barometer instead of a level for less accurate work.

II. The compilation of all available data—cadastral surveys, public land and county surveys, railroad and canal surveys—giving principally the horizontal distances and making a supplementary survey to supply the missing data, which in this case are principally elevations that may be supplied by leveling profiles, by trigonometric leveling, by interpolation and sketching.

III. The records of the survey are in the shape of field notes and sketches (tachymetry), the map being produced by platting the recorded data in the office:

(*a*) With a surveyor's compass and steel tape, locating the relative positions of characteristic points in the horizontal sense, while their relative elevations are ascertained by means of a level and minor details are sketched.

(*b*) By means of a transit and steel tape points are determined both geographically and hypsometrically (using vertical angles), and minor details by sketching.

(*c*) By means of a transit and telemeter or stadia rods.

(*d*) By means of a tachymeter and stadia rods (elevations being obtained mechanically with the instrument).

(*e*) By means of a transit with steel tape or telemeters, combined with a leveling instrument (for locating horizontal contours).

(*f*) By using a specially constructed aneroid barometer (Goldschmidt's) in place of the level for locating and tracing the horizontal contours in the field.

IV. The field records for developing the terrene are represented by photographic negatives, taken under special conditions (for phototopographic purposes) from known stations:

(*a*) With a camera or phototheodolite, telemeters, or other distance measures (and often a barometer for obtaining elevations).

(*b*) With a surveying camera, a separate theodolite, telemeters, and aneroid barometer.

(*c*) With a photographic plane-table, a distance measure, and aneroid barometer.

(*d*) With a surveying camera, a separate plane-table, and distance measure, frequently using an aneroid barometer for camera stations occupied without the plane-table.

V. The topographic survey may be accomplished by means of a specially constructed surveying camera attached to a free or captive balloon.

After the area which is to be surveyed has been covered with a net of triangles and polygons it will have been provided with a framework of lines of known lengths and direction (triangulation), forming a skeleton survey of the country, and after the natural and artificial features have been filled in by one of the numerous topographic methods (just mentioned) with more or less detail and accuracy we will have a topographic survey of the area of more or less precision.

A good example of changing the method with the locality may be cited in the new survey of Italy, where Paganini's results fully proved the efficiency of phototopography for alpine work (platted on a scale of 1 : 25000 and 1 : 50000) and led to the adoption of the phototheodolite as an auxiliary instrument to the plane-table, the latter being used for mapping the areas below 2,000 meters, while the phototheodolite, was exclusively used for the delineation of the terrene situated above that altitude.

Photogrammetry proper (or *metrophotography*) should be applied to the art of taking perspective views of buildings with a photographic camera for the purpose of constructing therefrom their elevations and ground plans, and it is used principally for architectural, archæological, and engineering purposes.

The term *phototopography* (or *topophotography*) should be generally adopted for all topographic surveys based on perspective views of the terrene obtained by means of the camera.

Under *photographic survey* we could then class all surveys based on photographic data which do not include the orographic delineation of the terrene.

Iconometry means the measuring of dimensions of objects from their

perspectives ("Bildmesskunst"), and this term could well be applied to those graphic constructions which serve to convert perspectives into horizontal projections; iconometry is the reverse of perspective drawing.

Photography has been very successfully employed for topographic surveys in Italy, Austria, and Canada, and for the production of the extensive topographic reconnaissance maps of southeastern Alaska.

Although this method, invented and elaborated by Colonel Laussedat, found its first application in France, still, both in France and in Germany, it was originally preempted by the military authorities, under whose auspices it was developed and chiefly used for so-called secret or military surveys; lately, however, photography has found a wider and more general application to surveying in those two countries, and we find this method now in use also in Greece, Spain, Portugal, Norway, Belgium, Mexico, Chile, Peru, Tonquin, Brazil, Argentine Republic, Switzerland, and England.

Although Lieut. Henry A. Reed has, for several years past, taught phototopography, theoretically and practically, at the United States Military Academy at West Point, there seems to be no record of any further work of this kind undertaken by others in the United States.

In the following paper we will treat principally of those photogrammetric methods which are applicable to topographic surveys, although the same principles underlie also the methods in use when applying photography to—

Geological surveys.—For the study of changes in glaciers (glacial motion or variation) based upon the comparison of glacier maps, obtained at stated time intervals from identical and known camera stations; for volcanic eruptions and their effects; for the study of periodical changes in sand dunes due to recurrent winds blowing from one direction at regular intervals, etc.

Meteorologic observations.—For the study of the higher aerial currents and cloud altitudes, based upon iconometric cloud charts, obtained by simultaneous photographic records on plates exposed at different stations at stated time intervals; for the study of the paths of lightning, their lengths, etc.

Hydrographic surveys.—For the location of rocks, buoys, etc.; for the study of fluvial currents, riparian changes due to corrosion, erosion, etc.; for obtaining coast views from points marked on the sailing charts to facilitate the locating of the position of vessels when approaching land, etc.

Engineering.—To estimate the amount of work done at any date by means of photographic surveys that show the status of the work (excavations, fills, structural buildings, etc.) at stated time intervals, etc.

Architectural purposes.—For constructing the ground plans and elevations of old buildings from their perspective views (photographs), for purposes of remodeling, renovation, or preservation.

Military and secret surveys, and so on.

CHAPTER I.

FUNDAMENTAL PRINCIPLES OF ICONOMETRY.

If only one perspective of an object, including its distance line, the principal point, and the horizon line is given—in other words, if the point of view and the central projection upon a vertical plane of an object are given—the object itself cannot yet be determined regarding its position and dimensions. In the same way the geographic position

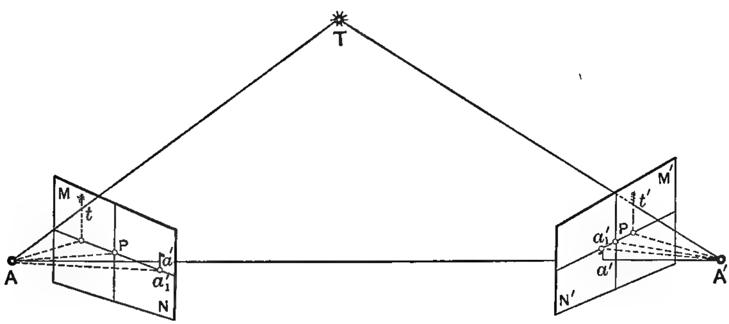


FIG. 286.

of a point cannot be located by means of the plane-table from a known station, unless direct measurements to ascertain the distance of such point from the station are resorted to.

If, however, two different perspectives (including their elements) of the same object, obtained from two different known stations, are given, the dimensions and the position of the object with reference to the two stations may be determined iconometrically in a manner analogous to that in which a point is located (by intersection) on the plane-table sheet by being observed upon from two different plane-table stations.

Referring to Fig. 286, the positions of the camera stations A and A', also the distance AA', may be given, and two photographs containing the image t of an object T, including the image a' of the other camera station, may have been obtained from the two stations.

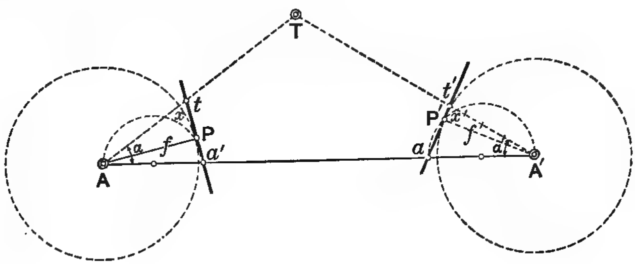


FIG. 287.

If the base line AA', Fig. 287, be laid down on paper, in reduced scale, and if the pictures MN and $M'N'$, Fig. 288, be brought into the same relative positions with reference to the platted line which they had at the time of their exposure in the field, the position T of the pictured point (with reference to the platted points A and A') may be located by drawing the rays At and $A't'$ to their intersection. To locate the platted position of T the horizontal projections of the rays

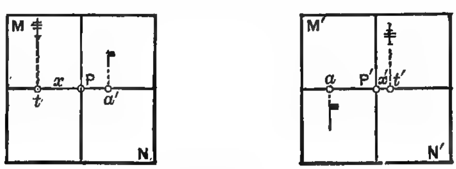


FIG. 288.

At and $A't'$ are brought to their intersection on the platting sheet, Fig. 289, which may be done by ascertaining the proper positions of the lines of intersection of the picture planes with the horizontal platting plane with reference to A and A' (by "orienting" the picture traces).

The map being the orthogonal projection of the terrene in horizontal plan, the horizontal projections of the perspectives (or picture planes exposed in the vertical plane) will appear as straight lines, termed "picture traces," Fig. 289.

The correct orientation of the picture traces forms the most important part of iconometric platting, the subsequent location of picture

points being accomplished by bringing the horizontal projections of the visual rays—lines of direction—drawn to identical points to their corresponding intersections.

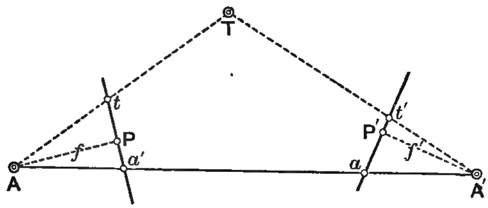


FIG. 289.

I. ORIENTING THE PICTURE TRACES ON THE WORKING SHEET.

(1) A base line AA', measured in the field, has been platted to scale, Fig. 287, and two pictures, MN and $M'N'$, Fig. 288, had been obtained from the camera stations A and A' respectively by means of a surveying camera. The focal lengths of the pictures ($=f$ and f' respectively), the positions of the principal points (P and P'), and the horizon lines may also be given.

It is desired to locate T with reference to AA' upon the working sheet.

The distances $AP = f$; $A'P' = f'$ (Fig. 289); tP, $t'P'$, Pa', and $P'a$ (to be measured on the pictures MN and $M'N'$ respectively), and the line AA' are given.

The distances Aa' and $A'a$ may be found graphically (by constructing the right-angle trangles APa' and $A'P'a$), or they may be computed from the equations

$$Aa' = \sqrt{(AP)^2 + (Pa')^2},$$
$$A'a = \sqrt{(A'P')^2 + (P'a)^2}.$$

These distances are now laid off upon AA' from A and A' respectively, semicircles are described over Aa' and $A'a$, and two circles are drawn about A and A' with f and f' respectively, as radii.

The intersections P and P' of these two pairs of circles locate the horizontal projections of the principal points on the two picture traces, the latter being represented by the tangents Pa' and $P'a$. The distances x ($= Pt$) and x' ($= P't'$) are now measured on the pictures and laid off on the tangents as indicated in Fig. 287, when the intersection

of the lines drawn from A and A' through the points (just found) t and t' will locate the horizontal projection of T with reference to A and A'.

(2) The instrument used was a camera or phototheodolite:

In this case the angles α and α' (Fig. 287) may be measured directly in the field.

We now plat the angles α and α' upon the base line AA' and make $AP = f$ and $A'P' = f'$.

The perpendiculars to AP and $A'P'$ in P and P', respectively, will represent the picture traces (ta' and $t'a$) in correct orientation.

(3) When several pictured points (triangulation points) and the base line are given on the working sheet, the orientation of the picture traces upon the map-projection may be accomplished as follows (Fig. 290):

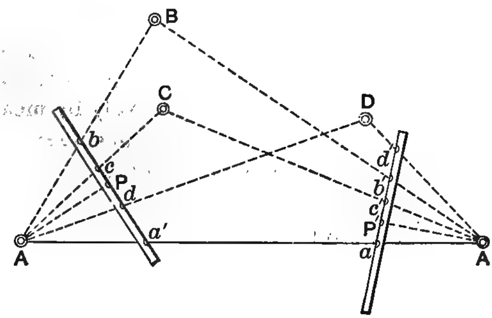


FIG. 290.

The rays AB, AC, AD, and $A'B$, $A'C$, $A'D$ are drawn upon the iconometric platting sheet, the points B, C, and D being already platted on the same.

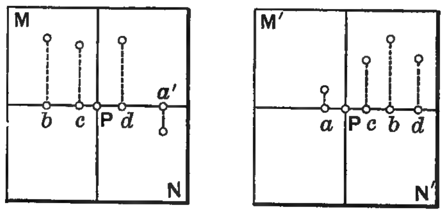


FIG. 291.

The points b, c, P, d, and a are transferred from the horizon line OO' of the negative MN (Fig. 291) upon the perfectly straight edge of

a strip of paper, which is placed upon the radials drawn from A (as center) to the points B, C, D. The strip is now moved about until

b falls upon the ray AB,
c falls upon the ray AC,
d falls upon the ray AD,
a' falls upon the line AA'.

The line AP should now be perpendicular to the straight edge of the paper strip, and the line $bcda'$ drawn upon the working sheet (along the straight edge of the paper strip) will represent the oriented picture trace of MN—

AP will be the distance line, and
P will be the horizontal projection of the principal point.

The same having been done regarding the point A' and its picture $M'N'$, both picture traces will be oriented and the positions of any additional points, that may be identified on both pictures, may be located by platting their abscissæ (measured on the horizon lines of the pictures, regarding P as the origin of the coordinates) upon the picture traces on the proper sides of the principal points. Lines drawn from the station points, A and A', through such corresponding points on the picture traces will locate the relative positions of such points on the platting sheet by their points of intersection.

II. ARITHMETICAL DETERMINATION OF THE PRINCIPAL AND HORIZON LINES ON THE PICTURES.

In the preceding it had been assumed that each perspective was provided with the principal and horizon lines, which would be the case when an *adjusted* surveying camera or phototheodolite had been used for obtaining the pictures. When an ordinary camera (with provisions to maintain the picture plane in a vertical position) or an unadjusted surveying camera is used, the correct position of the principal and horizon lines as well as the length of the distance line (focal length) must be ascertained, which may be accomplished in various ways:

(1) *Determination of the principal point and distance line of the perspective.*—A plumb-bob, suspended in such a way that the plumb-line will be photographed upon the negative, may serve to establish the direction of the principal line VV (Fig. 292) upon the trial plate.

The negative may also contain the images a, b, c, . . . of three or more points A, B, C, . . . of known positions. A line hh is drawn upon the negative perpendicular to VV, and the straight edge of a paper strip is placed upon this line. The pictured points a, b, c, . . . are now projected upon the straight edge of the paper by drawing parallels to VV through the points a, b c, . . . (Fig. 292).

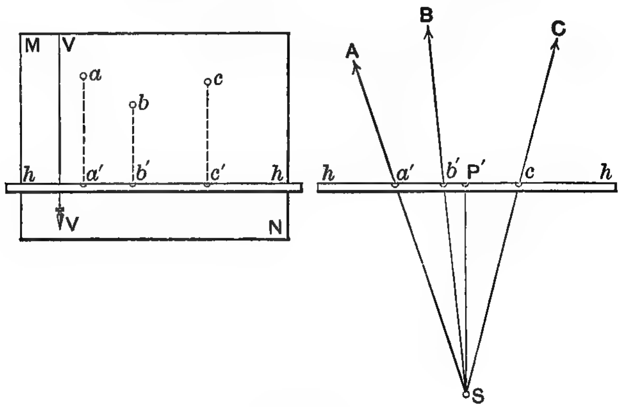


FIG. 292.

After having drawn radials from the platted station S to the points A', B', C', . . . the paper strip is adjusted over the former in such a way that the image projections a', b', c' will fall upon their corresponding radials, when the position (as indicated by the line hh) of the paper strip's edge will be the oriented picture trace. If we now draw a line (SP') from the platted station S perpendicular to hh, the point P' will be the horizontal projection of the principal point P, and SP' will be the distance line ($=f$) for the picture MN.

Whenever the positions of the points A, B, C, . . . with reference to the station S are not known, it will become necessary to observe the horizontal angles ASB, BSC, CSD, . . . instrumentally from the station S, and plat the same upon a sheet of paper in order to adjust the paper strip upon the radials, in the manner just described, to find the principal point and distance line (focal length).

(2) *Determination of the position of the horizon line on the perspective.*—When the elevations AA', BB', CC', . . . of the points A, B, C, . . . above the horizon of the station (S) are known, the position of the horizon line (oo') (Fig. 293) may be found by constructing or by

computing the lengths of the ordinates aa', bb', cc', . . . from the relations

$$aa' : AA' = Sa' : SA',$$
$$bb' : BB' = Sb' : SB',$$
$$cc' : CC' = Sc' : SC',$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

whence

$$aa' = \frac{Sa' . AA'}{SA'} = y',$$

$$bb' = \frac{Sb' . BB'}{SB'} = y', \text{ etc.}$$

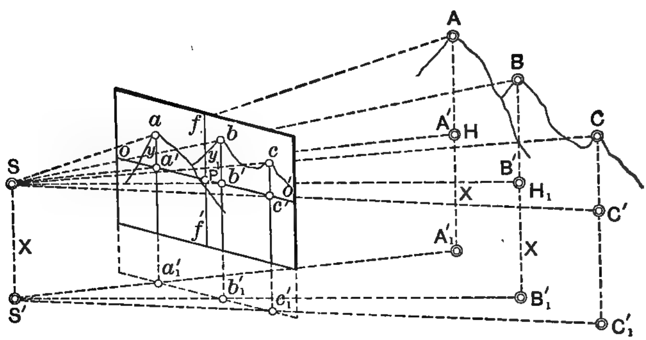


FIG. 293.

The distances Sa', Sb', Sc', . . . are taken from the platting sheet (Fig. 293) and the distances SA', SB', SC', . . . as well as the differences in elevation AA', BB', CC', . . . are known (if the points A, B, C, . . . had been located in the horizontal and vertical sense with reference to the station S).

For example:

Difference in elevation between A and $A' = 100^m$.
Distance of A' from the station $S = 1000^m$.
Distance Sa', measured on the platting sheet, $= 0.5^m$.

The ordinate $aa' = \dfrac{0.5 \times 100}{1000} = 0.05^m$.

The horizon line (oo') on the negative will be 50 mm. vertically below (parallel with VV) the pictured point a.

The direction of VV (the principal line) being parallel to the pictured plumb-line, this distance aa' is laid off in the same direction below a, and a line oo', drawn at right angles to VV through a', will locate the horizon line. The ordinates bb', cc', . . . of the other pictured points may well serve to check this position of oo'. The horizon line will be the tangent to the arcs described with aa', bb', cc', . . . about a, b, c, . . . respectively, as centers.

The principal point, P, may now be transferred to the negative by using the paper strip, and the line drawn through P perpendicular to oo' will be the principal line for the picture MN.

III. GRAPHIC METHOD FOR DETERMINING THE POSITIONS OF THE PRINCIPAL AND HORIZON LINES ON THE PERSPECTIVES.

The following graphic method for orienting the picture trace and locating the principal and the horizon lines was published by Prof. F. Schiffner in 1887; it is also mentioned by Prof. F. Steiner.

Three points, A, B, and C (Fig. 294), may be given with reference to the station S upon the platting sheet.

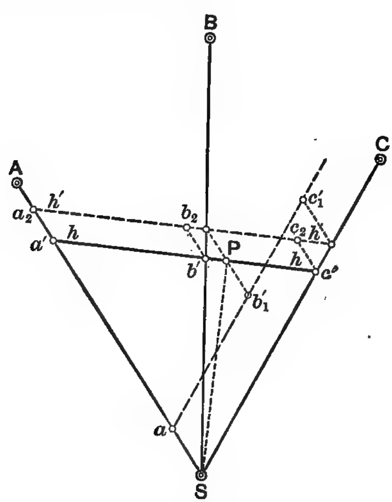


FIG. 294.

From S radials are drawn through A, B, and C. Through a point a on the ray SA a parallel to SC is drawn, and the distance $a'b'$ (taken from the negative MN) is laid off from a ($= ab_1'$) upon this parallel,

while the distance $b'c'$ is laid off upon the same line from b_1' $(= b_1'c_1')$. Parallels to the radial SA are now drawn through the points b_1' and c_1' and prolonged to intersect the radials SB and SC. The line $(h'h')$ connecting these two points of intersection will be parallel with the direction of the picture trace.

The same distances $a'b'$ and $b'c'$ (taken from the negative) are laid off upon this line $h'h'$ from a_2 $(= a_2b_2)$ and from b_2 $(= b_2c_2)$. The lines drawn through these points b_2 and c_2, and parallel with the radial SA, are brought to intersections with the radials SB and SC, when the line (hh) passing through these intersections will represent the picture trace correctly placed (oriented) with reference to S, A, B, and C.

The distance SP of S from hh represents the distance line (focal length) of the picture MN, while the point P' will be the horizontal projection of the principal point P.

After having transferred P' (with reference to a', b', and c'), by means of a paper strip, to the negative MN, a parallel to VV, drawn through the transferred point P, will locate the principal line upon the negative.

The horizon line may now be located in the same manner as shown under II, 2, adopting the graphic solution.

IV. THE FIVE-POINT PROBLEM (BY PROFESSOR STEINER).

In the methods just described it had been assumed that the position of the camera station was known with reference to the surrounding points A, B, C, . . .

In case the panorama pictures were taken from a camera station of unknown position and a series of known points are pictured upon the panorama views, the position of the camera station may be found (with reference to the surrounding points of known positions), and the orientation of the picture trace may be accomplished by means of Prof. F. Steiner's so-called "five-point problem" (Fig. 295), if one of the views contains the pictures of *five* or more points of known positions.

The panorama view MN may contain the images a, b, c, d, and e of the points A, B, C, D, and E (already plotted upon the working sheet), and also the picture of a suspended plumb-line or other vertical (or horizontal) line.

The points a, b, c, d, and e of the negative are again projected upon the straight edge of a paper strip $= a'$, b', c', d', and e'.

Radials are now drawn from one (A) of the five plotted points, as a center, to the other four, B, C, D, and E. The marked paper strip is then placed over the radials in such a way that

b' falls upon AB, d' falls upon AD, e' falls upon AE,

when the strip will have the position a_1, b_1, c_1, d_1, e_1. The line drawn through A and a_1 (the latter transferred by means of the strip) will be

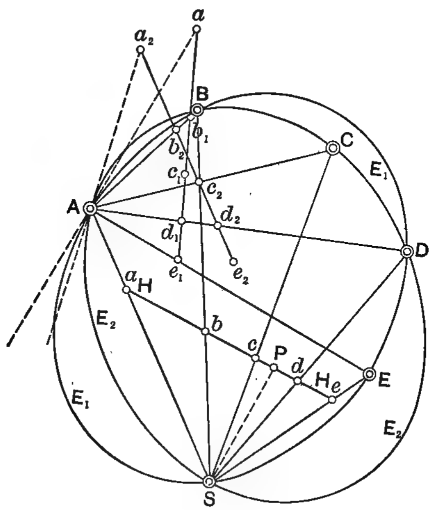


FIG. 295.

the tangent in A to the ellipse E_1 (passing through A, B, D, E, and through the station point S).

The paper strip is now placed over the radials AB, AC, and AD, so that

b' falls upon AB, c' falls upon AC, d' falls upon AD,

when the strip will have the position $a_2b_2c_2d_2e_2'$, and the line Aa_2 will be the tangent in A to the ellipse E_2 (passing through the points A, B, C, D, and the station point S).

The position of the station point S on the working sheet (with reference to the five points A, B, C, D, and E) will be identical with the point of intersection of the two ellipses E_1 and E_2.

(1) *Determination of the principal point and distance line in the perspective.*—The distance line and the principal point are now found by

drawing the radials SA, SB, SC, SD, and SE, and placing the paper strip over these in such a way that

a' falls upon SA, b' falls upon SB, c' falls upon SC,
d' falls upon SD, e' falls upon SE,

which position is indicated by the line HH. The perpendicular upon HH passing through S ($= \acute{S}P$) is the distance line and P is the principal point projected into horizontal plan, which may now be transferred

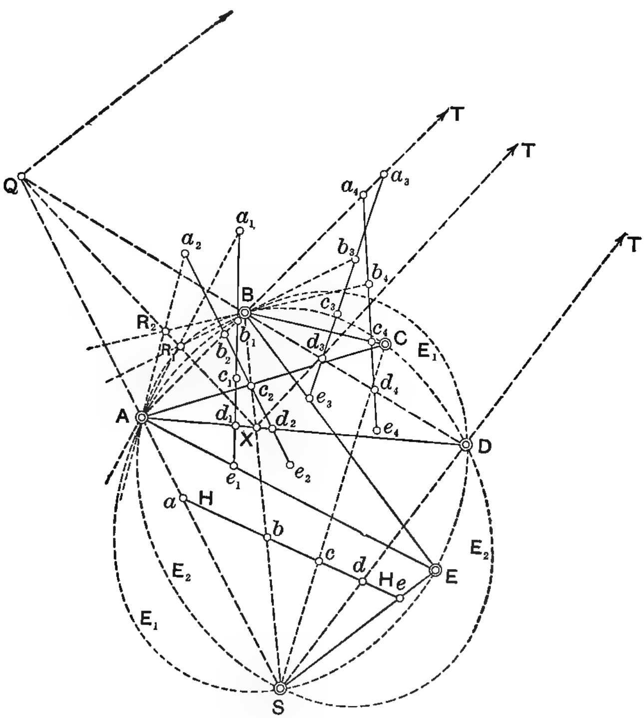


FIG. 296.

to the picture by means of the paper strip in order to locate the principal line in a similar manner to that mentioned in the preceding pages.

(2) *Simplified construction for locating the camera station by means of the five-point problem.*—The preceding method is rather complicated, but Professor Schiffner devised the following construction (Fig. 296), in which the drawing of the ellipses E_1 and E_2 is avoided:

The same five points, A, B, C, D, and E, with their images a, b, c, d, and e, on one plate MN, may be given.

The two lines, b_3B and b_4B, tangent in B to the two ellipses E_1 and E_2, respectively, are located precisely in the same manner as the two tangents a_1A and a_2A were found for the point A.

The intersections R_1 and R_2 of the tangent pairs a_1A, b_3B and a_2A, b_4B (belonging to the ellipses E_1 and E_2, respectively) are situated upon a line Qx, forming one side of the polar triangle QxT, common to both ellipses. This line Qx intersects the diagonal AD in x and the quadrilateral side BD in Q, and the lines drawn through Q from A and through x from B will intersect each other in the fourth point of intersection (S) of the two ellipses.

The quadrilateral $ABDS$, obtained by connecting the four points of intersection of the two ellipses, has the point x as the intersection of its diagonals. By prolonging the sides BD and AS to their point of intersection Q and the sides AB and SD to their point of intersection T, the three diagonal points QxT will form the polar triangle common to the two ellipses.

Also this method remains complicated and requires many lines to be drawn before the picture trace and the camera station (S) may be plotted.

(3) *Special application of the five-point problem for the case when the five points range themselves into a triangle.*—The application of the "five-point problem" becomes very much simplified, however, for the special case when the five points range themselves into a triangle, of which two sides (AC and CE) contain three points each (Fig. 297).

If we now place the strip of paper upon the radials drawn from A, so that

$$e' \text{ falls upon } AE, \quad d' \text{ falls upon } AD, \quad c' \text{ falls upon } AC,$$

it will have the position $a_2b_2c_2d_2e_2$, and the first ellipse (E_1) will resolve itself into the lines CE and Aa_2.

If we now place the paper strip $a'b'c'd'e'$ upon the radials drawn from E to A, B, and C, so that a' falls upon EA, b' upon EB, and c' upon EC, it will assume the position $a_1b_1c_1d_1e_1$, and the second ellipse (E_2) will have resolved itself into the lines AC and Ee_1.

The intersection S of the two lines Aa_2 and Ee_1 will locate the station point with reference to the five given points, and by placing the paper strip upon the radials SA, SB, SC, SD, and SE in such a way

that a' falls upon SA, b' upon SB, etc., its edge will locate the picture trace.

(4) *To find the elevation* (x) *of a camera station* (S) *that has been located by means of the "five-point problem."*—In order to ascertain the elevation of the unknown station S, platted after one of the preceding

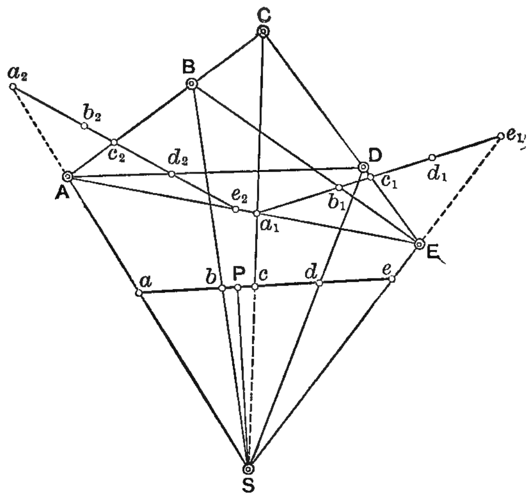


FIG. 297.

methods, it will become necessary to know the elevations of at least two of the five points.

Let the elevation of the station S, Fig. 293, be designated by x.

The elevation of $A = H$ and of $B = H_1$. The ordinates $aa' = y$ and $bb' = y_1$.

From the relation $S'a_1' : S'A_1' = aa' : AA'$, or

$$Sa' : SA' = y : (H - x),$$

we find

$$y = \frac{Sa'}{SA'}(H - x)$$

and

$$y_1 = \frac{Sb'}{SB'}(H_1 - x).$$

The difference between y and y_1 may be measured on the negative, hence

$$y - y_1 = m$$

is known, and the value for x may be found from the equation

$$y - y_1 = (H - x)\frac{Sa'}{SA'} - (H_1 - x)\frac{Sb'}{SB'} = m.$$

The values for Sa', SA', Sb', and SB' may be taken directly from the platting sheet, while those for H and H_1 are found in the triangulation records.

If we write the above equation in the general form

$$\frac{H - x}{n} - \frac{H_1 - x}{o} = m,$$

the elevation x of the camera station S may be computed from

$$x = \frac{mno - Ho + H_1 n}{n - o}.$$

The numerical values for the ordinates y and y_1 (locating the position of the horizon line on the perspective) may now be computed from the equations

$$y = \frac{H - x}{n}$$

and

$$y_1 = \frac{H_1 - x}{o}.$$

V. THE THREE-POINT PROBLEM.

If the triangulation points are not sufficiently close together that five or more points may be pictured on one perspective, and if stations are occupied with the camera that are not connected with the trigonometric survey, it will become necessary to employ other means to determine the position of the camera station with reference to the surrounding triangulation points.

In order to connect the camera station with the triangulation system by direct measurements and observations, made at the camera station, it will be requisite that at least three triangulation points be visible from such station, unless the location of the camera station is to be made by observations made from other stations. In the latter case the occupation of two (better three) triangulation points, if favorably located, would suffice to establish the ("concluded") position of the camera station.

The determination of the position of an occupied point by observing upon three fixed and known points is generally known as the "three-point problem," "station platting," "station pointing," or "*Pothenot's* method," although Snellius had used the same method in his trigonometric work in the Netherlands in the second decade of the seventeenth century. Let A, B, and C, Fig. 298, be the three points, the positions of which are known. A fourth undetermined point S may have been occupied from which the horizontal angles $ASB = M$ and $BSC = N$ may have been observed instrumentally. The position of S with reference to A, B, and C may then be ascertained in various ways.

(1) *Using the three-arm protractor (mechanical application of the three-point problem).*—The simplest (and crudest) method is purely mechanical in its application. The two horizontal angles M and N are laid off upon a three-arm protractor ("station pointer"), or upon a piece of tracing paper, moving the three radials SA, SB, and SC over the three fixed and platted points A, B, and C until the three radials SA, SB, and SC bisect their corresponding points A, B, and C. Holding the two angles M and N unchanged in this position, the point S is transferred to the working sheet.

(2) *Graphic solution of the three-point problem.*

(*a*) *Using the so-called "two-circle problem."*—Theoretically the best graphic method is that which locates the position of the fourth point S, Fig. 298, as the intersection of two circles, one passing through A and B and having all angles of circumference $= ASB = M$ over AB that may be drawn over the line AB as chord, the other circle passing through B and C and having over BC as chord all angles of circumference equal to $BSC = N$.

From the platted triangle side AB we lay off at A and B the angles BAc_1 and ABc_1 each equal to

$$\frac{180 - 2(ASB)}{2} = 90^\circ - ASB = 90^\circ - M,$$

and about the point c_1, thus obtained, a circle ABS is described with the radius $= c_1A = c_1B$. The observed angle $ASB = M$ will then be an angle of circumference over AB, and the point S will be located somewhere on the arc over the chord AB.

By means of the angle $BSC = N$ a second circle BCS is described over the triangle side BC, in a similar manner, about c_2 as center with the radius $c_2B = c_2C$. The observed second angle $BSC = N$ will be an

angle of circumference over the chord BC, hence the point S will be situated also upon the arc over the chord BC and the true position of S is at the point of intersection S of the two circles.

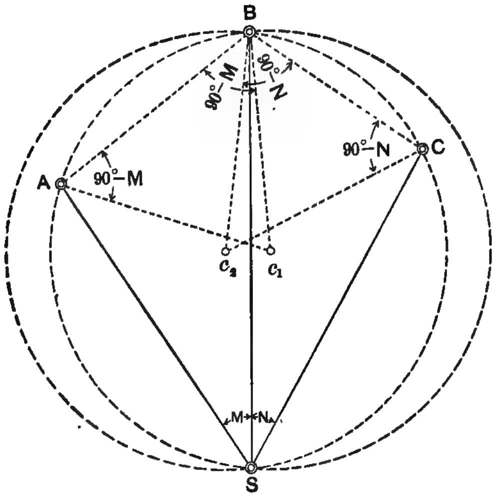


FIG. 298.

(*b*) *Using the method of Bohnenberger and Bessel.*—The following constructive method (devised by Bohnenberger and Bessel) is readily applied and of a very simple character, Fig. 299. If we describe a circle through two of the three given points A and B and the station S as the third point, the angles

$$ASB = ACB = M, \text{ and}$$
$$BSC = BAC = N$$

(being angles of circumference upon the same arcs AD and DC respectively).

Hence, if we lay off the observed horizontal angle M on the base line AC at C and the other horizontal angle N on AC at A, the point of intersection D of their convergent sides CD and AD will be on the line connecting the third point B and the platted station S. After having thus found the direction of the line DB the position of S on the line DB may be found as follows:

At any point x of the line DB the observed angles M and N are laid off to either side of DB, in the sense in which they were observed.

Lines AS and CS drawn through A and C parallel to xy and xz, respectively, will locate the position of the station S (upon DB) with reference to the three points A, B, and C.

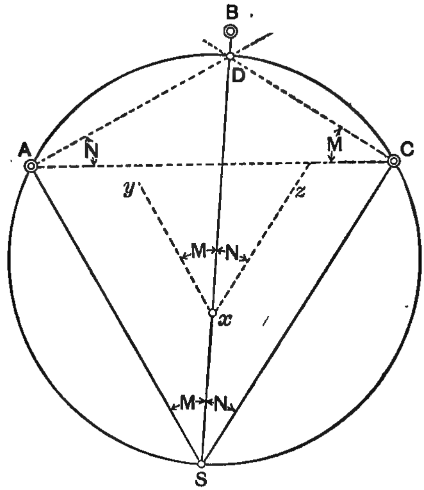


FIG. 299.

This construction is only recommended when BD is sufficiently long (in Fig. 299 it is evidently too short) to admit of a correct prolongation of its direction toward S.

The picture trace containing the horizontal projections of the pictured points a, b, and c may now be oriented in the known manner by adjusting the paper strip over the radials SA, SB, and SC.

CHAPTER II.

PHOTOGRAPHS ON INCLINED PLATES.

IN the preceding we have regarded photographic plates (perspectives) only that had been exposed in a vertical plane, and although the use of inclined plates for phototopographic purposes is not to be generally recommended (on account of the complications that will arise in the ordinarily simple constructions in iconometric platting from vertically exposed plates, and because the relations which exist between the elements of the perspective and the orthogonal projection in horizontal plan of the pictured objects will not be so readily recognized), still

occasions may arise where the selection of the available or accessible stations will be so circumscribed that the exposure of inclined plates will become necessary in order to control the inaccessible terrene (above or below the camera station).

Photographs may also have been obtained with an ordinary camera, without any device for adjusting the plate in vertical plane, or the use for iconometric platting of the photographs (perhaps taken only for illustrative purposes) may have been an afterthought.

With reference to Fig. 300 we have:

PP = principal plane;
HH = horizontal plane passing through the second nodal point of the camera lens (at the station S);
GG = ground plane;
MN = picture plane;
$O'P$ = trace of picture plane MN, in the horizon plane HH;
$O_0'P_0$ = ground line of picture plane MN;
S_0 = foot of the station S;
$P'P_0$ = principal line of the picture plane;
P' = principal point of the perspective MN;
SS_0 = vertical of the station; it will penetrate the picture plane MN above (or below) the horizon line at s. The trace s of this vertical sS_0 in the picture plane is the vanishing point for the perspective of all vertical lines that may be pictured on MN;
$P'SP = PsS = \alpha$ = angle of inclination of the plate MN;
SP = (horizontal) line from S perpendicular to horizon line $O'P$;
SA = line of direction from S to a point A, pictured in MN as a.

If we revolve SP, in the vertical plane PP, about P until SP falls within the picture plane, then the point S will fall into (S) and the line Sa will fall into $(S)a$.

The vertical plane containing the line SA and passing through SS_0 will intersect the ground plane in S_0a_0. If we now revolve the line S_0P_0, in the vertical plane PP, about P_0 until S_0P_0 falls within the picture plane MN, then the point S_0 will fall into (S_0) and the trace S_0a_0 will have assumed the position $(S_0)a_0$, and the intersection A of the trace S_0a_0 with the line of direction Sa will locate the platted position of the pictured point a in the ground plane GG.

The line sa intersects the ground line in a_0, and S_0a_0 will be the

radial in the ground plane to the platted position of A and passing through the foot S_0 of the station S.

To find A on S_0a_0 we first locate in the picture plane the intersection (A) of the revolved lines $(S)a$ and $(S_0)a_0$. This point (A) revolved in the vertical plane a_0S_0S about a_0 will locate A upon S_0a_0.

To locate the position of A in GG, in the manner just shown, we should know the position of the line $O'P$, as well as the points S

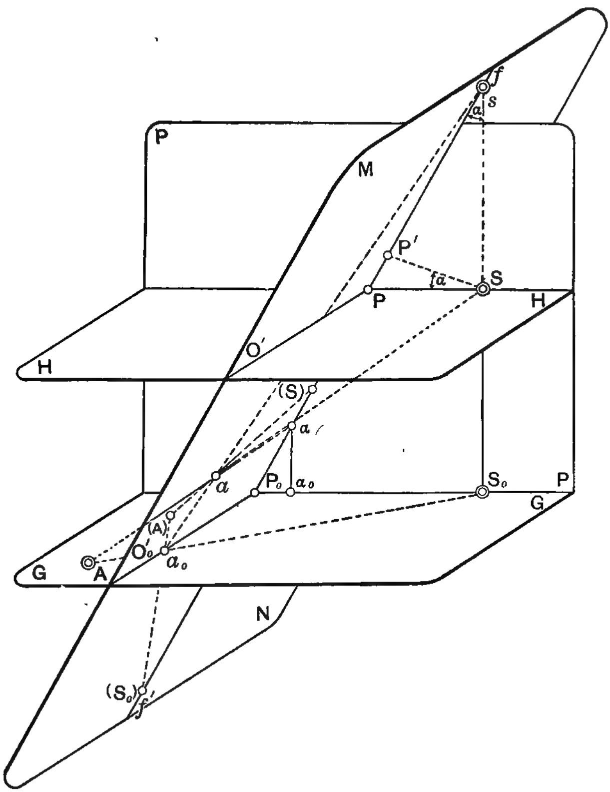


FIG. 300.

and P. These are known or may readily be found if the position of the principal point P', the length of the distance line SP', and the value of the angle of inclination (α) for the plate are known.

When a photographic plate in a surveying camera is intentionally exposed in an inclined position, it will generally be exposed in such a

way that the principal line ff' still coincides with the intersection of the picture plane MN and the principal plane PP, Fig. 300.

When the angle of inclination α is an angle of elevation (depression) the horizon line (intersection of horizon plane and inclined picture plane) will fall below (above) the line representing the horizon line on the plate when exposed vertically. In order to use the inclined plate for iconometric purposes the angle of inclination should be observed directly in the field, and, if the constant focal length of the camera $(=f)$ is known, the line SP, Fig. 300, may be found as the hypothenuse of the right-angle triangle with angle $= \alpha$ and adjoining side $= f$.

I. TO PLAT THE PICTURE TRACE OF AN INCLINED PLATE.

In order to plat the picture trace, the horizontal angle included between the optical axis of the inclined camera and the horizontal

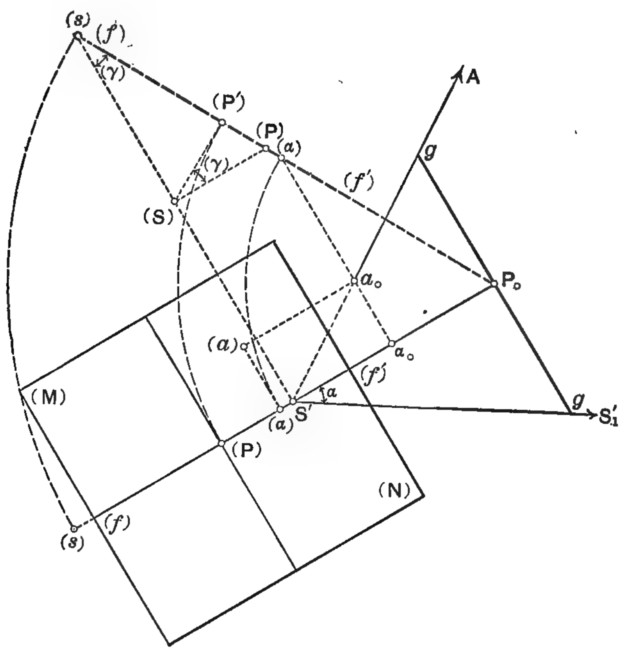


FIG. 301.

direction to some known point should be measured. Should the length SS' (elevation of station S above the foot of the station, Fig.

302), the position of the line connecting two camera stations, and also the position of a third point A (visible from both stations) be known, no horizontal angle α needs to be measured instrumentally, provided the plates containing the picture a of the third point A are oriented in such a way that the picture a be bisected by the vertical thread or principal line ff' of the perspective.

With reference to Fig. 301 we have

$$S' = \text{platted position of the station } S,$$
$$S'S_1' = \text{platted length and direction of the base line.}$$

The horizontal angle α (at S') included between this line of direction $S'S_1'$ and the principal plane (or horizontal projection of optical axis $S'P_0$) may have been observed in the field. The line $S'S$ in Fig. 302 represents the elevation of the station S (laid off in the platting scale). If we revolve this line $S'S$ about $S'P_0$ into the platting plane, it will assume the position shown as $S'(S)$ in Fig. 301. After erecting at (S) a line $(S)(P)$ perpendicular to $S'(S)$ the angle of inclination γ of the plate MN is laid off upon $(S)(P)$ from (S).

$(S)(P')$ is made equal to the constant focal length $(=f)$ of the camera, and the line drawn perpendicular to $(S)(P')$ through (P') will represent the principal line $(f)(f')$ of the perspective MN, revolved about $S'P_0$ into the platting plane. The point of intersection (s) of $(S)S'$ with $(f)(f')$ represents the vanishing point for all vertical lines shown on the picture.

The point of intersection P_0 of the line $(f)(f')$ and the horizontal projection of the optical axis $S'P_0$ will be the trace in the ground plane of the inclined principal line ff'.

The line P_0g, perpendicular to $S'P_0$ in P_0, is the ground line or the trace of the inclined picture plane MN in the platting plane GG.

II. PLATTING THE LINES OF DIRECTION TO POINTS PICTURED ON AN INCLINED PHOTOGRAPHIC PLATE.

The inclined picture plane MN, Fig. 302, is revolved about P_0g into the drawing or ground plane, when the picture will appear as $(M)(N)$, the principal point P falling upon $S'P_0 = (f)(f')$ in (P) and $(P)P_0 = PP_0$.

To plat the direction to a point A from S', we first locate the orthogonal projection a_0 (in the ground plane) of the picture point a, Fig. 301.

The image point a, Fig. 302, projected upon ff' or upon $PP_0 = \alpha$ and a circle described about P_0 with $P_0(\alpha)$ will locate the position (α) of the projected point on the principal line $(f)(f')$, revolved into the platting or ground plane.

The perpendicular to $S'P_0$ in α_0 and the vertical to the ground plane GG from a, Fig. 302, intersect each other in a_0, and $S'a_0$, Fig.

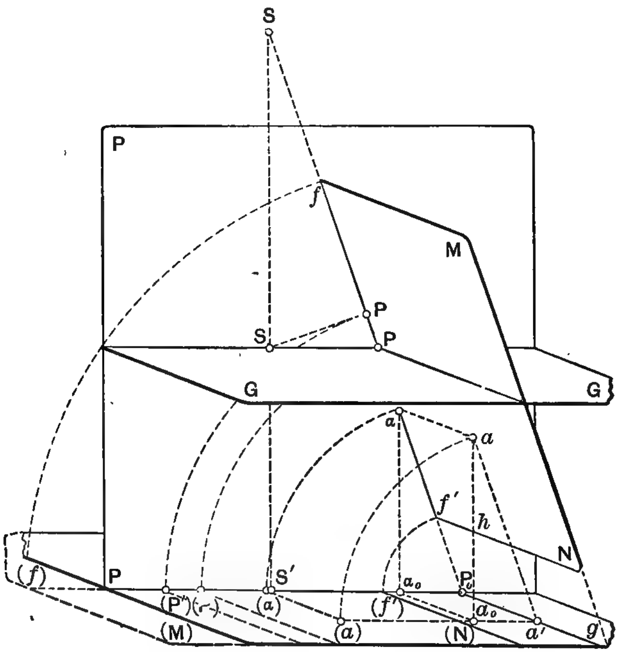


FIG. 302.

301, is the horizontal projection (in the ground or platting plane) of the line of direction or radial from S' to the point A.

III. DETERMINATION OF THE ALTITUDES OF POINTS PICTURED ON INCLINED PLATES.

We refer again to Fig. 302. It is desired to find the elevation H of the point A (pictured in a) above the ground plane GG.

Projecting a upon the principal plane PP we find α on ff'; the vertical through α intersects the horizontal projection of the principal

ray $S'P_0$ in α_0, Fig. 302; hence $a\alpha_0$ represents the elevation of the point A above GG, measured in the platting scale.

With reference to Fig. 301, this elevation aa_0 (Fig. 302) of a above the ground plane is found by projecting a upon $P'P_0$ ($= \alpha$ in Fig. 302); the corresponding point on the principal line revolved about P_0 into the platting plane is (α), and its orthogonal projection upon the principal plane, the latter revolved into the platting plane about $S'P_0$, Fig. 301, is (a); hence the elevation of A above the ground plane is $= (\alpha)\alpha_0 = h$, to be measured in the platting scale.

If $D =$ distance of the platted point from S', taken from the platting sheet, $H =$ elevation of the point A above the ground plane GG, $h = (\alpha)\alpha_0 = aa_0$, Fig. 302, $= (\alpha)\alpha_0$, Fig. 301, $= aa_0$, Fig. 303, $S'a_0 = d$ (Fig. 301), taken from the platting sheet, the elevation H of the point A may be found either graphically from a diagram, Fig. 303, or it may be computed from the relation

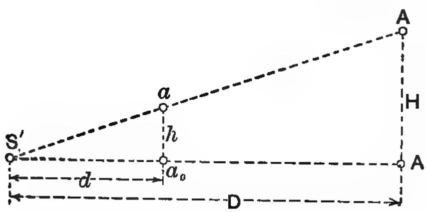


FIG. 303.

$$H = \frac{Dh}{d}.$$

CHAPTER III.

PHOTOTOPOGRAPHIC METHODS.

I. ANALYTICAL OR ARITHMETICAL METHODS.

(1) *General arithmetical method for finding the platted positions of points pictured on photographic perspectives (exposed in vertical plane).*—If we refer the pictured points to the principal point P by means of the rectangular system of coordinates formed by the principal line ff' and the horizon line OO', we will have with reference to Fig. 304:

S and S' = two camera stations; MN and $M'N'$ = two picture planes exposed in vertical plane, one from station S, the other from station S'; $aa'(= y)$ and $a'P(= x)$ = coordinates of pictured point a on MN; $a'a_1'(= y')$ and $a_1'P'(= x')$ = coordinates of a' pictured on $M'N'$; $f =$ focal length (the same for both pictures MN and $M'N'$); $D = S_0A_0$ = horizontal distance of A from station S; $D' = S_0'A_0$ = horizontal distance of A from station S'; $d = Sa' = S_0a_0$ = horizontal

distance of pictured point a from point of view S; $d' = S'a_1' = S_0'a_0' =$ horizontal distance of pictured point a' from point of view S'; $H =$ elevation of A above horizon plane of station S. $H' =$ elevation of A above horizon plane of station S'; $B = S_0S_0' =$ horizontal distance between the stations S and S'; α and $\alpha' =$ horizontal angles included between B and the principal planes passing through S and S', respectively.

If the camera (theodolite) was in perfect adjustment, if the base line B is known, and if the angles α and α' had been observed, we will

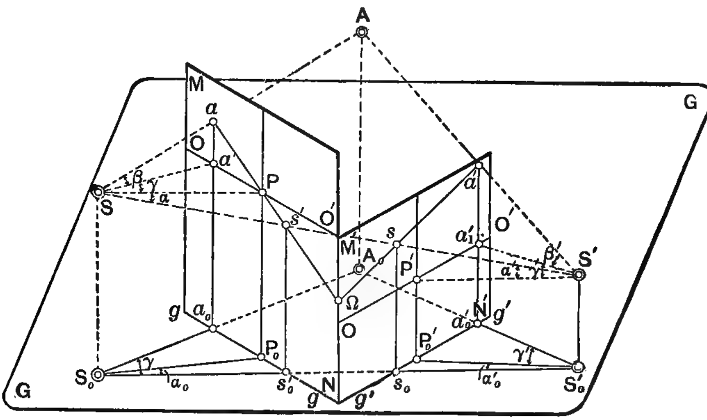


FIG. 304.

know the values of B, α, α', f, and the coordinates x, y, x', and y', the latter being obtained by direct measurement on the negatives.

We can now compute—

(1) The horizontal angle γ, included between the principal plane and any horizontal direction, Sa', Fig. 304, from the equation

$$\tan \gamma = \frac{x}{f} \quad \text{or} \quad \tan \gamma' = \frac{x'}{f}.$$

(2) The angle of elevation β of the line of direction Sa to any point, A, pictured as a on the photograph MN, from the equation

$$\tan \beta = \frac{y}{d} \quad \text{or} \quad \tan \beta' = \frac{y'}{d'}.$$

As

$$d = \sqrt{f^2 + x^2} \quad \text{or} \quad d' = \sqrt{f^2 + (x')^2},$$

we may also write

$$\tan \beta = \frac{y}{\sqrt{f^2 + x^2}}$$

or

$$\tan \beta' = \frac{y'}{\sqrt{f^2 + (x')^2}}.$$

We know the length S_0S_0' ($= B$) of the triangle $S_0A_0S_0'$, and the angles γ, α, γ', and α' also being given, we have

$$D : B = \sin(\gamma' + \alpha') : \sin[180° - (\gamma + \alpha + \gamma' + \alpha')]$$

$$= \frac{\sin(\gamma' + \alpha')}{\sin(\gamma + \alpha + \gamma' + \alpha')},$$

whence

$$S_0A_0 = D = \frac{B \sin(\gamma' + \alpha')}{\sin(\gamma + \alpha + \gamma' + \alpha')}$$

or

$$S_0'A_0 = D' = \frac{B \sin(\gamma + \alpha)}{\sin(\gamma + \alpha + \gamma' + \alpha')}.$$

The difference in elevation, H, between the point A and camera station S may be found from

$$\frac{H}{D} = \tan \beta,$$

whence

$$H = D \tan \beta$$

or

$$H' = D' \tan \beta.$$

(2) *General arithmetical method for finding the platted positions of points pictured on photographic perspectives for inclined picture planes.*—For inclined picture planes we will have to take into consideration the angle of inclination of the plate—the angle which is included between the optical axis of the inclined camera and the horizon plane of the camera station.

We have, with reference to Figs. 305 and 306:

α = horizontal angle between the principal plane of station S and the vertical plane passing through station S and the point A, pictured

as a on inclined picture MN; β = angle of elevation of the point A observed from S; γ = angle of inclination of the photographic plate MN; $\delta = 180^\circ - \gamma$; OO' = horizon line on MN when vertical, permanently marked on the camera; P = principal point for the vertical

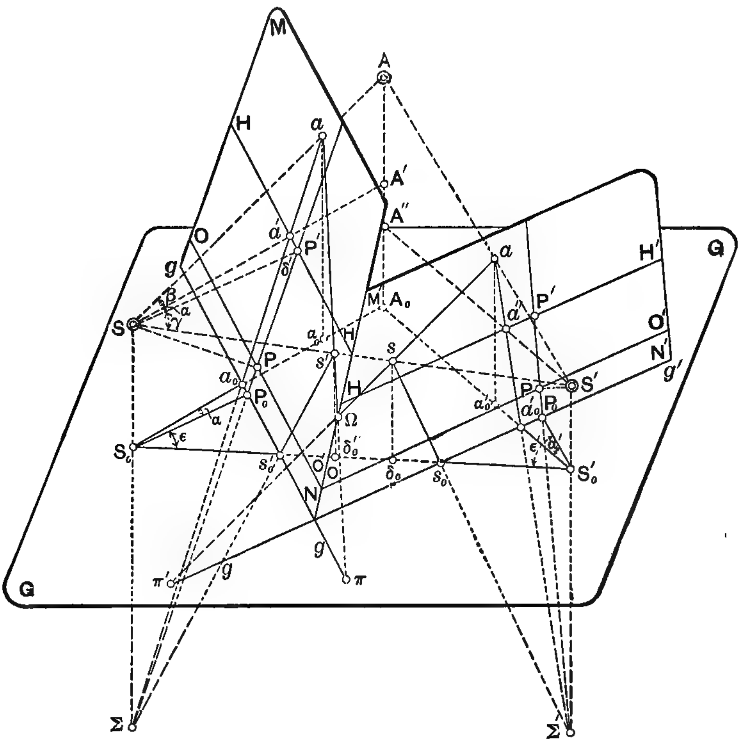


FIG. 305.

plate, also permanently marked as the intersection of the principal and horizon lines when the plate is vertical; $P\pi = y$ = ordinate of a on MN (Fig. 306); $a\pi = x$ = abscissa of a on MN, very nearly $= a'P'$; Σ = vanishing point for all vertical lines pictured on MN.

From inspection of Fig. 306 it will follow directly:

$$\tan\beta = \frac{a\alpha'}{S\alpha'} = \frac{\pi\pi'}{S\alpha'} = \frac{S\Pi}{S\alpha'}$$

$$= \frac{P\rho - P\Pi}{\sqrt{x^2 + (S\pi')^2}} = \frac{y \cos \gamma - f \sin \gamma}{\sqrt{x^2 + (S\Pi + \Pi\pi')^2}}$$

$$= \frac{y \cos \gamma - f \sin \gamma}{\sqrt{x^2 + (S\Pi + \rho\pi)^2}}$$

$$= \frac{y \cos \gamma - f \sin \gamma}{\sqrt{x^2 + (f \cos \gamma + y \sin \gamma)^2}}$$

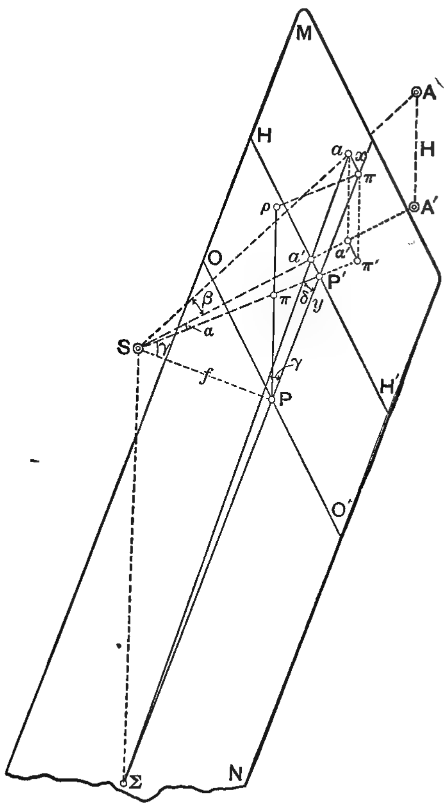


FIG. 306.

and

$$\tan \alpha = \frac{\alpha'\pi'}{S\pi'} = \frac{x}{S\Pi + \rho\pi} = \frac{x}{f \cos \gamma + y \sin \gamma}.$$

(We had found for the vertically exposed plate

$$\tan \beta = \frac{y}{\sqrt{x^2 + f^2}}$$

and

$$\tan \alpha = \frac{x}{f}.$$

The preceding formulas for tan α and tan β will assume the form of the latter if the angle of inclination γ is reduced to zero, as $\sin \gamma = \sin 0 = 0$ and $\cos \gamma = \cos 0 = 1$.)

After having thus found α and β (also α' and β') we can now compute the value for $D = S_0A_0$ and for $H = AA'$.

With reference to Fig. 305 we have

$$\frac{D}{B} = \frac{\sin (\epsilon' - \alpha')}{\sin [180° - (\alpha + \epsilon + \epsilon' - \alpha')]},$$

hence

$$D = \frac{B \sin (\epsilon' - \alpha')}{\sin (\alpha + \epsilon + \epsilon' - \alpha')},$$

and from $\tan \beta = \frac{H}{D}$, we find

$$H = D \tan \beta$$

$$= \frac{D(y \cos \gamma - f \sin \gamma)}{\sqrt{x^2 + (f \cos \gamma + y \sin \gamma)^2}}.$$

If an ordinary surveying camera, with a constant focal length, is used, and when it should become desirable to expose a photographic plate in an inclined plane, the complement δ of the angle of inclination of the optical axis ($= \gamma$) may be determined more readily (but only approximately) than the latter by carefully measuring the distances AD, Fig. 307 (in the direction of the line of a suspended plumb-bob), and DB, supposing AB to be parallel with the photographic plate.

(3) *General analytical determination of the elements of a photographic perspective.*—If, in addition to the photographs, data obtained by instrumental observations are given for a graphical determination of the focal lengths of the pictures, their horizon lines and principal points, then these elements may also be determined by computation.

A picture, MN, may contain the images a, b, and c of three known points, A, B, and C, the position of the camera station (whence this picture was obtained) being likewise known with reference to the three platted points A', B', and C', Fig. 308.

To orient the picture trace (or ground line) gg' with reference to the platted station S' and the platted points A', B', and C', the latter are preferably referred to a system of coordinates having the platted station S' as origin.

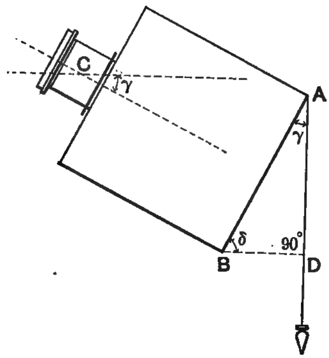


FIG. 307.

In Fig. 308, for example, a rectangular system of coordinates, $S'Y$ and $S'X$, has been adopted, with the origin in S', and axis of abscissa passing through one of the three triangulation points.

The coordinates of the three triangulation points A', B', and C', platted on the chart projection, are found by measurement $= X_1Y_1$, X_2Y_2, and X_3, respectively.

The coordinates of the orthogonal projections (on the picture trace gg') of the corresponding points, pictured on the photograph MN, may be designated by $x_I y_I$, $x_{II} y_{II}$, and x_{III}, respectively.

The horizontal distances between a and b, b and c, a and c (which are the same as those between a' and b', b' and c', a' and c' on the picture trace) may be m^I, m^{II}, and m^{III}, respectively.

We will find directly, from an inspection of Fig. 308:

$$(1)\ y_I : x_I = Y_1 : X_1.$$
$$(2)\ y_{II} : x_{II} = Y_2 : X_2.$$
$$(3)\ y_I : y_{II} = m^{III} : m^{II}.$$
$$(4)\ (x_{III} - x_I) : (x_{II} - x_I) = m^{III} : m^I.$$
$$(5)\ (x_{III} - x_I)^2 + y_I^2 = (m^{III})^2.$$

From these five equations the five unknown quantities x_I, y_I, x_{II}, y_{II}, and x_{III}—the coordinates of the points a', b', and c', which are to be located—may be computed.

From the area of the triangle $S'a'c'$,

$$\frac{y_{\rm I} \cdot x_{\rm III}}{2} = \frac{f \cdot m^{\rm III}}{2},$$

we find the focal length

$$f = \frac{y_{\rm I} \cdot x_{\rm III}}{m^{\rm III}}.$$

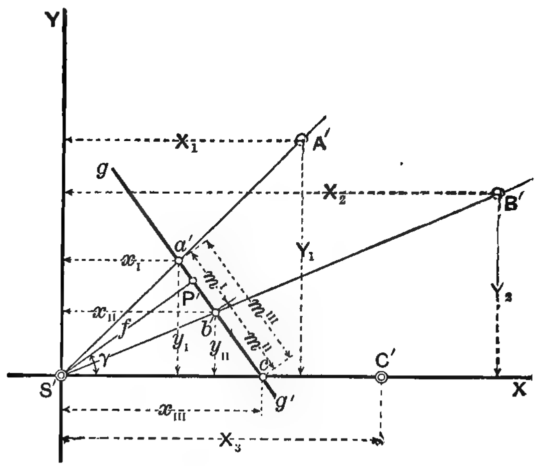


FIG. 308.

The horizontal angle of orientation γ—included between the principal ray $S'P'$ and the horizontal direction to C $(= S'C')$—may be found from the equation

$$\cos \gamma = \frac{f}{x_{\rm III}} \quad \text{or} \quad = \frac{y_{\rm I}}{m^{\rm III}}.$$

The principal point P' may now be located upon gg' from c' by making

$$P'c' = x_{\rm III} \sin \gamma.$$

The differences in elevation between the camera station S and the three triangulation points A, B, and C being known, it will now be an easy matter to draw the horizon line upon the photograph and mark the position of the principal point P on the same.

II. GRAPHICAL ICONOMETRIC METHODS.

(1) *Method of Col. A. Laussedat.*—Colonel Laussedat's methods of constructing topographic maps from perspective views of the terrene, having been widely published, form the groundwork for all subsequent work in this direction; they are chiefly of a graphical character and they are in harmony with the laws of perspective.

Laussedat considers two cases in reconnoissance surveys for geographic expeditions to which phototopographic methods may be applied with advantage:

(1) The explorer may remain sufficiently long in one locality to make a survey on a large scale, say 1 : 20 000, and even larger for special purposes.

(2) The explorer moves rapidly from place to place, gathering only the most necessary data on his itinerary to enable him to plat the topography of the traversed country as a "running survey" on a small scale—say 1 : 50 000 and even smaller—preserving and representing only the principal topographic features met with on the track survey.

In the first-mentioned case the explorer will measure one or more base lines, with as great an accuracy as the means at hand and the time at his disposal will admit. He will then cover the area to be mapped with a system of triangles, connected with (or founded upon) the base line, and, inasmuch as the triangulation stations will be occupied with the surveying camera, the scheme should be laid out with due reference to the subsequent iconometric platting of the topographic features.

When applying the ordinary surveying methods the triangulation scheme would probably be laid out with a view toward covering as large a territory as possible, occupying the least number of intervisible points. With the use of photography, however, the conditions are changed; every topographic feature that is to be platted iconometrically should be seen from two or more camera stations. The latter are to be triangulation stations, or they will have to be tied on to the general scheme by special supplementary instrumental observations. Still it is not always essential that the highest peaks, which may be included in the trigonometric survey (as concluded points), should also be occupied with the camera, as frequently other camera stations will answer the requirements just as well.

Regarding the second case, where the explorer follows a certain route, making only the most necessary (and at best but short) side

excursions, the phototopographic method is even of greater value than in the first case, particularly when traversing open and broken country. For this kind of reconnoissance it may be well claimed that the photographic method surpasses all other surveying methods regarding the amount of data which may be collected in a limited time period.

All topographic operations and instruments serve to measure vertical and horizontal angles, and a photographic perspective (of which the focal length and the positions of the horizon line and principal point are known) will give all the data needed to determine the vertical and horizontal angles of lines of direction drawn from the point of view to all points pictured on the photograph.

The points A and B, pictured on the vertical plate MN, Fig. 309, may represent the images of two distant mountain peaks; a and b will be their orthogonal projections upon the horizon line HH' (picture trace in horizontal plane HH).

$aSb = \alpha =$ horizontal angle between lines of direction from the station to the two peaks, A and B. SP (perpendicular to HH') $=$ distance line or focal length of the picture MN.

The vertical angles β and γ may be shown, in horizontal plan, by revolving the vertical planes passing through SA and SB about the lines

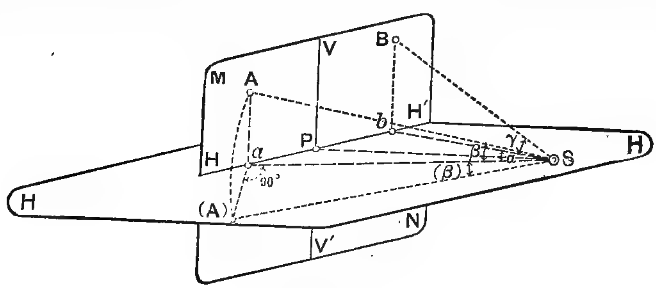


FIG. 309.

Sa and Sb, respectively, until they coincide with the horizon plane HH. This has been done in Fig. 309 for the vertical plane SaA:

$$aA = a(A) \quad \text{and} \quad (A)aS = AaS = 90^\circ,$$
$$ASa = (A)Sa = \beta.$$

The vertical angles β and γ may now be measured in horizontal plan as (β) and (γ).

To indicate the general method of iconometric platting, and to show

how the platted features of the terrene may be obtained from the photographs, we will refer to Figs. 310 and 311.

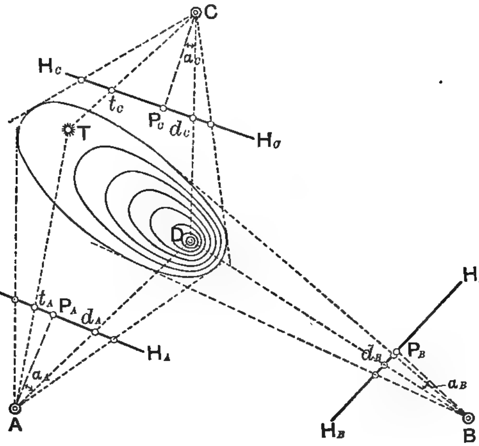


FIG. 310.

A, *B*, and *C* are three camera stations, platted in horizontal plan, whence three perspectives, I, II, and III, Fig. 311, of the same knoll *D* were obtained. The traces of these three pictures on the platting sheet, Fig. 310, may be H_AH_A, H_BH_B, H_CH_C. All three photographs may have been obtained with the same camera of constant focal length —the distance lines P_AA, P_BB, and P_CC are of equal length.

(*a*) *Locating points identified on several photographs on the platting sheet.*—The three stations *A*, *B*, and *C* are platted, either as parts of

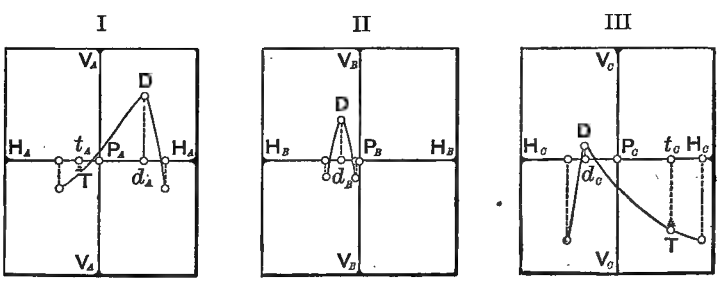


FIG. 311.

the triangulation system, or by measuring the base line *AB* on the ground and measuring the horizontal angles *CAB*, *CBA*, and *ACB*, after which the sides *AC* and *BC* may be found graphically (or by com-

putation) and the triangle ABC may now be platted upon the working plan. Horizontal angles or directions to D having also been observed from A, B, and C, its position with reference to those three points may also be platted. To plat the three picture traces HH we must know the horizontal angles PAd ($= \alpha$), which are observed in the field for each picture by means of the horizontal circle attached to the photo-theodolite.

The angles α are platted as α_A, α_B, and α_C (Fig. 310), and the constant focal length ($=f$) of the three negatives I, II, and III, Fig. 311, is laid off on the radials AP_A, BP_B, and CP_C. Perpendiculars erected to these lines in P_A, P_B, and P_C, respectively, will represent the oriented picture traces H_AH_A, H_BH_B, and H_CH_C, when the abscissæ P_Ad_A, P_Bd_B, P_Cd_C, measured on the negatives I, II, and III, should equal the lengths P_Ad_A, P_Bd_B, and P_Cd_C on the picture traces.

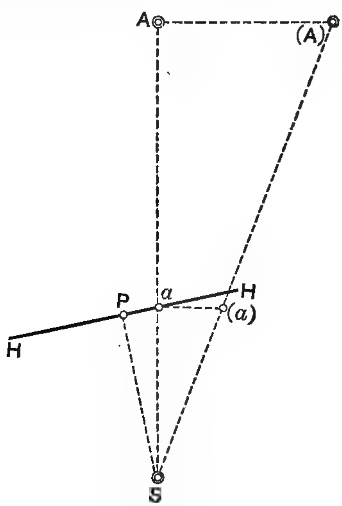


FIG. 312.

The point D is termed a "reference point." Every picture that is to be used in iconometric platting should contain the image of at least one such reference point of known position in both the horizontal and vertical sense.

After the picture traces HH have once been platted, any other point, T, of the terrene, shown on two or more photographs, may readily be platted from the photographs without requiring instrumental measurements in the field.

To locate the platted position of the point T, shown on two pictures, I and III, as t, the abscissæ, P_At_A and P^Ct_C, are laid off on the picture traces H_AH_A and H_CH_C, respectively, from P_A and P_C and on the proper side of P to correspond with the position of the image t with reference to the principal point, P, of the perspectives. Lines drawn from A and C through t_A and t_C, Fig. 310, represent the lines of horizontal directions to T, and their point of intersection locates the position of T on the plat with reference to A, B, and C.

(*b*) *Determination of the elevations of pictured points.*—The horizon line HH' of a perspective, Fig. 309, being the intersection of the vertical picture plane MN with the horizon plane (passing through the optical axis of the camera), will intersect points in the picture which in nature have the same elevation as the optical axis of the camera or as the point of view S.

The distances Sa and SA, Fig. 312, are measured on the platting sheet and the ordinate aA, Fig. 309, of the pictured point a, on the negative. Perpendiculars are then erected to SA in A and a and the latter is made equal to the ordinate of a taken from the picture $= Aa = a(a)$, Fig. 312. If we now draw the line $S(a)$ (to its intersection with the perpendicular in A), then the triangles $Sa(a)$ and $SA(A)$ will be similar and the angle $AS(A)$ will represent the vertical angle (of elevation) of the visual ray from S to A revolved about SA into the plane of the horizon or into the platting plan. From the similar triangles $Sa(a)$ and $SA(A)$ we derive the proportional equation

$$A(A) : SA = a(a) : Sa,$$

whence

$$A(A) = \frac{a(a) \,.\, SA}{sa}.$$

The value found for $A(A)$ measured on the platting scale will give the difference in elevation between camera station horizon and the point A.

In practical work the elevations of the camera stations are known, and by adding the height of the instrument including the value $A(A)$ to the elevation of S, Fig. 309, the absolute height of A will be found, which, however, is still to be corrected for curvature and refraction.

A second value for the elevation of A may be found in the same manner for another negative containing the image a (taken from another station), and the mean of several such determinations is adopted for the final value for the height of A.

(*c*) *Drawing the plan, including horizontal contours.*—After some little practice, points, pictured on different negatives but representing identical points in nature, will readily be identified by the observer, and he will soon be able to pick out the characteristic points to reproduce the watercourses, watersheds, roads, canals, etc., on the platting sheet. After these principal guide lines have been well located on the chart, buildings, outlines of woods, marshes, etc., are platted, including everything that is to be shown on the finished map.

Enough points should be platted iconometrically to give a good control for a correct delineation of the relief. When the number of points determined on the plan is sufficient, or if they are favorably located to give an adequate control only for the delineation in the horizontal sense, additional points should be platted in order to obtain an equally good control of the terrene in the vertical sense.

The planimetric work completed, elevations of as many of the platted points as seem necessary (or additional ones) are determined and inscribed on the chart. Horizontal and equidistant contours may now be drawn, by interpolation, to harmonize with the elevations suffixed on the chart to the points of control, conforming their courses (between the located points) to the configuration of the terrene, as it is shown on the photographs.

It cannot be denied that a certain amount of study and practical application are required to enable the draftsman to correctly interpret forms of the terrene, shown in perspective. Yet it should also be admitted that such translation or conversion of the relief of the terrene into the horizontal map projections may be far more accurately accomplished (at one's leisure) by means of geometrically correct perspectives than could be accomplished by sketching in the field. When topographic features are sketched, as seen from one direction, they will frequently be found to have been misconceived when they are seen again from another (not anticipated) point of view. Of course the platted forms may then be corrected in a measure, at least; still, many details are sketched which will not be seen again from other stations, and even those that are seen again under other conditions may not be modified to conform to their true shapes, unless the original station, whence they were first seen and sketched, could be reoccupied to verify the suggested changes and corrections. Generally speaking, topographers regard a second occupation of a station with little favor, it being considered too

great a waste of time, retarding progress, and considerably increasing the cost of the work.

In iconometric platting, however, it is always an easy matter to refer back again to panoramic views obtained from some other station, and the platting of topographic details should not be attempted without having first made a careful study of and a close comparison between the various pictures representing the same features but seen from different points of view.

CHAPTER IV.

PHOTOGRAMMETERS.

THE practical *value of a photogrammeter* (photographic surveying instrument) depends greatly upon the quality and general uniformity of its lens or lenses, upon the rigidity of the component parts of the apparatus, its easy transportability, and on the rapidity with which it may be put into adjustment.

A good *phototopographic lens* should be free from spherical aberration (or diffusion of the light rays); it should possess no chromatic aberration, nor should the image show distortion of any kind, and the field of view (the range of lens) should be large, rapidity of the lens being desirable, but less important than the other requirements just mentioned.

The *principal lenses in use* for phototopographic purposes are: *Dallmeyer's* rapid rectilinear, *Steinheil's* aplanat, *Bush's* pantoscopic, *Görz's* double anastigmat, and, more recently, *Zeiss's* anastigmat lens.

The nodal points, the focal length, arc of visibility, and the arc which is perfectly free from distortion of every kind should be known for every lens used for phototopographic purposes, and the manufacturers of all good lenses are best fitted to determine those values with great precision for every lens.

I. REQUIREMENTS TO BE FULFILLED BY A TOPOGRAPHIC SURVEYING CAMERA.

A good surveying camera or photogrammeter for topographic work should produce negatives which are geometrically true perspectives the elements of which should be known, and the following desiderata should be fulfilled:

First. The plates to be exposed should be adjustable into vertical plane.

Second. The distance between image point and sensitive plate should be maintained unchanged for all plates.

Third. This distance—the constant focal length—should be known or will have to be determined for every instrument.

Fourth. Means should be provided to trace or locate the horizon line upon every negative or print.

Fifth. Means should be provided for locating the principal point upon every negative.

Sixth. A ready orientation of the photographs (the picture traces) for iconometric platting should be provided for; and we may add as

Seventh. Enough characteristic stations (besides the triangulation points needed for the instrumental control) are to be occupied with the surveying camera to give a full development of the terrene which is to be mapped.

Until recently photographic surveying instruments were not procurable in open market. Nearly every observer who made practical application of the photographic methods for topographic surveys had an apparatus constructed for his particular need and according to his individual ideas.

In the following we will describe such photogrammeters as may be regarded as special types, constructed to fulfill different requirements.

II. ORDINARY CAMERAS ADAPTED FOR SURVEYING PURPOSES.

These cameras are generally supported by three leveling screws, and they are provided with a circular level, or with two cross levels, for adjusting the sensitive plate into vertical plane. The distance between lens and sensitive plate (focal distance) may be made invariable by means of two rods *Sp*, Fig. 313 (*Werner's* apparatus, made by R. Lechner, of Vienna, in Austria), or by means of two arms *H* and clamp screw *M*, after the bellows had been extended by aid of the pinion *K* and rack movement to that point indicated by the vernier *n*, Fig. 314, as the proper focal length for infinite distance. The arrangement shown in Fig. 314 represents the apparatus of Dr. *Vogel* and Professor *Doergens*, made by Stegemann, of Berlin, in Prussia.

Dr. *G. Le Bon* also used a similarly modified camera for his archæological researches in India (undertaken under the auspices of the French ministry of culture).

Short brass points *M*, Fig. 315, serve to locate the horizon and

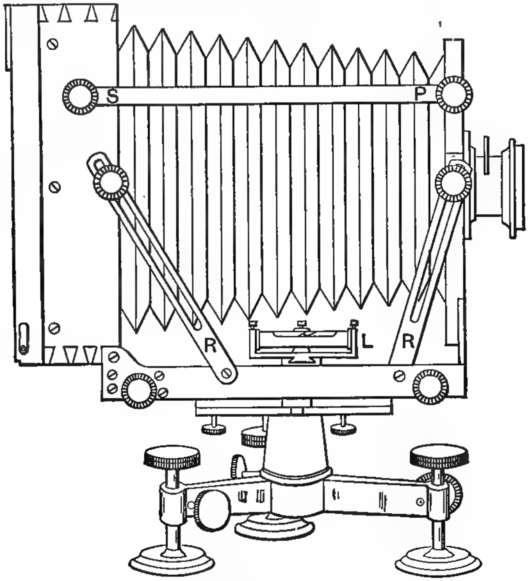


FIG. 313.

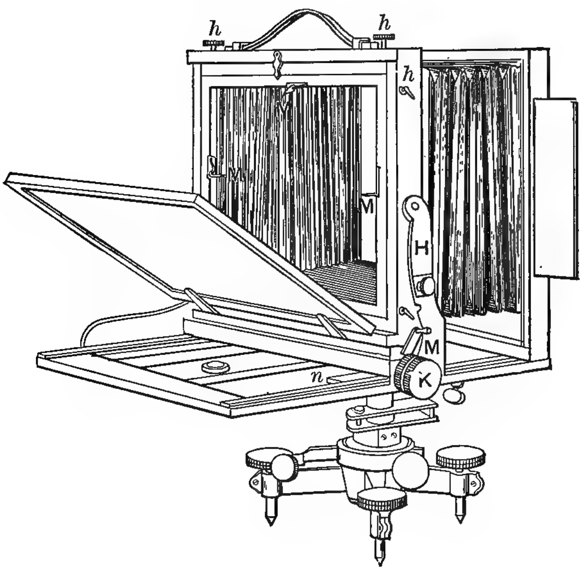


FIG. 314.

principal lines on the negatives by protecting the sensitive plates against the action of those light rays which they intercept. In some instances those points M may be brought into direct contact with the sensitive

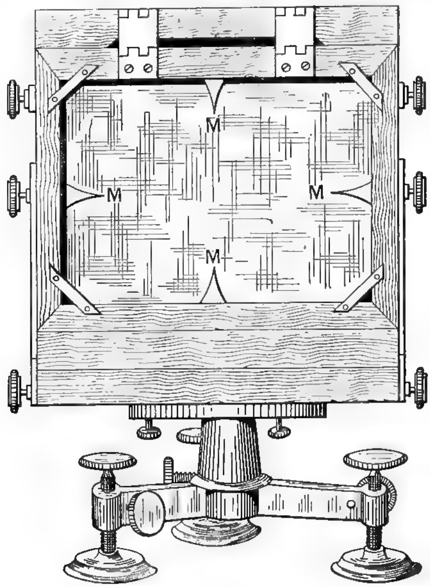


FIG. 315.

film surface of the plate by turning a button, thus producing a sharp, well-defined image of the outlines of the teeth on the negative.

The use of such modified cameras should not be extended beyond preliminary work; for extensive use the results will not be sufficiently uniform and accurate.

III. SURVEYING CAMERAS COMBINED WITH GEODETIC INSTRUMENTS.

(Phototheodolites, Photographic Plane Tables, etc.)

The data acquired in the field with photogrammeters of the class just described had to be supplemented with observations made in the field with some geodetic instrument (transit, plane table, etc.) in order to obtain complete topographic surveys of the regions traversed by the phototopographic surveying party.

The idea of combining surveying instruments with a photographic camera into single compact and serviceable instruments originated very

early with phototopographic workers, and refined phototheodolites and photographic plane tables are to this day the favorite phototopographic instruments in Europe, whence they are also exported to other countries.

These more or less complicated instruments have been devised to secure great precision in the work undertaken with them, and refined methods are employed for the field observations, for the culling of data from the photographic perspectives, and for the computations made in the office to increase the general precision of data derived from the operations executed in the field.

Generally speaking, the best results for topographic purposes are obtained by means of photography, if we bear in mind that phototopography essentially and primarily is a constructive and graphic art, based upon graphic or pictorial records (which are nothing more than central projections in vertical plan of objects and their dimensions, that are to be transposed graphically into orthogonal projections into horizontal plan). Instrumental observations being required only to furnish such elements as may be needed to make the graphic transpositions (iconometric platting in a reduced scale) of the lines of directions and distances, and also to obtain checks or a proper control for the work in its entirety.

Photographic surveys have been conducted principally in regions where other surveying methods are either precluded or where their application would entail great cost and consume too much time, and such regions are characterized chiefly by a rugged and broken topography.

The necessity, therefore, lies close at hand to devise instruments that will not readily get out of adjustment or drop to pieces when transported over rugged mountain trails, and the more simplified their structural composition the more available will they become for the production of rapid and accurate work.

It is at once evident that the combination of a camera and a surveying instrument into a well-united, well-balanced, easily manipulated, and essentially light and withal rigid instrument is not easily accomplished. It is not surprising, therefore, when searching the published descriptions of phototheodolites and other photogrammeters, to come upon a great number of types in which the many difficulties have been overcome, more or less successfully, by various devices.

We may find: A large-sized theodolite with a small camera, placed

centrally between the Y supports, after removal of the telescope from the latter, both being interchangeable;

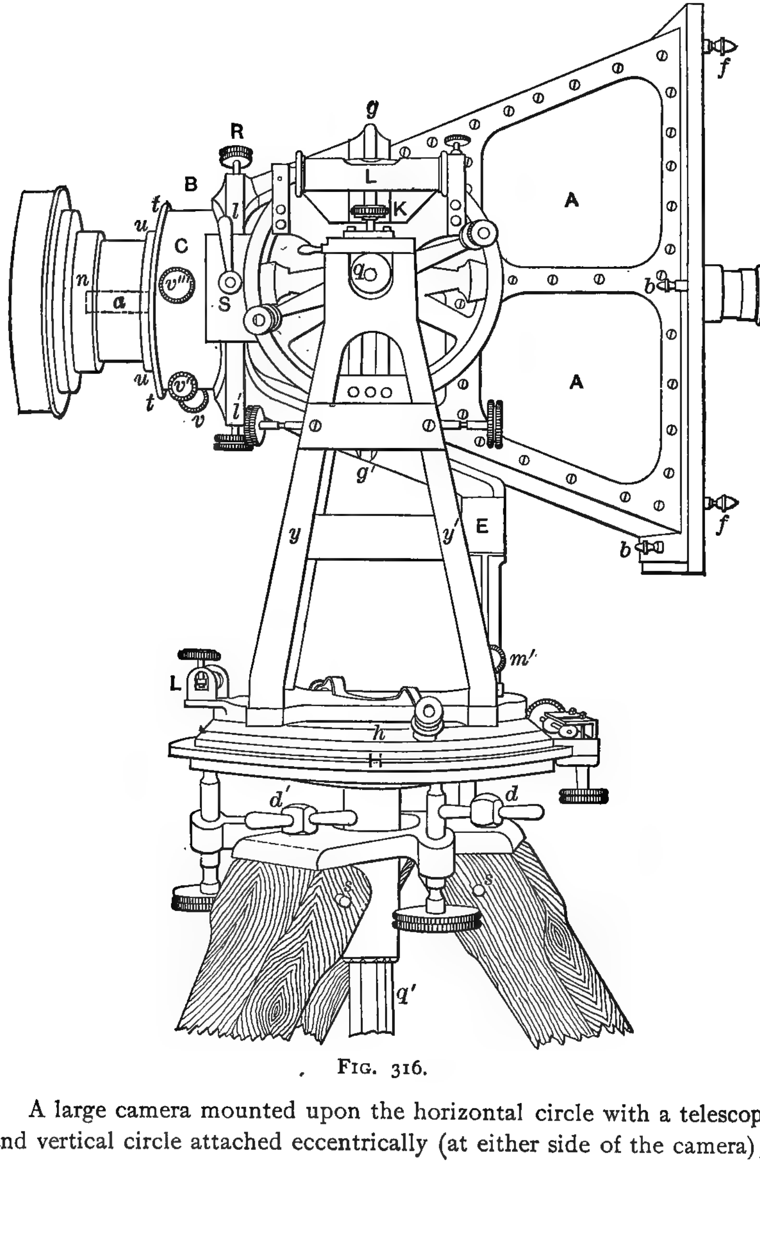


FIG. 316.

A large camera mounted upon the horizontal circle with a telescope and vertical circle attached eccentrically (at either side of the camera);

A large centrally located camera, the lens of which serves at the same time as objective of the telescope, the corresponding eyepiece being at the center of the frame that ordinarily supports the ground-glass plate (in this form the camera itself is the telescope);

Instruments where the board of the plane table has been replaced by a surveying camera, the upper face of which receives and supports the plane-table sheet and plane-table alidade; also various other combinations (some with compass attachments).

This class of instruments has been in use for large-scale surveys and where the instrumental outfit could readily be brought very near the stations to be occupied by convenient means of transportation, the instruments rarely being subjected to such primitive and rough methods of transportation over long distances, as generally has been the case on our continent when surveying cameras have been used.

(1) *The new Italian phototheodolite, devised by L. P. Paganini.*—Paganini's model of 1884 has been described in Appendix No. 3, United States Coast and Geodetic Survey Report for 1893.

The following description of Paganini's new phototheodolite, model of 1890, has been extracted from L. P. Paganini's "Nuovi appunti di fototopografia," Roma, 1894:

The general form and the dimensions of the camera box of Paganini's new phototheodolite remain about the same as with the older model, the principal change resting in the omission of the eccentric telescope which has been replaced by the centrally mounted camera, which may, at will of the observer, be converted into a telescope.

The telescopes which we generally find attached to surveying instruments consist of a tube, slightly conical in shape, having a positive lens or a system of convergent lenses at one end (the "objective") which produce within the telescope a real and inverted image—the same as the camera lens—of any object toward which the lens may be directed. The other, smaller end of the telescope tube has a still smaller tube inserted into it which may be moved in the direction of the axis of the tube. This second tube also contains a system of convergent lenses—so-called "ocular lens" or "eyepiece" of the telescope—which serve to project an enlargement of the image in the telescope upon the retina of the observer's eye. In the image plane of the objective (within the telescope) is the so-called diaphragm—a ring-shaped metal disk—to one side of which a pair of cross hairs—spider webs, cocoon threads, or lines cut into a thin piece of plate glass—is attached in such a way that the

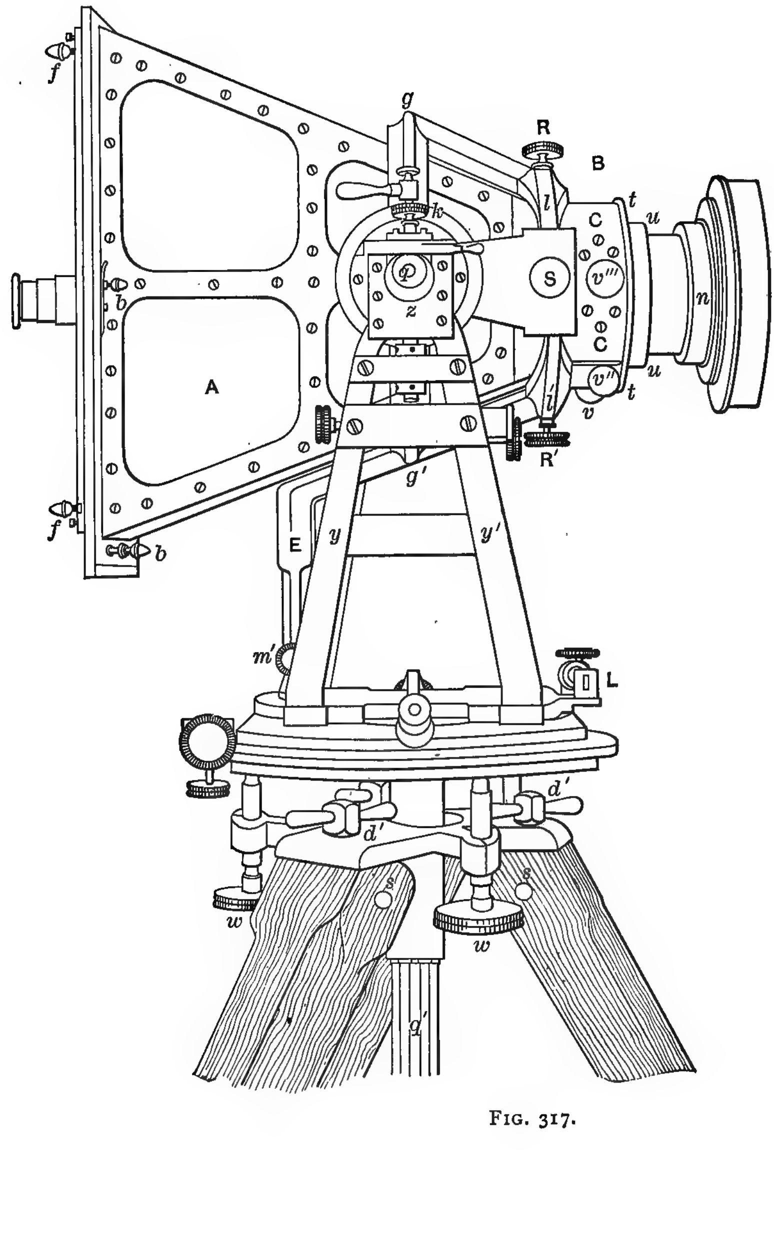


FIG. 317.

hairs fall within the image plane. One hair is vertical and the other horizontal, their point of intersection coinciding with the optical axis of the telescope.

The old camera was provided with the objective, and a corresponding eyepiece had only to be added to convert the camera into a surveying telescope. In the instrument under consideration the eyepiece consists of a positive lens set, known in optics as "Ramsden's ocular lens." The inner wall surfaces of the camera box should be well blackened to avoid any side reflection and a consequent dimness in the appearance of the cross wires.

The camera proper consists of two parts, a truncated pyramid *A*, Figs. 316 to 320, and a cylindrical attachment *B*, into which the tube *t* is inserted. A second tube within the cylinder *t* may be moved in the direction of the optical axis by means of a screw, the threads of which have a rise of one millimeter. By revolving the inner tube the lens is brought nearer to or farther from the image plane, the lens remaining parallel with the image plane at any position that may thus be given to the lens.

A scale *a*, Figs. 316 and 320, graduated to millimeters, is permanently attached to the tube *t* and it lies very close to the ring *n*, the circumference of which is divided into ten equal parts. (This graduated ring *n* is soldered upon the cylinder *u* containing the camera lens.) This scale *a* (extending in a direction parallel to the optical axis of the lens) has a mark, coinciding with the index rim of the ring *n*, thus indicating the focal length of the camera lens when focused upon objects at infinite distance. The millimeter graduation of the scale *a*, extending from the zero mark in the direction toward the ground glass, serves to ascertain the focal lengths for objects nearer the camera station. The circumferential graduation on the ring *n* serves to read one tenth of one revolution of the tube *u*, which is equal to an axial motion of the lens of 0.1 millimeter, hence the focal length for any object focused upon may be read to single millimeters on the scale *a* and to tenths of a millimeter on the graduated ring *n*.

The construction of this phototheodolite is such that the optical axis of the camera lens is always at right angles to the picture plane—the ground-glass surface or the sensitive film of the photographic plate. The intersection of the optical axis and the picture plane, the principal point, is marked by the intersection *P*, Fig. 319, of the two very fine platinum wires *OO'* and *ff'*, one horizontal and the other vertical when

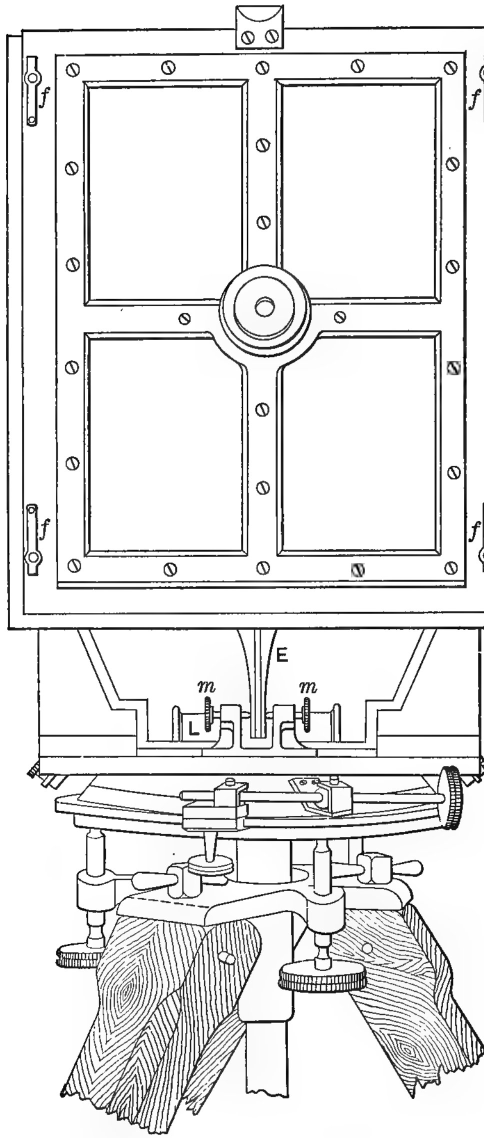


FIG. 318.

the instrument is in adjustment. These wires are stretched across the back of the camera box as close as possible to the picture plane. The buttons *b*, Figs. 316 and 317, serve to give tension to the wires. The wire OO' corresponds to the horizon line and the vertical wire ff' corresponds to the principal line of the perspective represented by the image on the ground-glass plate.

Fig. 318 shows the rear view of this instrument, the ground glass having been replaced by an opaque plate, strengthened by a metal frame and ribs, which supports the Ramsden eyepiece in the center, its optical axis coinciding with that of the camera lens. The cross wires OO', ff', at the rear of the camera, serve also for the astronomical telescope into which the camera may be converted by attaching the opaque plate with central eyepiece as shown in Fig. 318. The fitting of this eyepiece allows for axial motion to adjust its position to avoid parallax.

The rear opaque plate and the other sides of the camera box are made of cardboard (impregnated with chemicals to render it impervious to moisture), and they are stiffened by frames and ribs of metal as illustrated in Figs. 316 and 318.

The cylindrical part *B*, Figs. 316, 317, and 319, is inclosed by a solid metal collar *C*, which is held in position within the metal ring ll'

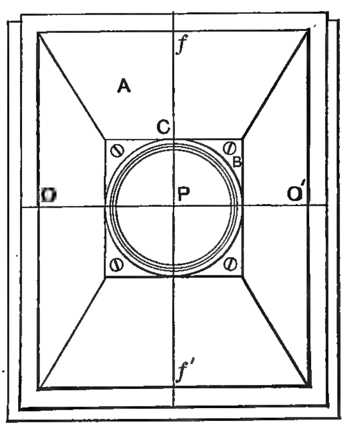


FIG. 319.

by four screws *R*, R', *S*, S'. This ring ll' is connected with the frame gg' by means of two arms lg and $l'g'$, all being cast into one piece. The frame gg' has pivots *q* attached to it which form the horizontal axis of rotation for the camera.

This instrument is provided with a vertical circle, Fig. 316, horizontal circle *H*, Figs. 316, 317, and 320, verniers, reading microscopes, levels *L*, Figs. 316 and 318, clamps, and slow-motion screws, forming a complete transit with centrally mounted "camera-telescope."

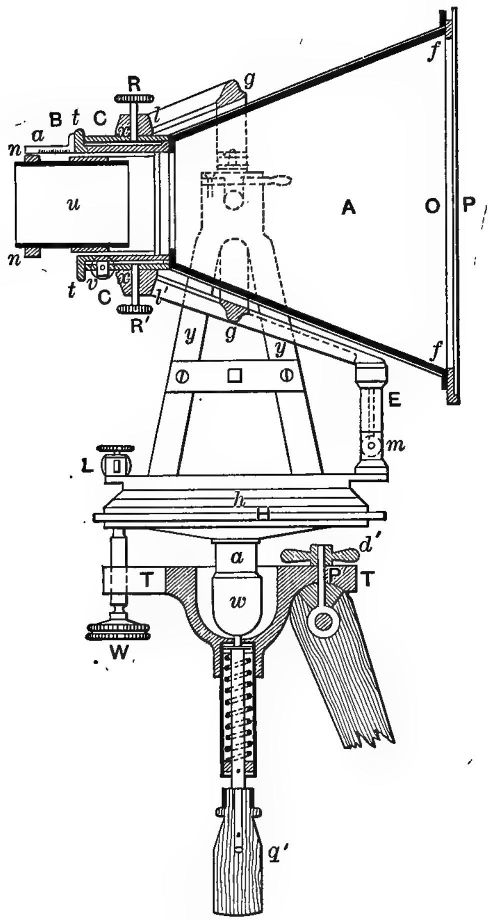


FIG. 320.

Fig. 320 represents a vertical section of this instrument. The scale *a*, already described, is here placed on top of the tube *u* to illustrate its function better. *yy* = uprights, supporting the horizontal axis of rotation of the "camera-telescope." *h* = alidade supplied with verniers.

H = lower limb or horizontal circle bearing the graduation; it is supported on the tripod head T by three leveling screws W. a = casing for conical centre. q' = central clamp-screw, firmly uniting T and H (it guards against an accidental falling off of the instrument from the tripod); it screws into a ball which is supported by the hemispherical socket w of the lower part of a.

The horizontal circle has a diameter of 10.5 centimeters. It is graduated into thirty minutes, and its verniers read to single minutes.

The photographic plates are 18 by 24 centimeters, the same size as for the 1884 model camera.

The objective lens was an aplanat of Steinheil of 237.7 millimeters focal length. More recently, however, the Italian phototheodolites have been provided with anastigmats of Zeiss.

The column E, Figs. 316, 317, 318, and 320, forming a prolongation of the lower arm $l'g'$, is held in place by two counter-screws m and m', Fig. 318, which serve to hold the horizontal axis of rotation of the camera in a fixed position, avoiding accidental changes during the execution of a set of panorama pictures.

After unscrewing the nuts d', Fig. 320, the tripod legs may be removed. They serve as "alpenstocks" when the instrument is being transported from station to station. The camera-telescope is lifted out of the wyes and packed in a separate case; the lower part of the instrument is packed in another case, and the plate holders and plates are transported in a third case.

TABLES.

TABLE I.*

ASTRONOMICAL MEAN TIMES† OF THE ELONGATIONS AND CULMINATIONS OF POLARIS IN THE YEAR 1901, WITH PRECEPTS FOR THEIR DETERMINATION IN SUBSEQUENT YEARS.

(Computed for latitude + 40° and longitude 6 h. west from Greenwich.)

Date.	Elongation.	Time.† h.	Time.† m.	Time of Upper Culmination.† h.	Time of Upper Culmination.† m.	Diff. for 1 Day. m.
1901. Jan. 1	W.	12	34.7	6	39.7	
" 15	W.	11	39.4	5	44.4	3.95
Feb. 1	W.	10	32 3	4	37.3	3.95
" 15	W.	9	37.0	3	42.1	3.95
Mch. 1	W.	8	41.8	2	46.8	3.94
" 15	W.	7	46.6	1	51.7	3.94
April 1	W.	6	39.7	0	44.7	3.94
" 15	W.	5	44.6	23	45.8	3.93
	E.	17	50.8			3.93
May 1	E.	16	48.0	22	43.0	
" 15	E.	15	53.1	21	48.0	3.92
June 1	E.	14	46.5	20	41.4	3.92
" 15	E.	13	51.7	19	46.6	3.92
July 1	E.	12	49.0	18	44.0	3.91
" 15	E.	11	54.2	17	49.2	3.91
Aug. 1	E.	10	47.7	16	42.6	3 92
" 15	E.	9	52.8	15	47.8	3.92
Sep. 1	E.	8	46.2	14	41.2	3.92
" 15	E.	7	51.3	13	46.3	3.92
Oct. 1	E.	6	48.5	12	43.5	3.92
" 15	E.	5	53.5	11	48.5	3.93
	W.	17	43.4			3.93
Nov. 1	W.	16	36.6	10	41.6	
" 15	W.	15	41.5	9	46.5	3.94
Dec. 1	W.	14	38.5	8	43.5	3.94
" 15	W.	13	43.2	7	48.3	3.94
1902. Jan. 1	W.	12	36.2	6	41.2	3.95

Lower culmination occurs 11 h. 58.1 m. before or after upper culmination.

* Arranged from the Manual of Surveying Instructions of 1894.

† The astronomical mean time is reckoned from *noon* up to 24 hours. Thus, the time of western elongation on Jan. 1 is given as 12 h. 34.7 m., indicating that it occurs 12 h. 34.7 m. after noon on Jan. 1, or at 12 h. 34.7 m. A.M. of Jan. 2.

(*a*) To obtain the times for the year 1902 add $1^m.5$ to the tabular values; for 1903 add $2^m.9$; for 1904 add $4^m.4$ before March 1 and $0^m.4$ after March 1; for 1905 add $1^m.9$; for 1906 add $3^m.3$; for 1907 add $4^m.8$; for 1908 add $6^m.2$ before March 1 and $2^m.3$ after March 1; for 1909 add $3^m.7$; for 1910 add $5^m.2$; for 1911 add $6^m.6$.

(*b*) To refer to any calendar day other than the 1st or the 15th of each month, subtract the "Difference for 1 day" for every day between it and the preceding tabular day, or add the "Difference for 1 day" for every day between it and the succeeding tabular day. The interval between two consecutive upper culminations equals 24 h. less the "Difference for 1 day," and hence there may be two upper culminations on the same day,—somewhere between April 1 and April 15,—in 1901 on April 12 at 0 h. 1.5 m. and 23 h. 57.6 m. respectively.

(c) To refer to any latitude between 25° and 50° north other than 40°, *add* to the time of *west* elongation $0^m.12$ for every degree *south* of 40°, and *subtract* from the time of *west* elongation $0^m.17$ for every degree *north* of 40°; reverse these signs for the corrections to the times of east elongation. The time of culmination is independent of the latitude.

(d) To refer to any longitude other than 6^h west, subtract $0^m.16$ for each hour west of the 6^h meridian, and add $0^m.16$ for each hour east.

(e) If the "standard time" is desired, find the difference between the longitude expressed in degrees, and the *nearest* multiple of 15; multiply this difference by 4 to get a correction expressed in minutes of time, and add this result to the local time if the west longitude exceeds the nearest multiple of 15, and subtract it if the longitude is less.

Example.—What will be the standard time of the visible elongation at Salem, Oregon, on the night of June 12-13, 1904?

Using the best map available, the latitude and longitude of Salem are found to be, say, 44°.9 N. and 123°.1 or $8^h.21$ W. respectively (15° correspond to 1 hour).

From the table we find that *eastern* elongation takes place on

June 15, 1901, at..................................	13^h	$51^m.7$
By precept (a), the correction to 1904 is.............		$+\ 0\ .4$
	13	52 .1
By precept (b): $15-12=3:3\times 3.92$ is..............		$+\ 11\ .8$
	14	3 .9
By precept (c): $44.9-40=4.9:4.9\times 0.17$ is.........		$+\ 0\ .8$
	14	4 .7
By precept (d): $8.2-6\doteq 2.2:2.2\times 0.16$ is		$-\ 0\ .4$
Local time of elongation is......................	14	4 .3
By precept (e): $123^\circ.1-120^\circ=3^\circ.1:3.1\times 4$ is.......		$+\ 12\ .4$
	14	16 .7

Hence the standard time at which the eastern elongation occurs is $14^h\ 16^m.7$ after noon on June 12: this is what we call, in common life, $2^h\ 16^m.7$ A.M. on June 13,—the answer desired.

TABLE II.

The mean polar distance of Polaris for Jan. 1, 1900, is

$$1^\circ\ 13'.55.$$

The mean polar distance for any other date may be found by subtracting $0'.311$ multiplied by the time expressed in years that has elapsed since that date.

Example.—Find the mean polar distance of Polaris for July 1, 1903.

$$1^\circ\ 13'.55 - 0'.311\ .\ 3.5 = 1^\circ\ 12'.46.$$

TABLE III.

REFRACTION CORRECTION TO BE SUBTRACTED FROM THE OBSERVED ALTITUDE OF POLARIS.

Alt..............	20°	25°	30°	35°	40°	50°	60°
Ref..............	2'.59	2'.03	1'.64	1'.36	1'.13	0'.80	0'.55

TABLE IV.*

AZIMUTHS OF POLARIS WHEN AT ELONGATION FOR ANY YEAR BETWEEN 1900 AND 1910 AND FOR ANY LATITUDE BETWEEN 25° AND 72° NORTH.

Lat.	1900.	1901.	1902.	1903.	1904.	1905.	1906.	1907.	1908.	1909.	1910.
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
25	1 21.2	1 20.8	1 20.5	1 20.1	1 19.8	1 19.4	1 19.1	1 18.7	1 18.4	1 18.1	1 17.7
26	21.8	21.5	21.1	20.8	20.5	20.1	19.8	19.4	19.1	18.7	18.4
27	22.5	22.2	21.9	21.5	21.2	20.8	20.5	20.1	19.8	19.4	19.1
28	23.3	23.0	22.6	22.2	21.9	21.6	21.3	20.9	20.5	20.1	19.8
29	24.1	23.8	23.4	23.0	22.7	22.4	22.1	21.7	21.3	20.9	20.5
30	1 24.9	1 24.6	1 24.2	1 23.9	1 23.5	1 23.1	1 22.8	1 22.4	1 22.1	1 21.7	1 21.3
31	25.8	25.5	25.1	24.7	24.4	24.0	23.6	23.2	22.9	22.5	22.2
32	26.7	26.4	26.0	25.6	25.3	24.9	24.5	24.1	23.8	23.4	23.1
33	27.7	27.3	27.0	26.6	26.2	25.9	25.5	25.1	24.7	24.3	24.0
34	28.7	28.4	28.0	27.6	27.2	26.9	26.5	26.1	25.7	25.3	25.0
35	1 29.8	1 29.4	1 29.0	1 28.7	1 28.3	1 27.9	1 27.5	1 27.1	1 26.8	1 26.4	1 26.0
36	30.9	30.5	30.1	29.8	29.4	29.0	28.6	28.2	27.9	27.5	27.1
37	32.1	31.7	31.3	30.9	30.5	30.1	29.7	29.3	29.0	28.6	28.2
38	33.4	33.0	32.6	32.2	31.8	31.4	31.0	30.6	30.2	29.8	29.4
39	34.7	34.3	33.9	33.5	33.1	32.7	32.3	31.8	31.4	31.0	30.6
40	1 36.0	1 35.6	1 35.2	1 34.8	1 34.4	1 34.0	1 33.6	1 33.2	1 32.8	1 32.4	1 32.0
41	37.5	37.1	36.7	36.2	35.8	35.4	35.0	34.6	34.2	33.8	33.4
42	39.0	38.6	38.2	37.7	37.3	36.9	36.5	36.0	35.6	35.2	34.8
43	40.6	40.2	39.8	39.3	38.9	38.5	38.1	37.6	37.2	36.8	36.3
44	42.3	41.8	41.4	41.0	40.5	40.1	39.7	39.2	38.8	38.4	37.9
45	1 44.0	1 43.6	1 43.2	1 42.7	1 42.3	1 41.8	1 41.4	1 40.9	1 40.5	1 40.1	1 39.6
46	45.9	45.5	45.0	44.6	44.2	43.7	43.2	42.7	42.3	41.9	41.4
47	47.9	47.4	46.9	46.5	46.0	45.6	45.1	44.6	44.2	43.7	43.3
48	49.9	49.5	49.0	48.6	48.1	47.7	47.2	46.7	46.3	45.8	45.3
49	52.1	51.7	51.2	50.7	50.2	49.8	49.3	48.8	48.4	47.9	47.4
50	1 54.4	1 54.0	1 53.5	1 53.0	1 52.5	1 52.0	1 51.5	1 51.0	1 50.6	1 50.1	1 49.6
51	56.9	56.4	55.9	55.4	54.9	54.4	54.0	53.5	53.0	52.5	52.0
52	59.5	59.0	58.5	58.0	57.5	57.0	56.4	55.9	55.4	54.9	54.4
53	2 02.2	2 01.7	2 01.2	2 00.7	2 00.2	59.6	59.1	58.6	58.1	57.6	57.1
54	05.1	04.6	04.1	03.5	03.0	2 02.5	2 02.0	2 01.5	2 00.9	2 00.4	59.9
55	2 08.3	2 07.8	2 07.2	2 06.6	2 06.1	2 05.6	2 05.0	2 04.4	2 03.9	2 03.4	2 02.8
56	11.6	11.0	10.5	09.9	09.4	08.8	08.2	07.7	07.1	06.6	06.0
57	15.1	14.5	14.0	13.4	12.8	12.2	11.7	11.1	10.5	10.0	09.4
58	18.8	18.2	17.6	17.1	16.5	15.9	15.3	14.7	14.2	13.6	13.0
59	22.8	22.2	21.6	21.0	20.4	19.8	19.2	18.6	18.0	17.4	16.8
60	2 27.1	2 26.5	2 25.9	2 25.2	2 24.6	2 24.0	2 23.4	2 22.8	2 22.1	2 21.5	2 20.9
61	31.7	31.1	30.4	29.8	29.1	28.5	27.9	27.2	26.6	25.9	25.3
62	36.7	36.0	35.4	34.7	34.1	33.4	32.7	32.1	31.4	30.8	30.1
63	42.1	41.4	40.7	40.0	39.3	38.6	38.0	37.3	36.6	35.9	35.2
64	47.8	47.1	46.4	45.7	45.0	44.3	43.6	42.9	42.2	41.5	40.8
65	2 54.1	2 53.4	2 52.6	2 51.9	2 51.2	2 50.4	2 49.7	2 49.0	2 48.3	2 47.5	2 46.8
66	3 00.9	3 00.1	59.4	58.6	57.9	57.1	56.3	55.6	54.8	54.1	53.3
67	08.3	07.5	3 06.7	3 05.9	3 05.1	3 04.4	3 03.6	3 02.8	3 02.0	3 01.2	3 00.4
68	16.4	15.6	14.8	13.9	13.1	12.3	11.5	10.7	09.8	09.0	08.2
69	25.3	24.4	23.6	22.7	21.9	21.0	20.1	19.3	18.4	17.6	16.7
70	3 35.2	3 34.3	3 33.4	3 32.5	3 31.6	3 30.6	3 29.7	3 28.8	3 27.9	3 27.0	3 26.1
71	46.1	45.1	44.2	43.2	42.3	41.3	40.3	39.4	38.4	37.5	36.5
72	58.2	57.2	56.2	55.2	54.2	53.2	52.1	51.1	50.1	49.1	48.1

* From the Manual of 1894, issued by the General Land Office.

TABLE V.*

CORRECTION OF AZIMUTHS FROM POLARIS OBSERVATIONS FOR EACH MONTH.

Table IV was computed with the mean place (declination) of Polaris for each year. A closer result will be had by applying to the tabular results the following correction, which depends upon the difference of the mean and apparent declinations of the star. The tabular azimuth thus corrected may generally be depended upon with no greater error than $\pm$ 0′.2. except for high latitude, where it must be somewhat increased,

For Middle of —	Latitude.				For Middle of —	Latitude.			
	25°	40°	55°	70°		25°	40°	55°	70°
	′	′	′	′		′	′	′	′
January...	− 0.3	− 0.4	− 0.5	− 0.9	July......	+ 0.2	+ 0.3	+ 0.4	+ 0.6
February..	− 0.3	− 0.3	− 0.4	− 0.7	August...	+ 0.1	+ 0.1	+ 0.2	+ 0.3
March.....	− 0.1	− 0.2	− 0.2	− 0.4	September	0.0	− 0.1	− 0.1	− 0.1
April......	0.0	0.0	0.0	0.0	October...	− 0.2	− 0.3	− 0.4	− 0.6
May.......	+ 0.2	+ 0.2	+ 0.2	+ 0.4	November	− 0.5	− 0.6	− 0.7	− 1.1
June.......	+ 0.2	+ 0.3	+ 0.4	+ 0.6	December.	− 0.6	− 0.8	− 0.9	− 1.5

* From the Manual of 1894.

Table VI gives in Part III azimuths of Polaris for various times during the years 1901–1911. The argument is the time that has elapsed since the last upper culmination of the star, the times of upper culmination being determined from Parts I and II. Thus the time of upper culminat on on Dec. 15, 1901, is from Part I, 7 hrs. 48.1 m. Supposing an observation to be made at 9 hrs. 52.1 m. on the same date, the argument for entrance to Part III would be 9 hrs. 52.1 m. — 7 hrs. 48.1 m. = 2 hrs. 4 m., and, on reference to Part III, the azimuth opposite this argument and for latitude, say 42°, is found to be 0° 51′. If the time elapsed since upper culmination is greater than 11 hrs. 58 m., the time argument for entrance into Table VI is 23 hrs. 56.1 m. — the time elapsed. To determine the meridian the azimuth is to be laid off to the east or the west of the star as the time elapsed is less or greater than 11 hrs. 58 m.

TABLE VI.*—PART I.†—LOCAL MEAN (ASTRONOMICAL) TIME OF THE UPPER CULMINATION OF POLARIS, COMPUTED FOR LONGITUDE 108° (7H. 12M.) WEST OF GREENWICH.

[The time on line with any date in Part I is the hours and minutes elapsed (common watch time) since the preceding noon.]

PART I.

Date.	1901.		1902.		1903.		1904.		1905.		1906.		1907.		1908.		Diff. 1 Day.
	h.	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*	*m.*
Jan. 1	6	39.5	6	41.0	6	42 4	6	43.9	6	41.4	6	42.8	6	44.3	6	45.7	3.95
15	5	44.2	5	45.7	5	47.1	5	48.6	5	46.1	5	47.5	5	49.0	5	50.4	3.95
Feb. 1	4	37.1	4	38.6	4	40.0	4	41.5	4	39.0	4	40.4	4	41.9	4	43.3	3.95
15	3	41.9	3	43.4	3	44.8	3	46.3	3	43.8	3	45.2	3	46.7	3	48.1	3.95
Mar. 1	2	46.0	2	48.1	2	49.5	2	47.0	2	48.5	2	49.9	2	51.4	2	48.9	3.94
15	1	51.5	1	53.0	1	54.4	1	51.9	1	53.4	1	54.8	1	56.3	1	53.8	3.94
Apr. 1	0	44.6	0	46 1	0	47.5	0	45.0	0	46.5	0	47.9	0	49 4	0	46.8	3.94
15	23	45.6	23	47.1	23	48.5	23	46.0	23	47.5	23	48.9	23	50.4	23	47.8	3.93
May 1	22	42.8	22	44.3	22	45.7	22	43.2	22	44 7	22	46.1	22	47.6	22	45.1	3.93
15	21	47.8	21	49.3	21	50.7	21	48.2	21	49.7	21	51.1	21	52.6	21	50.1	3.92
June 1	20	41.2	20	42.7	20	44.1	20	41.6	20	43.1	20	44.5	20	46.0	20	43 5	3.92
15	19	46.4	19	47.9	19	49.3	19	46.8	19	48.3	19	49.7	19	51.2	19	48.7	3.91
July 1	18	43.8	18	45.3	18	46.7	18	44.2	18	45.7	18	47.1	18	48.6	18	46.1	3.91
15	17	49.0	17	50.1	17	51.9	17	49.4	17	50.9	17	52.3	17	53.8	17	51.3	3.92
Aug. 1	16	42.4	16	43 9	16	45.3	16	42.8	16	44.3	16	45 7	16	47 2	16	44.7	3.92
15	15	47.6	15	49.1	15	50.5	15	48.0	15	49.5	15	50 9	15	52.4	15	49.9	3.92
Sept. 1	14	41.0	14	42.5	14	43.9	14	41.4	14	42.9	14	44.3	14	45.8	14	43.3	3.92
15	13	46.1	13	47.6	13	49.0	13	46.5	13	48.0	13	49.4	13	50.9	13	48.4	3.93
Oct. 1	12	43.3	12	44.8	12	46.2	12	43 7	12	45.2	12	46.6	12	48.1	12	45.6	3.93
15	11	48.3	11	49.8	11	51.2	11	48.7	11	50.2	11	51.6	11	53.1	11	50.6	3.93
Nov. 1	10	41.4	10	42.9	10	44.3	10	41.8	10	43.3	10	44.7	10	46.2	10	43.7	3.93
15	9	46 4	9	47.9	9	49.3	9	46.8	9	48.3	9	49.7	9	51.2	9	48.7	3.94
Dec. 1	8	43.3	8	44.8	8	46.2	8	43 7	8	45.2	8	46.6	8	48 1	8	45.6	3.94
15	7	48.1	7	49.6	7	51.0	7	48.5	7	50.0	7	51.4	7	52 9	7	50.4	3.95

PART I.—*Continued.*

Date.	1909.		1910.		1911.		Diff. for 1 Day.
	h.	*m.*	*h.*	*m.*	*h.*	*m.*	*m.*
Jan. 1	6	43.2	6	44.7	6	46.1	3.95
15	5	47.9	5	49.4	5	50.8	3.95
Feb. 1	4	40.8	4	42.3	4	43.7	3.95
15	3	45.6	3	47.1	3	48.5	3.95
Mar. 1	2	50.3	2	51.8	2	53.2	3.94
15	1	55.2	1	56.7	1	58.1	3.94
Apr. 1	0	48.3	0	49.8	0	51.2	3.94
15	23	49.3	23	50.8	23	52.2	3 93
May 1	22	46.5	22	48.0	22	49.4	3.93
15	21	51.5	21	53.0	21	54.4	3.92
June 1	20	44.9	20	46.4	20	47.8	3.92
15	19	50.1	19	51.6	19	53.0	3.91
July 1	18	47.5	18	49.0	18	50.4	3.91
15	17	52.7	17	54.2	17	55.6	3.92
Aug. 1	16	46.1	16	47.6	16	49.0	3.92
15	15	51.3	15	52.8	15	54.2	3.92
Sept. 1	14	44.7	14	46.2	14	47.6	3.92
15	13	49.8	13	51.3	13	52.7	3.93
Oct. 1	12	47.0	12	48.5	12	49.9	3.93
15	11	52.0	11	53.5	11	54.9	3.93
Nov. 1	10	45.1	10	46.6	10	48.0	3.93
15	9	50.1	9	51.6	9	53.0	3.94
Dec. 1	8	47.0	8	48.5	8	49.9	3.94
15	7	51.8	7	53.3	7	54.7	3.95

PART II.

Reduction of tabular times to *intermediate dates.* Subtract the reduction when computing from a *preceding*, or add it when working from a *following* date.

Day of Month.	Reduc. Arg.—"Diff. for 1 day" *m.* 3.91	*m.* 3.92	*m.* 3 93	*m.* 3.94	*m.* 3.95	No. of Days Elapsed
	m.	*m.*	*m.*	*m.*	*m.*	
2 or 16	3 9	3.9	3.9	3.9	3.9	1
3 or 17	7.8	7.8	7.9	7.9	7.9	2
4 or 18	11.7	11.8	11.8	11.8	11.8	3
5 or 19	15.6	15.7	15.7	15.8	15.8	4
6 or 20	19.5	19.6	19.6	19.7	19.7	5
7 or 21	23.5	23.5	23.6	23.6	23.7	6
8 or 22	27.4	27.4	27.5	27.6	27.6	7
9 or 23	31.3	31.4	31.4	31.5	31.6	8
10 or 24	35.2	35.3	35.4	35.5	35.5	9
11 or 25	39.1	39.2	39.3	39.4	39.5	10
12 or 26	43.0	43.1	43.2	43.3	43.4	11
13 or 27	46.9	47.0	47.2	47.2	47.4	12
14 or 28	50.8	51.0	51.1	51.2	51.3	13
29	54.7	54.9	55.0	55.2	55.3	14
30	58.6	58.8	58.9	59.1	59.2	15
31	62.6	62.7	62.9	63.0	63.2	16

* From advanced sheets of the Manual of 1902, issued by the General Land Office

† To be corrected for longitude as directed under Table I.

PART III.—AZIMUTH OF POLARIS

[The hour angles are expressed in *mean solar* time. The occurrence of a period after

Star and Azimuth.	Polaris *above* the Pole.
W. of N. when hour angle is *less* than $11^h\ 58^m$. E. of N. when hour angle is *greater* than $11^h\ 58^m$. Time argument, the star's hour angle (or $23^h\ 56^m.1$ *minus* the star's hour angle), for the year—	To determine the true meridian, the azimuth will be laid off to the *east* when the hour angle is *less* than $11^h\ 58^m$, and to the *west* when *greater* than $11^h\ 58^m$.

Hours.	1901.	1902.	1903.	1904.	1905.	1906.	1907.	1908.	1909.	1910.	1911.	Azimuths for Latitude— 30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
h.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	′	′	′	′	′	′	′	′	′	′	′
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4.	4.	4.	5	5	5	5	5	5	5	5	2	2	2	2	2	[illegible]	2	2	2	2.	2.
	9.	9.	9.	9.	9.	9.	9.	9.	10	10	10	3.	3.	3.	4	4	4	4	4.	4	4.	5
	14	14	14	14.	14.	14.	14.	14.	14.	14.	15	5.	5.	5.	5.	6	6	6	6.	6.	7	7
	19	19	19	19	19	19	19.	19.	19.	19.	19.	7	7	7.	7.	8	8	8.	8.	9	9	9.
	23.	23.	23.	24	24	24	24	24.	24.	24.	24.	9	9	9	9.	9.	10	10.	10.	11	11.	12
	28.	28.	28.	28.	29	29	29	29	29.	29.	29.	10.	11	11	11.	11.	12	12.	13	13.	14	14.
	33	33	33.	33.	33.	34	34	34	34.	34.	34.	12.	12.	13	13	13.	14	14.	15	15.	16	17
	38	38	38	38.	38.	38.	39	39	39	39.	39.	14	14.	14.	15	15.	16	16.	17	18	18.	19
	42.	42.	43	43	43.	43.	44	44	44	44.	44.	16	16	16.	17	17.	18	18.	19.	20	21	21.
	47.	47.	48	48	48	48.	48.	49	49	49.	49.	17.	18	18.	19	19.	20	20.	21.	22.	23	24
	52	52.	52.	53	53	53.	53.	54	54	54.	54.	19.	20	20.	21	21.	22	22.	23.	24.	25.	26.
	57	57.	57.	58	58	58.	58.	59	59	59.	0	21.	21.	22	22.	23.	24	25	26	27	28	29
1	2	2	2.	3	3	3.	3.	4	4.	4.	5	23	23.	24	24.	25	26	27	28	29	30	31.
	7	7	7.	7.	8	8.	8.	9	9.	9.	10	25	25.	26	26.	27	28	29	30	31.	32.	34
	12	12	12.	13	13	13.	14	14	14	15	15	27	27	27.	28.	29	30	31	32	33.	35	36.
	17	17	17.	18	18	18.	19	19.	19.	20	20.	28.	29	29.	30	31	32	33	34.	36	37.	39
	22	22	22.	23	23	23.	24	24.	25	25.	26	30.	31	31.	32	33	34	35	36.	38	39.	41.
	27	27	27.	28	28.	29	29.	29.	30	30.	31	32	32.	33	34	35	36	37	38.	40	42	43.
	32	32.	33	33.	33.	34	34.	35	35.	36	36	33.	34.	35	36	37	38	39.	41	42.	44	46
	37	37.	38	38.	39	39.	40	40.	41	41.	42	35	36	37	38	39	40	41.	43	44.	46.	48.
	42.	43	43.	44	44.	45	45.	46	46.	47	47.	37	38	38.	39.	40.	42	43.	45	46.	48.	50.
	47.	48	48.	49	50	50.	51	51.	52	52.	53	39	39.	40.	41.	42.	44	45.	47	49	51	53
	53	53.	54	54.	55	55.	56.	57	57.	58	58.	40.	41.	42.	43.	44.	46	47	49	51	53	55.
	58.	59	59.	0	0.	1.	[illegible]	2.	3	4	4.	42.	43	44	45.	46.	48	49.	51.	53.	55.	57.
2	4	4.	5	6	6.	7	7.	8.	9	9.	10.	44	45	46	47	48.	50	51.	53.	55.	57.	60
	[illegible]	10	10.	11.	12	12.	13.	14	15	15.	16.	46	47	48	49	50.	52	53.	55.	57.	60	62.
	15	16	16.	17	18	18.	19.	20	21	21.	22.	47.	48.	49.	51	52.	54	56	57.	60	62	64.
	21	21.	22.	23	24	24.	25.	26	27	28	28.	49.	50.	51.	53	54.	56	58	60	62	64.	67
	27	27.	28.	29	30	30.	31.	32.	33	34	35	51	52	53.	55	56.	58	60	62	64.	66.	69.
	33	33.	34.	35	36	37	38	38.	39.	40.	41.	53	54	55.	56.	58.	60	62	64	66.	69	72
	39	40	40.	41.	42.	43.	44.	45	46	47	48	54.	56	57	58.	60	62	64	66	68.	71.	74.
	45	46	47	48	49	50	51	52	53	54	55	56.	57.	59	60.	62	64	66	68.	71	73.	76.
	51.	52.	53.	54.	55.	56.	57.	58.	59.	1	2	58	59.	61	62.	64	66	68	70.	73	76	79
	58.	59.	0.	1.	2.	3.	4.	6	7	8	9	60	61.	63	64.	66	68	70	72.	75	78	81.
3	5.	6.	7.	8.	10	11	12	13	14.	15.	17	61.	63	64.	66	68	70	72	74.	77.	80.	84
	12.	13.	14.	16	17	18.	19.	21	22	23.	25	63.	65	66.	68	70	72	74.	77	79.	82.	86
	19.	21	22	23.	25	26	27.	29	30.	31.	33	65	66.	68.	70	72	74	76.	79	82	85	88.
	27.	28.	30	31.	33	34.	35.	37	38.	40.	42	67	68.	70	72	74	76	78.	81	84	87	91
	35.	37	38.	39.	41	43	44.	46	47	49.	51	69	70.	72	74	76	78	80.	83	86	89.	93.
	44	45.	47	48.	50	52	53.	55	57	59	0.	70.	72	74	75.	77.	80	82.	85.	88.	91.	95.
	53	54.	56	58	59.	1.	3.	5.	7.	9.	11.	72.	74	76	77.	79.	82	84.	87.	90.	94	98
4	2.	4.	6.	8	10	12.	14.	16.	19	21	23.	74	76	77.	79.	81.	84	86.	89.	92.	96	100.
	13	15	17.	19.	22	24	26.	29	32	34.	37.	76	77.	79.	81.	83.	86	88.	91.	95	98.	103
	24.	27	29.	32	34.	37.	40.	43.	46.	50	53	77.	79.	81.	83.	85.	88	90.	94	97	101	105
	38	40.	43.	46.	50	53.	57.	2	6.	11	16.	79.	81.	83	85	87.	90	93	96	99.	103	107.
	54	57.	1.	5	10	16	23.	32	42.			81	83	85	87	89.	92	95	98	101.	105	110.
5	16	22	29.	40.								83	85	87	89	91.	94	97	100	103.	107.	112

FOR THE USE OF LAND SURVEYORS.

minutes of time or of an hour angle indicates that its value is $0^m.5$ greater than printed.]

Star and Azimuth.

W. of N. when hour angle is *less* than $11^h\ 58^m$.
E. of N. when hour angle is *greater* than $11^h\ 58^m$.

Time argument, the star's hour angle (or $23^h\ 56^m.1$ *minus* the star's hour angle), for the year—

Polaris *below* the Pole.

To determine the true meridian, the azimuth will be laid off to the *east* when the hour angle is *less* than $11^h\ 58^m$, and to the *west* when *greater* than $11^h\ 58^m$.

Hours.	1901.	1902.	1903.	1904.	1905.	1906.	1907.	1908.	1909.	1910.	1911.	Azimuths for Latitude— 30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
h.	m.	m.	m.	m.								′	′	′	′	′	′	′	′	′	′	′
6	34	28	20.	9.	m.	m.	m.	m.	m.			83	85	87	89	91.	94	97	100	103.	107.	112
	56	52.	48.	45	40.	34	27	18	8	m.	m.	81.	83	85	87	89.	92	95	98	101.	105.	109.
7	12.	10	7	4	0.	56.	52.	48.	44	39	34	79.	81.	83	85	87.	90	93	96	99.	103	107
	26	23.	21	18.	16	13	10	7	4	0.	57	78	79.	81.	83.	85.	88	90.	93.	97	100.	104.
	37.	35.	33.	31.	29	26.	24	21.	19	16	13	76	77.	79.	81.	83.	86	88.	91.	95	98.	102
	48.	46.	44.	42.	40.	38.	36.	34.	32	29.	27	74.	76	77.	79.	81.	84	86.	89.	92.	96	100
	58	56.	55	53	51.	49.	47.	45.	43.	41.	39.	72.	74	76	77.	79.	82	84.	87.	90.	94	97.
8	7.	6	4	2.	1	59.	57.	56	54	52.	50.	71	72.	74	76	77.	80	82.	85.	88.	91.	95
	16	14	13	11.	10	8.	7	5.	4	2.	1	69	70.	72	74	75.	78	80.	83	86	89	92.
	24	23	21.	20	18.	17.	16	14.	13	11.	10	67.	68.	70.	72	74	76	78.	81	84	87	90
	32	31	29.	28	27	25.	24.	23	21.	20.	19	65.	67	68.	70	72	74	76.	79	81.	84.	88
	39.	38.	37	36	35	33.	32.	31	30	28.	27.	64	65	66.	68	70	72	74.	77	79.	82.	85.
	47	46	44.	43.	42.	41.	40	39	38	36.	35.	62	63.	64.	66.	68	70	72	74.	77.	80	83
	54	53	52	50.	49.	48.	47.	46	45.	44.	43	60.	61.	63	64.	66	68	70	72.	75	78	80.
9	0.	59.	58.	57.	56.	55.	54.	53.	52.	51.	50.	58.	59.	61	62.	64	66	68	70.	73	75.	78
	7.	6.	5.	4.	3.	2.	1.	1	0	59	58	57	58	59	60.	62.	64	66	68	70.	73	76
	13.	13	12	11	10.	9.	8.	7.	6	5.	5	55	56	57.	59	60.	62	64	66	68.	71	73.
	20	19	18.	17.	17	16	15	14.	13.	12.	11.	53.	54.	55.	57	58.	60	62	64	66	68.	71
	26	25.	24.	24	23	[illegible]	21.	20.	20	19	18	51.	52.	53.	55	56.	58	60	62	64	66.	68.
	32	31.	31	30	29.	28.	28	27	26.	25.	24.	49.	50.	52	53	54.	56	58	59.	62	64	66.
	38	37.	37	36	35.	35	34	33.	32.	32	31	48	49	50	51	52.	54	55.	57.	59.	61.	64
	44	43.	43	42	41.	41	40	39.	38.	38	37.	46	47	48	49.	50.	52	53.	55.	57.	59.	61.
	50	49	48.	48	47.	47	46	45.	45	44	43.	44.	45.	46.	47.	48.	50	51.	53.	55	57	59
	55.	55	54.	54	53	52.	52	51.	51	50	49.	42.	43.	44.	45.	46.	48	49.	51	53	55	57
10	1	0.	0	59.	59	58.	57.	57.	56.	56	55	41	41.	42.	43.	44.	46	47.	49	50.	52.	54.
	6.	6	5.	5	4.	4	3.	3	2.	2	1.	39	40	40.	41.	43	44	45.	47	48.	50	52
	12	11.	11	10.	10	9.	9	8	8	7.	7	37.	38	39	40	41	42	43.	45	46.	48	49
	17.	17	16.	16	15.	15	14.	14	13.	13	12.	35.	36	37	38	39	40	41	42.	44	45.	47.
	23	22.	22	21.	21	20.	20	19	19.	19	18.	34	34.	35	36	37	38	39	40.	42	43.	45
	28	27.	27.	27	26.	26	25.	25	25	24.	24	32	32.	33.	34	35	36	37	38.	39.	41	42.
	33.	33	32.	32	32	31.	31	30.	30	30	29.	30	31	31.	32	33	34	35	36	37.	39	40
	38.	38	38	37.	37	36.	36	36	35.	35	35	28.	29	29.	30.	31	32	33	34	35.	36.	38
	43.	43.	43	42.	42.	42	41.	41.	41	40.	40	26.	27.	28	28.	29	30	31	32	33	34	35.
	49	48.	48	48	47.	47.	47	46.	46.	46	45.	25	25.	26	26.	27	28	29	30	31	32	33
	54	53.	53	53	52.	52.	52	52	51.	51	51	23	23.	24	24.	25.	26	27	27.	28	29.	31
	59	58.	58.	58	58	57.	57.	57	57	56.	56	21.	22	22	22.	23	24	25	25.	26.	27.	28.
11	[illegible]	3.	3.	3	3	2.	2.	2	2	1.	1.	19.	20	20.	21	21.	22	22.	23.	24	25	26
	9	8.	8.	8.	8	8	7.	7.	7	7	6.	18	18	18.	19	19.	20	20.	21.	22	23	23.
	14	13.	13.	13.	13	13	12.	12.	12.	12	12	16	16.	16.	17	17.	18	18.	19	20	20.	21.
	19	18.	18.	18.	18	18	18	17.	17.	17.	17	14	14.	15	15	15.	16	16.	17	17.	18	19
	24	23.	23.	23.	23	23	23	22.	22.	22.	22	12.	12.	13	13.	13.	14	14.	15	15.	16	16.
	28.	28.	28.	28	28.	28	28	28	27.	27.	27.	10.	11	11	11.	11.	12	12.	12.	13	13.	14
	33.	33.	33.	33.	33	33	33	33	33	32.	32.	9	[illegible]	9	9.	9.	10	10.	10.	11	11.	12
	38.	38.	38.	38.	38	38	38	38	38	38	37.	7	7	7.	7.	8	8	8	8.	9	9	9.
	43.	43.	43.	43.	43	43	43	43	43	43	43	5.	5.	5.	5.	6	6	6	6.	6.	7	7
	48.	48.	48	48	48	48	48	48	48	48	48	3.	3.	3.	4	4	4	4	4	4.	4.	4.
	53	53	53	53	53	53	53	53	53	53	53	2	2	2	2	2	2	2	2	2	2.	2.
	58	58	58	58	58	58	58	58	58	58	58	0	0	0	[illegible]	0	[illegible]	0	0	0	0	0

TABLE VII.*

CONTAINING $z = 60521.5(1 + .001017 \times 36°) \log \frac{30}{h}$. ARGUMENT h.

h Ins.	z Feet.	Diff. for .01 Feet.	h Ins.	z Feet.	Diff. for .01 Feet.	h Ins.	z Feet.	Diff for .01 Feet.	h Ins.	z Feet.	Diff for .01 Feet.
11.0	27336	—24.6	16.0	17127	—16.9	21.0	9718	—12.9	26.0	3899	—10.5
11.1	27090	24.4	16.1	16958	16.9	21.1	9589	12.9	26.1	3794	10.4
11.2	26846	24.2	16.2	16789	16.8	21.2	9460	12.8	26.2	3690	10.4
11.3	26604	24.0	16.3	16621	16.7	21.3	9332	12.8	26.3	3586	10.3
11.4	26364	23.8	16.4	16454	16.6	21.4	9204	12.7	26.4	3483	10.3
11.5	26126	23.6	16.5	16288	16.4	21.5	9077	12.6	26.5	3380	10.3
11.6	25890	23.4	16.6	16124	16.3	21.6	8951	12.6	26.6	3277	10.2
11.7	25656	23.2	16.7	15961	16.3	21.7	8825	12.5	26.7	3175	10.2
11.8	25424	23.0	16.8	15798	16.2	21.8	8700	12.5	26.8	3073	10.1
11.9	25194	22.8	16.9	15636	16.0	21.9	8575	12.4	26.9	2972	10.1
12.0	24966	22.6	17.0	15476	16.0	22.0	8451	12.4	27.0	2871	10.1
12.1	24740	22.4	17.1	15316	15.9	22.1	8327	12.3	27.1	2770	10.0
12.2	24516	22.2	17.2	15157	15.8	22.2	8204	12.2	27.2	2670	10.0
12.3	24294	22.1	17.3	14999	15.7	22.3	8082	12.2	27.3	2570	10.0
12.4	24073	21.9	17.4	14842	15.6	22.4	7960	12.2	27.4	2470	9.9
12.5	23854	21.7	17.5	14686	15.5	22.5	7838	12.1	27.5	2371	9.9
12.6	23637	21.6	17.6	14531	15.4	22.6	7717	12.0	27.6	2272	9.9
12.7	23421	21.4	17.7	14377	15.4	22.7	7597	12.0	27.7	2173	9.8
12.8	23207	21.2	17.8	14223	15.3	22.8	7477	11.9	27.8	2075	9.8
12.9	22995	21.0	17.9	14070	15.2	22.9	7358	11.9	27.9	1977	9.7
13.0	22785	20.9	18.0	13918	15.1	23.0	7239	11.8	28.0	1880	9.7
13.1	22576	20.8	18.1	13767	15.0	23.1	7121	11.7	28.1	1783	9.7
13.2	22368	20.6	18.2	13617	14.9	23.2	7004	11.7	28.2	1686	9.7
13.3	22162	20.4	18.3	13468	14.9	23.3	6887	11.7	28.3	1589	9.6
13.4	21958	20.1	18.4	13319	14.7	23.4	6770	11.6	28.4	1493	9.6
13.5	21757	20.0	18.5	13172	14.7	23.5	6654	11.6	28.5	1397	9.5
13.6	21557	19.9	18.6	13025	14.6	23.6	6538	11.5	28.6	1302	9.5
13.7	21358	19.8	18.7	12879	14.6	23.7	6423	11.5	28.7	1207	9.5
13.8	21160	19.8	18.8	12733	14.4	23.8	6308	11.4	28.8	1112	9.4
13.9	20962	19.7	18.9	12589	14.4	23.9	6194	11.4	28.9	1018	9.4
14.0	20765	19.5	19.0	12445	14.3	24.0	6080	11.3	29.0	924	9.4
14.1	20570	19.3	19.1	12302	14.2	24.1	5967	11.3	29.1	830	9.4
14.2	20377	19.1	19.2	12160	14.2	24.2	5854	11.3	29.2	736	9.3
14.3	20186	18.9	19.3	12018	14.1	24.3	5741	11.2	29.3	643	9.3
14.4	19997	18.8	19.4	11877	14.0	24.4	5629	11.1	29.4	550	9.2
14.5	19809	18.6	19.5	11737	13.9	24.5	5518	11.1	29.5	458	9.2
14.6	19623	18.6	19.6	11598	13.9	24.6	5407	11.1	29.6	366	9.2
14.7	19437	18.5	19.7	11459	13.8	24.7	5296	11.0	29.7	274	9.2
14.8	19252	18.4	19.8	11321	13.7	24.8	5186	10.9	29.8	182	9.1
14.9	19068	18.2	19.9	11184	13.7	24.9	5077	10.9	29.9	91	9.1
15.0	18886	18.1	20.0	11047	13.6	25.0	4968	10.9	30.0	00	9.1
15.1	18705	18.0	20.1	10911	13.5	25.1	4859	10.8	30.1	—91	9.0
15.2	18525	17.9	20.2	10776	13.4	25.2	4751	10.8	30.2	181	9.0
15.3	18346	17.8	20.3	10642	13.4	25.3	4643	10.8	30.3	271	9.0
15.4	18168	17.6	20.4	10508	13.3	25.4	4535	10.7	30.4	361	9.0
15.5	17992	17.5	20.5	10375	13.3	25.5	4428	10.7	30.5	451	8.9
15.6	17817	17.4	20.6	10242	13.2	25.6	4321	10.6	30.6	540	8.9
15.7	17643	17.3	20.7	10110	13.1	25.7	4215	10.6	30.7	629	8.8
15.8	17470	17.2	20.8	9979	13.1	25.8	4109	10.5	30.8	717	8.8
15.9	17298	—17.1	20.9	9848	—13.0	25.9	4004	—10.5	30.9	805	—8.8
16.0	17127		21.0	9718		26.0	3899		31.0	—893	

* From Appendix X, U. S. C. and G. Survey, report for 1881.

TABLE VIII.*

TEMPERATURE AND HUMIDITY FACTORS.

$t_1 + t_2$.	C.	$t_1 + t_2$.	C.
0°	— .1025	100°	0.0049
10	— .0915	110	.0156
20	— .0806	120	.0262
30	— .0698	130	.0368
40	— .0592	140	.0472
50	— .0486	150	.0575
60	— .0380	160	.0677
70	— .0273	170	.0779
80	— .0166	180	.0879
90	— .0058		

* Computed from Tables I and IV, Appendix 10, Report for 1881, U. S. C. & G. Survey.

TABLE IX.*

STADIA REDUCTIONS FOR A CONSTANT OF 100.

ROD VERTICAL.

Minutes.	0°		1°		2°		3°		4°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	100.00	.00	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
2	100.00	.06	99.97	1.80	99.87	3.55	99.72	5.28	99.51	7.02
4	100.00	.12	99.97	1.86	99.87	3.60	99.71	5.34	99.50	7.07
6	100.00	.17	99.96	1.92	99.87	3.66	99.71	5.40	99.49	7.13
8	100.00	.23	99.96	1.98	99.86	3.72	99.70	5.46	99.48	7.19
10	100.00	.29	99.96	2.04	99.86	3.78	99.69	5.52	99.47	7.25
12	100.00	.35	99.96	2.09	99.85	3.84	99.69	5.57	99.46	7.30
14	100.00	.41	99.95	2.15	99.85	3.90	99.68	5.63	99.46	7.36
16	100.00	.47	99.95	2.21	99.84	3.95	99.68	5.69	99.45	7.42
18	100.00	.52	99.95	2.27	99.84	4.01	99.67	5.75	99.44	7.48
20	100.00	.58	99.95	2.33	99.83	4.07	99.66	5.80	99.43	7.53
22	100.00	.64	99.94	2.38	99.83	4.13	99.66	5.86	99.42	7.59
24	100.00	.70	99.94	2.44	99.82	4.18	99.65	5.92	99.41	7.65
26	99.99	.76	99.94	2.50	99.82	4.24	99.64	5.98	99.40	7.71
28	99.99	.81	99.93	2.56	99.81	4.30	99.63	6.04	99.39	7.76
30	99.99	.87	99.93	2.62	99.81	4.36	99.63	6.09	99.38	7.82
32	99.99	.93	99.93	2.67	99.80	4.42	99.62	6.15	99.38	7.88
34	99.99	.99	99.93	2.73	99.80	4.48	99.62	6.21	99.37	7.94
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27	99.36	7.99
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33	99.35	8.05
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38	99.34	8.11
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44	99.33	8.17
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50	99.32	8.22
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56	99.31	8.28
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61	99.30	8.34
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67	99.29	8.40
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73	99.28	8.45
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78	99.27	8.51
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84	99.26	8.57
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90	99.25	8.63
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96	99.24	8.68
$c = .75$	.75	.01	.75	.02	.75	.03	.75	.05	.75	.06
$c = 1.15$	1.15	.01	1.15	.03	1.15	.05	1.15	.07	1.15	.09
$c = 1.90$	1.90	.02	1.90	.05	1.90	.08	1.90	.12	1.89	.15

* From the catalogue of the A. Lietz Company, San Francisco, Cal.

TABLE IX.—*Continued.*

STADIA REDUCTIONS FOR A CONSTANT OF 100.

ROD VERTICAL.

Minutes.	5°		6°		7°		8°		9°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78	97.55	15.45
2	99.23	8.74	98.90	10.45	98.50	12.15	98.05	13.84	97.53	15.51
4	99.22	8.80	98.88	10.51	98.48	12.21	98.03	13.89	97.52	15.56
6	99.21	8.85	98.87	10.57	98.47	12.26	98.01	13.95	97.50	15.62
8	99.20	8.91	98.86	10.62	98.46	12.32	98.00	14.01	97.48	15.67
10	99.19	8.97	98.85	10.68	98.44	12.38	97.98	14.06	97.46	15.73
12	99.18	9.03	98.83	10.74	98.43	12.43	97.97	14.12	97.44	15.78
14	99.17	9.08	98.82	10.79	98.41	12.49	97.95	14.17	97.43	15.84
16	99.16	9.14	98.81	10.85	98.40	12.55	97.93	14.23	97.41	15.89
18	99.15	9.20	98.80	10.91	98.39	12.60	97.92	14.28	97.39	15.95
20	99.14	9.25	98.78	10.96	98.37	12.66	97.90	14.34	97.37	16.00
22	99.13	9.31	98.77	11.02	98.36	12.72	97.88	14.40	97.35	16.06
24	99.11	9.37	98.76	11.08	98.34	12.77	97.87	14.45	97.33	16.11
26	99.10	9.43	98.74	11.13	98.33	12.83	97.85	14.51	97.31	16.17
28	99.09	9.48	98.73	11.19	98.31	12.88	97.83	14.56	97.29	16.22
30	99.08	9.54	98.72	11.25	98.29	12.94	97.82	14.62	97.28	16.28
32	99.07	9.60	98.71	11.30	98.28	13.00	97.80	14.67	97.26	16.33
34	99.06	9.65	98.69	11.36	98.27	13.05	97.78	14.73	97.24	16.39
36	99.05	9.71	98.68	11.42	98.25	13.11	97.76	14.79	97.22	16.44
38	99.04	9.77	98.67	11.47	98.24	13.17	97.75	14.84	97.20	16.50
40	99.03	9.83	98.65	11.53	98.22	13.22	97.73	14.90	97.18	16.55
42	99.01	9.88	98.64	11.59	98.20	13.28	97.71	14.59	97.16	16.61
44	99.00	9.94	98.63	11.64	98.19	13.33	97.69	15.01	97.14	16.66
46	98.99	10.00	98.61	11.70	98.17	13.39	97.68	15.06	97.12	16.72
48	98.98	10.05	98.60	11.76	98.16	13.45	97.66	15.12	97.10	16.77
50	98.97	10.11	98.58	11.81	98.14	13.50	97.64	15.17	97.08	16.83
52	98.96	10.17	98.57	11.87	98.13	13.56	97.62	15.23	97.06	16.88
54	98.94	10.22	98.56	11.93	98.11	13.61	97.61	15.28	97.04	16.94
56	98.93	10.28	98.54	11.98	98.10	13.67	97.59	15.34	97.02	16.99
58	98.92	10.34	98.53	12.04	98.08	13.73	97.57	15.40	97.00	17.05
60	98.91	10.40	98.51	12.10	98.06	13.78	97.55	15.45	96.98	17.10
$c =$ .75	.75	.07	.75	.08	.74	.10	.74	.11	.74	.12
$c =$ 1.15	1.14	.11	1.14	.13	1.14	.15	1.14	.17	1.13	.19
$c =$ 1.90	1.89	.18	1.89	.21	1.88	.25	1.88	.28	1.87	.31

TABLE IX.—*Continued.*

STADIA REDUCTIONS FOR A CONSTANT OF 100.

ROD VERTICAL.

Minutes.	10°		11°		12°		13°		14°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0'	96.98	17.10	96.36	18.73	95.68	20.34	94.94	21.92	94.15	23.47
2	96.96	17.16	96.34	18.78	95.65	20.39	94.91	21.97	94.12	23.52
4	96.94	17.21	96.32	18.84	95.63	20.44	94.89	22.02	94.09	23.58
6	96.92	17.26	96.29	18.89	95.61	20.50	94.86	22.08	94.07	23.63
8	96.90	17.32	96.27	18.95	95.58	20.55	94.84	22.13	94.04	23.68
10	96.88	17.37	96.25	19.00	95.56	20.60	94.81	22.18	94.01	23.73
12	96.86	17.43	96.23	19.05	95.53	20.66	94.79	22.23	93.98	23.78
14	96.84	17.48	96.21	19.11	95.51	20.71	94.76	22.28	93.95	23.83
16	96.82	17.54	96.18	19.16	95.49	20.76	94.73	22.34	93.93	23.88
18	96.80	17.59	96.16	19.21	95.46	20.81	94.71	22.39	93.90	23.93
20	96.78	17.65	96.14	19.27	95.44	20.87	94.68	22.44	93.87	23.99
22	96.76	17.70	96.12	19.32	95.41	20.92	94.66	22.49	93.84	24.04
24	96.74	17.76	96.09	19.38	95.39	20.97	94.63	22.54	93.81	24.09
26	96.72	17.81	96.07	19.43	95.36	21.03	94.60	22.60	93.79	24.14
28	96.70	17.86	96.05	19.48	95.34	21.08	94.58	22.65	93.76	24.19
30	96.68	17.92	96.03	19.54	95.32	21.13	94.55	22.70	93.73	24.24
32	96.66	17.97	96.00	19.59	95.29	21.18	94.52	22.75	93.70	24.29
34	96.64	18.03	95.98	19.64	95.27	21.24	94.50	22.80	93.67	24.34
36	96.62	18.08	95.96	19.70	95.24	21.29	94.47	22.85	93.65	24.39
38	96.60	18.14	95.93	19.75	95.22	21.34	94.44	22.91	93.62	24.44
40	96.57	18.19	95.91	19.80	95.19	21.39	94.42	22.96	93.59	24.49
42	96.55	18.24	95.89	19.86	95.17	21.45	94.39	23.01	93.56	24.55
44	96.53	18.30	95.86	19.91	95.14	21.50	94.36	23.06	93.53	24.60
46	96.51	18.35	95.84	19.96	95.12	21.55	94.34	23.11	93.50	24.65
48	96.49	18.41	95.82	20.02	95.09	21.60	94.31	23.16	93.47	24.70
50	96.47	18.46	95.79	20.07	95.07	21.66	94.28	23.22	93.45	24.75
52	96.45	18.51	95.77	20.12	95.04	21.71	94.26	23.27	93.42	24.80
54	96.42	18.57	95.75	20.18	95.02	21.76	94.23	23.32	93.39	24.85
56	96.40	18.62	95.72	20.23	94.99	21.81	94.20	23.37	93.36	24.90
58	96.38	18.68	95.70	20.28	94.97	21.87	94.17	23.42	93.33	24.95
60	96.36	18.73	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
c = .75	.74	.14	.73	.15	.73	.16	.73	.17	.73	.19
c = 1.15	1.13	.21	1.13	.23	1.12	.25	1.12	.27	1.11	.29
c = 1.90	1.87	.35	1.86	.38	1.85	.41	1.85	.44	1.84	.48

TABLE IX.—*Continued.*

STADIA REDUCTIONS FOR A CONSTANT OF 100.

ROD VERTICAL.

Minutes.	15°		16°		17°		18°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0'	93.30	25.00	92.40	26.50	91.45	27.96	90.45	29.39
2	93.27	25.05	92.37	26.55	91.42	28.01	90.42	29.44
4	93.24	25.10	92.34	26.59	91.39	28.06	90.38	29.48
6	93.21	25.15	92.31	26.64	91.35	28.10	90.35	29.53
8	93.18	25.20	92.28	26.69	91.32	28.15	90.31	29.58
10	93.16	25.25	92.25	26.74	91.29	28.20	90.28	29.62
12	93.13	25.30	92.22	26.79	91.26	28.25	90.24	29.67
14	93.10	25.35	92.19	26.84	91.22	28.30	90.21	29.72
16	93.07	25.40	92.15	26.89	91.19	28.34	90.18	29.76
18	93.04	25.45	92.12	26.94	91.16	28.39	90.14	29.81
20	93.01	25.50	92.09	26.99	91.12	28.44	90.11	29.86
22	92.98	25.55	92.06	27.04	91.09	28.49	90.07	29.90
24	92.95	25.60	92.03	27.09	91.06	28.54	90.04	29.95
26	92.92	25.65	92.00	27.13	91.02	28.58	90.00	30.00
28	92.89	25.70	91.97	27.18	90.99	28.63	89.97	30.04
30	92.86	25.75	91.93	27.23	90.96	28.68	89.93	30.09
32	92.83	25.80	91.90	27.28	90.92	28.73	89.90	30.14
34	92.80	25.85	91.87	27.33	90.89	28.77	89.86	30.19
36	92.77	25.90	91.84	27.38	90.86	28.82	89.83	30.23
38	92.74	25.95	91.81	27.43	90.82	28.87	89.79	30.28
40	92.71	26.00	91.77	27.48	90.79	28.92	89.76	30.32
42	92.68	26.05	91.74	27.52	90.76	28.96	89.72	30.37
44	92.65	26.10	91.71	27.57	90.72	29.01	89.69	30.41
46	92.62	26.15	91.68	27.62	90.69	29.06	89.65	30.46
48	92.59	26.20	91.65	27.67	90.66	29.11	89.61	30.51
50	92.56	26.25	91.61	27.72	90.62	29.15	89.58	30.55
52	92.53	26.30	91.58	27.77	90.59	29.20	89.54	30.60
54	92.49	26.35	91.55	27.81	90.55	29.25	89.51	30.65
56	92.46	26.40	91.52	27.86	90.52	29.30	89.47	30.69
58	92.43	26.45	91.48	27.91	90.48	29.34	89.44	30.74
60	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
$c = .75$	.72	.20	.72	.21	.72	.23	.71	.24
$c = 1.15$	1.11	.31	1.10	.33	1.10	.35	1.09	.37
$c = 1.90$	1.83	.51	1.82	.54	1.81	.57	1.80	.60

TABLE IX.—*Continued.*

STADIA REDUCTIONS FOR A CONSTANT OF 100.

ROD VERTICAL.

Minutes.	19°		20°		21°		22°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist	Diff. Elev.
0′	89.40	30.78	88.30	32.14	87.16	33.46	85.97	34.73
2	89.36	30.83	88.26	32.18	87.12	33.50	85.93	34.77
4	89.33	30.87	88.23	32.23	87.08	33.54	85.89	34.82
6	89.29	30.92	88.19	32.27	87.04	33.59	85.85	34.86
8	89.26	30.97	88.15	32.32	87.00	33.63	85.80	34.90
10	89.22	31.01	88.11	32.36	86.96	33.67	85.76	34.94
12	89.18	31.06	88.08	32.41	86.92	33.72	85.72	34.98
14	89.15	31.10	88.04	32.45	86.88	33.76	85.68	35.02
16	89.11	31.15	88.00	32.49	86.84	33.80	85.64	35.07
18	89.08	31.19	87.96	32.54	86.80	33.84	85.60	35.11
20	89.04	31.24	87.93	32.58	86.77	33.89	85.56	35.15
22	89.00	31.28	87.89	32.63	86.73	33.93	85.52	35.19
24	88.96	31.33	87.85	32.67	86.69	33.97	85.48	35.23
26	88.93	31.38	87.81	32.72	86.65	34.01	85.44	35.27
28	88.89	31.42	87.77	32.76	86.61	34.06	85.40	35.31
30	88.86	31.47	87.74	32.80	86.57	34.10	85.36	35.36
32	88.82	31.51	87.70	32.85	86.53	34.14	85.31	35.40
34	88.78	31.56	87.66	32.89	86.49	34.18	85.27	35.44
36	88.75	31.60	87.62	32.93	86.45	34.23	85.23	35.48
38	88.71	31.65	87.58	32.98	86.41	34.27	85.19	35.52
40	88.67	31.69	87.54	33.02	86.37	34.31	85.15	35.56
42	88.64	31.74	87.51	33.07	86.33	34.35	85.11	35.60
44	88.60	31.78	87.47	33.11	86.29	34.40	85.07	35.64
46	88.56	31.83	87.43	33.15	86.25	34.44	85.02	35.68
48	88.53	31.87	87.39	33.20	86.21	34.48	84.98	35.72
50	88.49	31.92	87.35	33.24	86.17	34.52	84.94	35.76
52	88.45	31.96	87.31	33.28	86.13	34.57	84.90	35.80
54	88.41	32.01	87.27	33.33	86.09	34.61	84.86	35.85
56	88.38	32.05	87.24	33.37	86.05	34.65	84.82	35.89
58	88.34	32.09	87.20	33.41	86.01	34.69	84.77	35.93
60	88.30	32.14	87.16	33 46	85.97	34.73	84 73	35.97
$c =$ .75	.71	.25	.70	.26	.70	.27	.69	.29
$c =$ 1.15	1.08	.38	1.08	.40	1.07	.42	1.06	.44
$c =$ 1.90	1.79	.63	1.78	.66	1.77	.70	1.76	.73

Table IX.—*Continued.*

STADIA REDUCTIONS FOR A CONSTANT OF 100.

ROD VERTICAL.

Minutes.	23°		24°		25°		26°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	84.73	35.97	83.46	37.16	82.14	38.30	80.78	39.40
2	84.69	36.01	83.41	37.20	82.09	38.34	80.74	39.44
4	84.65	36.05	83.37	37.23	82.05	38.38	80.69	39.47
6	84.61	36.09	83.33	37.27	82.01	38.41	80.65	39.51
8	84.57	36.13	83.28	37.31	81.96	38.45	80.60	39.54
10	84.52	36.17	83.24	37.35	81.92	38.49	80.55	39.58
12	84.48	36.21	83.20	37.39	81.87	38.53	80.51	39.61
14	84.44	36.25	83.15	37.43	81.83	38.56	80.46	39.65
16	84.40	36.29	83.11	37.47	81.78	38.60	80.41	39.69
18	84.35	36.33	83.07	37.51	81.74	38.64	80.37	39.72
20	84.31	36.37	83.02	37.54	81.69	38.67	80.32	39.76
22	84.27	36.41	82.98	37.58	81.65	38.71	80.28	39.79
24	84.23	36.45	82.93	37.62	81.60	38.75	80.23	39.83
26	84.18	36.49	82.89	37.66	81.56	38.78	80.18	39.86
28	84.14	36.53	82.85	37.70	81.51	38.82	80.14	39.90
30	84.10	36.57	82.80	37.74	81.47	38.86	80.09	39.93
32	84.06	36.61	82.76	37.77	81.42	38.89	80.04	39.97
34	84.01	36.65	82.72	37.81	81.38	38.93	80.00	40.00
36	83.97	36.69	82.67	37.85	81.33	38.97	79.95	40.04
38	83.93	36.73	82.63	37.89	81.28	39.00	79.90	40.07
40	83.89	36.77	82.58	37.93	81.24	39.04	79.86	40.11
42	83.84	36.80	82.54	37.96	81.19	39.08	79.81	40.14
44	83.80	36.84	82.49	38.00	81.15	39.11	79.76	40.18
46	83.76	36.88	82.45	38.04	81.10	39.15	79.72	40.21
48	83.72	36.92	82.41	38.08	81.06	39.18	79.67	40.24
50	83.67	36.96	82.36	38.11	81.01	39.22	79.62	40.28
52	83.63	37.00	82.32	38.15	80.97	39.26	79.58	40.31
54	83.59	37.04	82.27	38.19	80.92	39.29	79.53	40.35
56	83.54	37.08	82.23	38.23	80.87	39.33	79.48	40.38
58	83.50	37.12	82.18	38.26	80.83	39.36	79.44	40.42
60	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
c = .75	.69	.30	.68	.31	.68	.32	.67	.33
c = 1.15	1.05	.46	1.05	.48	1.04	.50	1.03	.51
c = 1.90	1.74	.76	1.73	.79	1.72	.82	1.70	.85

TABLE IX.—*Continued.*

STADIA REDUCTIONS FOR A CONSTANT OF 100.

ROD VERTICAL.

Minutes.	27°		28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30
2	79.34	40.49	77.91	41.48	76.45	42.43	74.95	43.33
4	79.30	40.52	77.86	41.52	76.40	42.46	74.90	43.36
6	79.25	40.55	77.81	41.55	76.35	42.49	74.85	43.39
8	79.20	40.59	77.77	41.58	76.30	42.53	74.80	43.42
10	79.15	40.62	77.72	41.61	76.25	42.56	74.75	43.45
12	79.11	40.66	77.67	41.65	76.20	42.59	74.70	43.47
14	79.06	40.69	77.62	41.68	76.15	42.62	74.65	43.50
16	79.01	40.72	77.57	41.71	76.10	42.65	74.60	43.53
18	78.96	40.76	77.52	41.74	76.05	42.68	74.55	43.56
20	78.92	40.79	77.48	41.77	76.00	42.71	74.49	43.59
22	78.87	40.82	77.42	41.81	75.95	42.74	74.44	43.62
24	78.82	40.86	77.38	41.84	75.90	42.77	74.39	43.65
26	78.77	40.89	77.33	41.87	75.85	42.80	74.34	43.67
28	78.73	40.92	77.28	41.90	75.80	42.83	74.29	43.70
30	78.68	40.96	77.23	41.93	75.75	42.86	74.24	43.73
32	78.63	40.99	77.18	41.97	75.70	42.89	74.19	43.76
34	78.58	41.02	77.13	42.00	75.65	42.92	74.14	43.79
36	78.54	41.06	77.09	42.03	75.60	42.95	74.09	43.82
38	78.49	41.09	77.04	42.06	75.55	42.98	74.04	43.84
40	78.44	41.12	76.99	42.09	75.50	43.01	73.99	43.87
42	78.39	41.16	76.94	42.12	75.45	43.04	73.93	43.90
44	78.34	41.19	76.89	42.15	75.40	43.07	73.88	43.93
46	78.30	41.22	76.84	42.19	75.35	43.10	73.83	43.95
48	78.25	41.26	76.79	42.22	75.30	43.13	73.78	43.98
50	78.20	41.29	76.74	42.25	75.25	43.16	73.73	44.01
52	78.15	41.32	76.69	42.28	75.20	43.18	73.68	44.04
54	78.10	41.35	76.64	42.31	75.15	43.21	73.63	44.07
56	78.06	41.39	76.59	42.34	75.10	43.24	73.58	44.09
58	78.01	41.42	76.55	42.37	75.05	43.27	73.52	44.12
60	77.96	41.45	76.50	42.40	75.00	43.30	73.47	44.15
$c = .75$	.66	.35	.66	.36	.65	.37	.65	.38
$c = 1.15$	1.02	.53	1.01	.55	1.00	.57	.99	.58
$c = 1.90$	1.69	.88	1.67	.91	1.65	.94	1.64	.95

TABLE X.*

LENGTH OF A DEGREE OF LONGITUDE.

Lat.	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	Lat.
′	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains*	′
0	4843.17	4795.82	4747.01	4696.75	4645.06	4591.96	4537.45	4481.56	4424.29	4365.68	**0**
1	42.40	95.02	46.19	95.90	44.19	91.06	36.53	80.61	23.33	64.69	**1**
2	41.62	94.22	45.36	95.05	43.32	90.16	35.61	79.67	22.36	63.70	**2**
3	40.84	93.42	44.53	94.20	42.44	89.26	34.69	78.73	21.40	62.72	**3**
4	40.06	92.61	43.71	93.35	41.57	88.37	33.77	77.78	20.43	61.73	**4**
5	39.28	91.81	42.88	92.50	40.69	87.47	32.84	76.84	19.46	60.74	**5**
6	38.50	91.01	42.05	91.65	39.82	86.57	31.92	75.89	18.49	59.75	**6**
7	37.72	90.20	41.22	90.80	38.94	85.67	31.00	74.95	17.53	58.76	**7**
8	36.94	89.40	40.39	89.94	38.06	84.77	30.08	74.00	16.56	57.77	**8**
9	36.16	88.59	39.56	89.09	37.19	83.87	29.15	73.05	15.59	56.77	**9**
10	35.38	87.79	38.73	88.24	36.31	82.97	28.23	72.11	14.62	55.78	**10**
11	34.60	86.98	37.90	87.38	35.43	82.07	27.30	71.16	13.65	54.79	**11**
12	33.82	86.18	37.07	86.53	34.55	81.17	26.38	70.21	12.68	53.80	**12**
13	33.04	85.37	36.24	85.67	33.68	80.26	25.46	69.26	11.71	52.81	**13**
14	32.26	84.56	35.41	84.82	32.80	79.36	24.53	68.32	10.74	51.81	**14**
15	31.47	83.76	34.58	83.96	31.92	78.46	23.60	67.37	09.77	50.82	**15**
16	30.69	82.95	33.75	83.11	31.04	77.56	22.68	66.42	08.80	49.83	**16**
17	29.91	82.14	32.92	82.25	30.16	76.65	21.75	65.47	07.82	48.83	**17**
18	29.12	81.33	32.08	81.40	29.28	75.75	20.83	64.52	06.85	47.84	**18**
19	28.34	80.52	31.25	80.54	28.40	74.85	19.90	63.57	05.88	46.84	**19**
20	27.55	79.71	30.42	79.68	27.52	73.94	18.97	62.62	04.91	45.85	**20**
21	26.77	78.90	29.58	78.82	26.64	73.04	18.04	61.67	03.93	44.85	**21**
22	25.98	78.09	28.75	77.97	25.75	72.13	17.11	60.72	02.96	43.85	**22**
23	25.20	77.28	27.92	77.11	24.87	71.23	16.19	59.77	01.98	42.86	**23**
24	24.41	76.47	27.08	76.25	23.99	70.32	15.26	58.81	01.01	41.86	**24**
25	23.62	75.66	26.25	75.39	23.11	69.41	14.33	57.86	4400.04	40.86	**25**
26	22.83	74.85	25.41	74.53	22.22	68.51	13.40	56.91	4399.06	39.87	**26**
27	22.05	74.04	24.57	73.67	21.34	67.60	12.47	55.96	98.08	38.87	**27**
28	21.26	73.22	23.74	72.81	20.45	66.69	11.54	55.00	97.11	37.87	**28**
29	20.47	72.41	22.90	71.95	19.57	65.78	10.61	54.05	96.13	36.87	**29**
30	19.68	71.60	22.06	71.09	18.69	64.88	09.67	53.09	95.16	35.87	**30**
31	18.89	70.78	21.22	70.22	17.80	63.97	08.74	52.14	94.18	34.87	**31**
32	18.10	69.97	20.39	69.36	16.91	63.06	07.81	51.19	93.20	33.87	**32**
33	17.31	69.16	19.55	68.50	16.03	62.15	06.88	50.23	92.22	32.87	**33**
34	16.52	68.34	18.71	67.64	15.14	61.24	05.94	49.27	91.25	31.87	**34**
35	15.73	67.53	17.87	66.77	14.26	60.33	05.01	48.32	90.27	30.87	**35**
36	14.94	66.71	17.03	65.91	13.37	59.42	04.08	47.36	89.29	29.87	**36**
37	14.15	65.89	16.19	65.05	12.48	58.51	03.14	46.41	88.31	28.87	**37**
38	13.35	65.08	15.35	64.18	11.59	57.60	02.21	45.45	87.33	27.87	**38**
39	12.56	64.26	14.51	63.32	10.70	56.68	01.28	44.49	86.35	26.87	**39**
40	11.77	63.44	13.67	62.45	09.81	55.77	4500.34	43.53	85.37	25.86	**40**
41	10.98	62.52	12.82	61.59	08.93	54.86	4499.40	42.57	84.39	24.86	**41**
42	10.18	61.81	11.98	60.72	08.04	53.95	98.47	41.62	83.41	23.86	**42**
43	09.39	60.99	11.14	59.85	07.15	53.03	97.53	40.66	82.42	22.85	**43**
44	08.59	60.17	10.30	58.99	06.26	52.12	96.59	39.70	81.44	21.85	**44**
45	07.80	59.35	09.45	58.12	05.36	51.21	95.66	38.74	80.46	20.85	**45**
46	07.00	58.53	08.61	57.25	04.47	50.29	94.72	37.78	79.48	19.84	**46**
47	06.21	57.71	07.76	56.38	03.58	49.38	93.78	36.82	78.49	18.84	**47**
48	05.41	56.89	06.92	55.51	02.69	48.46	92.84	35.86	77.51	17.83	**48**
49	04.61	56.07	06.07	54.65	01.80	47.55	91.91	34.89	76.53	16.82	**49**
50	03.82	55.25	05.23	53.78	00.90	46.63	90.97	33.93	75.54	15.82	**50**
51	03.02	54.43	04.38	52.91	4600.01	45.71	90.03	32.97	74.56	14.81	**51**
52	02.22	53.60	03.54	52.04	4599.12	44.80	89.09	32.01	73.57	13.80	**52**
53	01.42	52.78	02.69	51.17	98.22	43.88	88.15	31.04	72.59	12.80	**53**
54	4800.62	51.96	01.84	50.30	97.33	42.96	87.21	30.08	71.60	11.79	**54**
55	4799.82	51.13	01.00	49.42	96.44	42.04	86.27	29.12	70.62	10.78	**55**
56	99.02	50.31	4700.15	48.55	95.54	41.13	85.32	28.15	69.63	09.77	**56**
57	98.22	49.49	4699.30	47.68	94.64	40.21	84.38	27.19	68.64	08.76	**57**
58	97.42	48.66	98.45	46.81	93.75	39.29	83.44	26.22	67.66	07.75	**58**
59	96.62	47.84	97.60	45.94	92.85	38.37	82.50	25.26	66.67	06.74	**59**
60	4795.82	4747.01	4696.75	4645.06	4591.96	4537.45	4481.56	4424.29	4365.68	4305.73	**60**

* From the Manual of Instructions of 1894.

Table X.—*Continued.*

LENGTH OF A DEGREE OF LONGITUDE.

Lat.	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	Lat.
′	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	′
0	4305.73	4244.47	4181.91	4118.06	4052.96	3986.62	3919.05	3850.28	3780.33	3709.22	**0**
1	04.72	43.44	80.85	16.99	51.87	85.50	17.91	49.12	79.15	08.03	**1**
2	03.71	42.41	79.80	15.91	50.77	84.38	16.78	47.97	77.98	06.83	**2**
3	02.70	41.37	78.75	14.84	49.67	83.27	15.64	46.81	76.80	05.63	**3**
4	01.69	40.34	77.69	13.76	48.58	82.15	14.50	45.65	75.63	04.44	**4**
5	4300.68	39.31	76.64	12.69	47.48	81.03	13.36	44.50	74.45	03.24	**5**
6	4299.67	38.27	75.58	11.61	46.38	79.91	12.23	43.34	73.27	02.05	**6**
7	98.65	37.24	74.52	10.53	45.28	78.79	11.09	42.18	72.00	3700.85	**7**
8	97.64	36.20	73.47	09.46	44.19	77.68	09.95	41.02	70.92	3699.65	**8**
9	96.63	35.17	72.41	08.38	43.09	76.56	08.81	39.86	69.74	98.46	**9**
10	95.61	34.13	71.36	07.30	41.99	75.44	07.67	38.70	68.56	97.26	**10**
11	94.60	33.10	70.30	06.22	40.89	74.32	06.53	37.54	67.38	96.06	**11**
12	93.59	32.06	69.24	05.14	39.79	73.20	05.39	36.38	66.20	94.86	**12**
13	92.57	31.02	68.18	04.07	38.69	72.08	04.25	35.22	65.02	93.66	**13**
14	91.56	29.99	67.12	02.99	37.59	70.96	03.11	34.06	63.84	92.46	**14**
15	90.54	28.95	66.07	01.91	36.49	69.84	01.97	32.90	62.66	91.26	**15**
16	89.52	27.91	65.01	4100.83	35.39	68.72	3900.83	31.74	61.48	90.06	**16**
17	88.51	26.87	63.95	4099.75	34.29	67.59	3899.69	30.58	60.30	88.86	**17**
18	87.49	25.84	62.89	98.67	33.19	66.47	98.54	29.42	59.12	87.66	**18**
19	86.48	24.80	61.83	97.58	32.09	65.35	97.40	28.26	57.94	86.46	**19**
20	85.46	23.76	60.77	96.50	30.98	64.23	96.26	27.09	56.76	85.26	**20**
21	84.44	22.72	59.71	95.42	29.88	63.11	95.12	25.93	55.57	84.06	**21**
22	83.42	21.68	58.65	94.34	28.78	61.98	93.97	24.77	54.39	82.86	**22**
23	82.40	20.64	57.58	93.26	27.67	60.86	92.83	23.60	53.21	81.66	**23**
24	81.39	19.60	56.52	92.17	26.57	59.73	91.68	22.44	52.02	80.46	**24**
25	80.37	18.56	55.46	91.09	25.47	58.61	90.54	21.28	50.84	79.25	**25**
26	79.35	17.52	54.40	90.01	24.36	57.49	89.40	20.11	49.66	78.05	**26**
27	78.33	16.48	53.44	88.92	23.26	56.36	88.25	18.95	48.47	76.85	**27**
28	77.31	15.43	52.27	87.84	22.15	55.24	87.11	17.78	47.29	75.64	**28**
29	76.29	14.39	51.21	86.75	21.05	54.11	85.96	16.62	46.10	74.44	**29**
30	75.27	13.35	50.14	85.67	19.94	52.98	84.81	15.45	44.92	73.24	**30**
31	74.24	12.31	49.08	84.58	18.84	51.86	83.67	14.29	43.73	72.03	**31**
32	73.22	11.26	48.02	83.50	17.73	50.73	82.52	13.12	42.55	70.83	**32**
33	72.20	10.22	46.95	82.41	16.62	49.60	81.37	11.05	41.30	69.62	**33**
34	71.18	09.18	45.89	81.33	15.52	48.48	80.23	10.79	40.18	68.42	**34**
35	70.16	08.13	44.82	80.24	14.41	47.35	79.08	09.62	38.99	67.21	**35**
36	69.13	07.09	43.75	79.15	13.30	46.22	77.93	08.45	37.80	66.01	**36**
37	68.11	06.04	42.69	78.07	12.19	45.09	76.78	07.28	36.62	64.80	**37**
38	67.09	05.00	41.62	76.98	11.09	43.96	75.63	06.11	35.43	63.59	**38**
39	66.06	03.95	40.55	75.89	09.98	42.83	74.48	04.95	34.24	62.39	**39**
40	65.04	02.90	39.49	74.80	08.87	41.71	73.34	03.78	33.05	61.18	**40**
41	64.01	01.86	38.42	73.71	07.76	40.58	72.19	02.61	31.86	59.97	**41**
42	62.99	4200.81	37.35	72.62	06.65	39.45	71.04	01.44	30.67	58.76	**42**
43	61.96	4199.76	36.28	71.53	05.54	38.32	69.89	3800.27	29.48	57.56	**43**
44	60.93	98.72	35.21	70.44	04.43	37.18	68.74	3799.10	28.30	56.35	**44**
45	59.91	97.67	34.14	69.35	03.32	36.05	67.58	97.93	27.11	55.14	**45**
46	58.88	96.62	33.08	68.26	02.21	34.92	66.43	96.76	25.92	53.93	**46**
47	57.85	95.57	32.01	67.17	4001.10	33.79	65.28	95.59	24.73	52.72	**47**
48	56.83	94.52	30.93	66.08	3999.98	32.66	64.13	94.41	23.53	51.51	**48**
49	55.80	93.47	29.86	64.99	98.87	31.53	62.98	93.24	22.34	50.30	**49**
50	54.77	92.42	28.79	63.90	97.76	30.39	61.82	92.07	21.15	49.09	**50**
51	53.74	91.37	27.72	62.81	96.65	29.26	60.67	90.90	19.96	47.88	**51**
52	52.71	90.32	26.65	61.71	95.53	28.13	59.52	89.72	18.77	46.67	**52**
53	51.68	89.27	25.58	60.62	94.42	26.99	58.36	88.55	17.58	45.46	**53**
54	50.66	88.22	24.51	59.53	93.31	25.86	57.21	87.38	16.38	44.25	**54**
55	49.63	87.17	23.43	58.43	92.19	24.73	56.06	86.20	15.19	43.03	**55**
56	48.59	86.12	22.36	57.34	91.08	23.59	54.90	85.03	14.00	41.82	**56**
57	47.56	85.07	21.29	56.25	89.96	22.46	53.75	83.86	12.80	40.61	**57**
58	46.53	84.02	20.21	55.15	88.85	21.32	52.59	82.68	11.61	39.40	**58**
59	45.50	82.96	19.14	54.06	87.73	20.19	51.44	81.51	10.41	38.18	**59**
60	4244.47	4181.91	4118.06	4052.96	3986.62	3919.05	3850.28	3780.33	3709.22	3636.97	**60**

Table XI.*

LENGTH OF A DEGREE OF LATITUDE.

Lat.	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	Lat.
′	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	′
0	5509.15	5509.97	5510.82	5511.67	5512.55	5513.44	5514.34	5515.25	5516.18	5517.11	0
1	09.16	09.99	10.83	11.69	12.56	13.45	14.35	15.27	16.19	17.13	1
2	09.17	10.00	10.84	11.70	12.58	13.47	14.37	15.28	16.21	17.14	2
3	09.19	10.01	10.86	11.72	12.59	13.48	14.38	15.30	16.22	17.16	3
4	09.20	10.03	10.87	11.73	12.61	13.50	14.40	15.31	16.24	17.17	4
5	09.21	10.04	10.89	11.75	12.62	13.51	14.42	15.33	16.25	17.19	5
6	09.23	10.06	10.90	11.76	12.64	13.53	14.43	15.34	16.27	17.20	6
7	09.24	10.07	10.91	11.78	12.65	13.54	14.45	15.36	16.28	17.22	7
8	09.25	10.08	10.93	11.79	12.67	13.56	14.46	15.38	16.30	17.23	8
9	09.27	10.10	10.94	11.81	12.68	13.57	14.48	15.39	16.32	17.25	9
10	09.28	10.11	10.96	11.82	12.70	13.59	14.49	15.41	16.33	17.27	10
11	09.30	10.13	10.97	11.83	12.71	13.60	14.51	15.42	16.35	17.28	11
12	09.31	10.14	10.99	11.85	12.73	13.62	14.52	15.44	16.36	17.30	12
13	09.32	10.15	11.00	11.86	12.74	13.63	14.54	15.45	16.38	17.31	13
14	09.34	10.17	11.01	11.88	12.76	13.65	14.55	15.47	16.39	17.33	14
15	09.35	10.18	11.03	11.89	12.77	13.66	14.57	15.48	16.41	17.34	15
16	09.36	10.19	11.04	11.91	12.79	13.68	14.58	15.50	16.42	17.36	16
17	09.38	10.21	11.06	11.92	12.80	13.69	14.60	15.51	16.44	17.38	17
18	09.39	10.22	11.07	11.94	12.81	13.71	14.61	15.53	16.46	17.39	18
19	09.41	10.24	11.09	11.95	12.83	13.72	14.63	15.54	16.47	17.41	19
20	09.42	10.25	11.10	11.96	12.84	13.74	14.64	15.56	16.49	17.42	20
21	09.43	10.26	11.11	11.98	12.86	13.75	14.66	15.57	16.50	17.44	21
22	09.45	10.28	11.13	11.99	12.87	13.77	14.67	15.59	16.52	17.45	22
23	09.46	10.29	11.14	12.01	12.89	13.78	14.69	15.61	16.53	17.47	23
24	09.47	10.31	11.16	12.02	12.90	13.80	14.70	15.62	16.55	17.49	24
25	09.49	10.32	11.17	12.04	12.92	13.81	14.72	15.64	16.56	17.50	25
26	09.50	10.33	11.19	12.05	12.93	13.83	14.73	15.65	16.58	17.52	26
27	09.51	10.35	11.20	12.07	12.95	13.84	14.75	15.67	16.60	17.53	27
28	09.53	10.36	11.21	12.08	12.96	13.86	14.76	15.68	16.61	17.55	28
29	09.54	10.38	11.23	12.10	12.98	13.87	14.78	15.70	16.63	17.56	29
30	09.56	10.39	11.24	12.11	12.99	13.89	14.79	15.71	16.64	17.58	30
31	09.57	10.41	11.26	12.12	13.01	13.90	14.81	15.73	16.66	17.60	31
32	09.58	10.42	11.27	12.14	13.02	13.92	14.82	15.74	16.67	17.61	32
33	09.60	10.44	11.29	12.15	13.04	13.93	14.84	15.76	16.69	17.63	33
34	09.61	10.45	11.30	12.17	13.05	13.95	14.86	15.77	16.70	17.64	34
35	09.63	10.46	11.31	12.18	13.07	13.96	14.87	15.79	16.72	17.66	35
36	09.64	10.48	11.33	12.20	13.08	13.98	14.89	15.81	16.74	17.67	36
37	09.65	10.49	11.34	12.21	13.10	13.99	14.90	15.82	16.75	17.69	37
38	09.67	10.50	11.36	12.22	13.11	14.01	14.92	15.84	16.77	17.71	38
39	09.68	10.52	11.37	12.24	13.13	14.02	14.93	15.85	16.78	17.72	39
40	09.69	10.53	11.39	12.26	13.14	14.04	14.95	15.87	16.80	17.74	40
41	09.71	10.55	11.40	12.27	13.16	14.05	14.96	15.88	16.81	17.75	41
42	09.72	10.56	11.42	12.29	13.17	14.07	14.98	15.90	16.83	17.77	42
43	09.74	10.57	11.43	12.30	13.18	14.08	14.99	15.91	16.84	17.78	43
44	09.75	10.59	11.44	12.31	13.20	14.10	15.01	15.93	16.86	17.80	44
45	09.76	10.60	11.46	12.33	13.21	14.11	15.02	15.94	16.88	17.82	45
46	09.78	10.62	11.47	12.34	13.23	14.13	15.04	15.95	16.89	17.83	46
47	09.79	10.63	11.49	12.36	13.24	14.14	15.05	15.98	16.91	17.85	47
48	09.80	10.65	11.50	12.37	13.26	14.16	15.07	15.99	16.92	17.86	48
49	09.82	10.66	11.52	12.39	13.27	14.17	15.08	16.01	16.94	17.88	49
50	09.83	10.67	11.53	12.40	13.29	14.19	15.10	16.02	16.95	17.89	50
51	09.85	10.69	11.54	12.42	13.30	14.20	15.11	16.04	16.97	17.91	51
52	09.86	10.70	11.56	12.43	13.32	14.22	15.13	16.05	16.98	17.93	52
53	09.87	10.72	11.57	12.45	13.33	14.23	15.15	16.07	17.00	17.94	53
54	09.89	10.73	11.59	12.46	13.35	14.25	15.16	16.08	17.02	17.96	54
55	09.90	10.74	11.60	12.48	13.36	14.26	15.18	16.10	17.03	17.97	55
56	09.92	10.76	11.62	12.49	13.38	14.28	15.19	16.11	17.05	17.99	56
57	09.93	10.77	11.63	12.51	13.39	14.29	15.21	16.13	17.06	18.00	57
58	09.94	10.79	11.65	12.52	13.41	14.31	15.22	16.15	17.08	18.02	58
59	09.96	10.80	11.66	12.53	13.42	14.32	15.24	16.16	17.09	18.04	59
60	5509.97	5510.82	5511.67	5512.55	5513.44	5514.34	5515.25	5516.18	5517.11	5518.05	60

* From the Manual of Instructions of 1894.

Table XI.—*Continued.*

LENGTH OF A DEGREE OF LATITUDE.

Lat.	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	Lat.
′	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	*Chains.*	′
0	5518.05	5519.00	5519.96	5520.92	5521.88	5522.85	5523.81	5524.78	5525.75	5526.72	0
1	18.07	19.02	19.97	20.93	21.90	22.86	23.83	24.80	25.77	26.73	1
2	18.08	19.03	19.99	20.95	21.91	22.88	23.85	24.82	25.78	26.75	2
3	18.10	19.05	20.00	20.96	21.93	22.89	23.86	24.83	25.80	26.76	3
4	18.11	19.06	20.02	20.98	21.94	22.91	23.88	24.85	25.82	26.78	4
5	18.13	19.08	20.04	21.00	21.96	22.93	23.90	24.86	25.83	26.80	5
6	18.15	19.10	20.05	21.01	21.98	22.94	23.91	24.88	25.85	26.81	6
7	18.16	19.11	20.07	21.03	21.99	22.96	23.93	24.90	25.86	26.83	7
8	18.18	19.13	20.08	21.04	22.01	22.98	23.94	24.91	25.88	26.84	8
9	18.19	19.14	20.10	21.06	22.02	22.99	23.96	24.93	25.90	26.86	9
10	18.21	19.16	20.12	21.08	22.04	23.01	23.98	24.94	25.91	26.88	10
11	18.22	19.18	20.13	21.09	22.06	23.02	23.99	24.96	25.93	26.89	11
12	18.24	19.19	20.15	21.11	22.07	23.04	24.01	24.98	25.94	26.91	12
13	18.26	19.21	20.16	21.12	22.09	23.06	24.02	24.99	25.96	26.92	13
14	18.27	19.22	20.18	21.14	22.11	23.07	24.04	25.01	25.98	26.94	14
15	18.29	19.24	20.20	21.16	22.12	23.09	24.06	25.03	25.99	26.96	15
16	18.30	19.25	20.21	21.17	22.14	23.10	24.07	25.04	26.01	26.97	16
17	18.32	19.27	20.23	21.19	22.15	23.12	24.09	25.06	26.02	26.99	17
18	18.34	19.29	20.24	21.20	22.17	23.14	24.11	25.07	26.04	27.00	18
19	18.35	19.30	20.26	21.22	22.19	23.15	24.12	25.09	26.06	27.02	19
20	18.37	19.32	20.28	21.24	22.20	23.17	24.14	25.11	26.07	27.04	20
21	18.38	19.33	20.29	21.25	22.22	23.19	24.15	25.12	26.09	27.05	21
22	18.40	19.35	20.31	21.27	22.23	23.20	24.17	25.14	26.10	27.07	22
23	18.41	19.37	20.32	21.29	22.25	23.22	24.19	25.15	26.12	27.09	23
24	18.43	19.38	20.34	21.30	22.27	23.23	24.20	25.17	26.14	27.10	24
25	18.45	19.40	20.36	21.32	22.28	23.25	24.22	25.19	26.15	27.12	25
26	18.46	19.41	20.37	21.33	22.30	23.27	24.23	25.20	26.17	27.13	26
27	18.48	19.43	20.39	21.35	22.31	23.28	24.25	25.22	26.19	27.15	27
28	18.49	19.45	20.40	21.36	22.33	23.30	24.27	25.23	26.20	27.17	28
29	18.51	19.46	20.42	21.38	22.35	23.31	24.28	25.25	26.22	27.18	29
30	18.53	19.48	20.44	21.40	22.36	23.33	24.30	25.27	26.23	27.20	30
31	18.54	19.49	20.45	21.41	22.38	23.35	24.32	25.28	26.25	27.21	31
32	18.56	19.51	20.47	21.43	22.40	23.36	24.33	25.30	26.27	27.23	32
33	18.57	19.53	20.48	21.45	22.41	23.38	24.35	25.32	26.28	27.25	33
34	18.59	19.54	20.50	21.46	22.43	23.40	24.36	25.33	26.30	27.26	34
35	18.60	19.56	20.52	21.48	22.44	23.41	24.38	25.35	26.31	27.28	35
36	18.62	19.57	20.53	21.49	22.46	23.43	24.40	25.36	26.33	27.29	36
37	18.64	19.59	20.55	21.51	22.48	23.44	24.41	25.38	26.35	27.31	37
38	18.65	19.60	20.56	21.53	22.49	23.46	24.43	25.40	26.36	27.33	38
39	18.67	19.62	20.58	21.54	22.51	23.48	24.44	25.41	26.38	27.34	39
40	18.68	19.64	20.60	21.56	22.52	23.49	24.46	25.43	26.39	27.36	40
41	18.70	19.65	20.61	21.57	22.54	23.51	24.48	25.44	26.41	27.37	41
42	18.72	19.67	20.63	21.59	22.56	23.52	24.49	25.46	26.43	27.39	42
43	18.73	19.68	20.64	21.61	22.57	23.54	24.51	25.48	26.44	27.41	43
44	18.75	19.70	20.66	21.62	22.59	23.56	24.52	25.49	26.46	27.42	44
45	18.76	19.72	20.68	21.64	22.60	23.57	24.54	25.51	26.47	27.44	45
46	18.78	19.73	20.69	21.65	22.62	23.59	24.56	25.52	26.49	27.45	46
47	18.79	19.75	20.71	21.67	22.64	23.60	24.57	25.54	26.51	27.47	47
48	18.81	19.76	20.72	21.69	22.65	23.62	24.59	25.56	26.52	27.49	48
49	18.83	19.78	20.74	21.70	22.67	23.64	24.61	25.57	26.54	27.50	49
50	18.84	19.80	20.76	21.72	22.69	23.65	24.62	25.59	26.56	27.52	50
51	18.86	19.81	20.77	21.74	22.70	23.67	24.64	25.61	26.57	27.53	51
52	18.87	19.83	20.79	21.75	22.72	23.69	24.65	25.62	26.59	27.55	52
53	18.89	19.84	20.80	21.77	22.73	23.70	24.67	25.64	26.60	27.57	53
54	18.91	19.86	20.82	21.78	22.75	23.72	24.69	25.65	26.62	27.58	54
55	18.92	19.88	20.84	21.80	22.77	23.73	24.70	25.67	26.64	27.60	55
56	18.94	19.89	20.85	21.82	22.78	23.75	24.72	25.69	26.65	27.61	56
57	18.95	19.91	20.87	21.83	22.80	23.77	24.73	25.70	26.67	27.63	57
58	18.97	19.92	20.88	21.85	22.81	23.78	24.75	25.72	26.68	27.65	58
59	18.98	19.94	20.90	21.86	22.83	23.80	24.77	25.73	26.70	27.66	59
60	5519.00	5519.96	5520.92	5521.88	5522.85	5523.81	5524.78	5525.75	5526.72	5527.68	60

TABLE XIa.

LENGTH IN FEET OF 1′ ARCS OF LATITUDE AND LONGITUDE.

Lat.	1′ Lat.	1′ Long.	Lat.	1′ Lat.	1′ Long.	Lat.	1′ Lat.	1′ Long.
1°	6045	6085	21°	6053	5684	41°	6072	4600
2	6045	6083	22	6054	5646	42	6073	4530
3	6045	6078	23	6054	5605	43	6074	4458
4	6045	6071	24	6055	5563	44	6075	4385
5	6045	6063	25	6056	5519	45	6076	4311
6	6045	6053	26	6057	5474	46	6077	4235
7	6046	6041	27	6058	5427	47	6078	4158
8	6046	6027	28	6059	5378	48	6079	4080
9	6046	6012	29	6060	5327	49	6080	4001
10	6047	5994	30	6061	5275	50	6081	3920
11	6047	5975	31	6061	5222	51	6082	3838
12	6048	5954	32	6062	5166	52	6084	3755
13	6048	5931	33	6063	5109	53	6085	3671
14	6049	5907	34	6064	5051	54	6086	3586
15	6049	5880	35	6065	4991	55	6087	3499
16	6050	5852	36	6066	4930	56	6088	3413
17	6050	5822	37	6067	4867	57	6089	3323
18	6051	5790	38	6068	4802	58	6090	3233
19	6052	5757	39	6070	4736	59	6091	3142
20	6052	5721	40	6071	4669	60	6092	3051

TABLE XII.*

INCLINATION AND CONVERGENCY OF THE MERIDIANS.

Lat.	Inclination for One Mile.	Inclination for Six Miles.	Convergency for One Township of 36 Miles.	Lat.	Inclination for One Mile.	Inclination for Six Miles.	Convergency for One Township of 36 Miles.	Lat.	Inclination for One Mile.	Inclination for Six Miles.	Convergency for One Township of 36 Miles.
°	″	′ ″	Links	°	″	′ ″	Links	°	′ ″	′ ″	Links
10	9.18	55	13.0	27	26.52	2 39	36.9	44	50.19	5 01	70.1
11	10.13	1 01	14.2	28	27.66	2 46	38.6	45	52.00	5 12	72.6
12	11.07	1 06	15.5	29	28.85	2 53	40.2	46	53.83	5 23	75.2
13	12.02	1 12	16.8	30	30.03	3 00	41.9	47	55.67	5 34	77.8
14	12.98	1 18	18.1	31	31.26	3 07	43.6	48	57.67	5 46	80.6
15	13.96	1 24	19.4	32	32.49	3 15	45.4	49	59.83	5 59	83.5
16	14.93	1 30	20.7	33	33.83	3 23	47.2	50	1 02.00	6 12	86.5
17	15.92	1 36	22.0	34	35.17	3 31	49.1	51	1 04.17	6 25	89.7
18	16.91	1 41	23.4	35	36.50	3 39	50.9	52	1 06.67	6 40	93.0
19	17.93	1 47	24.9	36	37.83	3 46	52.7	53	1 09.17	6 55	96.4
20	18.94	1 54	26.5	37	39.17	3 55	54.7	54	1 16.67	7 10	100.0
21	19.98	2 00	27.8	38	40.67	4 04	56.8	55	1 14.33	7 26	103.7
22	21.02	2 06	29.3	39	42.17	4 13	58.8	56	1 17.17	7 43	107.6
23	22.10	2 13	30.8	40	43.67	4 22	60.9	57	1 20.00	8 00	111.8
24	23.17	2 19	32.3	41	45.17	4 31	63.1	58	1 22.00	8 19	116.2
25	24.30	2 26	33.8	42	46.85	4 41	65.4	59	1 26.66	8 40	120.9
26	25.38	2 32	35.4	43	48.52	4 51	67.7	60	1 30.00	9 00	125.7

* From the catalogue of C. L. Berger & Sons, Boston.

Table XIII.

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
1	0432	0475	0518	0561	0604	0647	0689	0732	0775	0817
2	0860	0903	0945	0988	1030	1072	1115	1157	1199	1242
3	1284	1326	1368	1410	1452	1494	1536	1578	1620	1662
4	1703	1745	1787	1828	1870	1912	1953	1995	2036	2078
5	2119	2160	2202	2243	2284	2325	2366	2407	2449	2490
6	2531	2572	2612	2653	2694	2735	2776	2816	2857	2898
7	2938	2979	3019	3060	3100	3141	3181	3222	3262	3302
8	3342	3383	3423	3463	3503	3543	3583	3623	3663	3703
9	3743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
1	4532	4571	4610	4650	4689	4727	4766	4805	4844	4883
2	4922	4961	4999	5038	5077	5115	5154	5192	5231	5269
3	5308	5346	5385	5423	5461	5500	5538	5576	5614	5652
4	5690	5729	5767	5805	5843	5881	5918	5956	5994	6032
5	6070	6108	6145	6183	6221	6258	6296	6333	6371	6408
6	6446	6483	6521	6558	6595	6633	6670	6707	6744	6781
7	6819	6856	6893	6930	6967	7004	7041	7078	7115	7151
8	7188	7225	7262	7298	7335	7372	7408	7445	7482	7518
9	7555	7591	7628	7664	7700	7737	7773	7809	7846	7882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
1	8279	8314	8350	8386	8422	8458	8493	8529	8565	8600
2	8636	8672	8707	8743	8778	8814	8849	8884	8920	8955
3	8991	9026	9061	9096	9132	9167	9202	9237	9272	9307
4	9342	9377	9412	9447	9482	9517	9552	9587	9621	9656
5	9691	9726	9760	9795	9830	9864	9899	9934	9968	10003
6	10037	10072	10106	10140	10175	10209	10243	10278	10312	0346
7	0380	0415	0449	0483	0517	0551	0585	0619	0653	0687
8	0721	0755	0789	0823	0857	0890	0924	0958	0992	1025
9	1059	1093	1126	1160	1193	1227	1261	1294	1327	1361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
1	1727	1760	1793	1826	1860	1893	1926	1959	1992	2024
2	2057	2090	2123	2156	2189	2222	2254	2287	2320	2352
3	2385	2418	2450	2483	2516	2548	2581	2613	2646	2678
4	2710	2743	2775	2808	2840	2872	2905	2937	2969	3001
5	3033	3066	3098	3130	3162	3194	3226	3258	3290	3322
6	3354	3386	3418	3450	3481	3513	3545	3577	3609	3640
7	3672	3704	3735	3767	3799	3830	3862	3893	3925	3956
8	3988	4019	4051	4082	4114	4145	4176	4208	4239	4270
9	4301	4333	4364	4395	4426	4457	4489	4520	4551	4582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
1	4922	4953	4983	5014	5045	5076	5106	5137	5168	5198
2	5229	5259	5290	5320	5351	5381	5412	5442	5473	5503
3	5534	5564	5594	5625	5655	5685	5715	5746	5776	5806
4	5836	5866	5897	5927	5957	5987	6017	6047	6077	6107
5	6137	6167	6197	6227	6256	6286	6316	6346	6376	6406
6	6435	6465	6495	6524	6554	6584	6613	6643	6673	6702
7	6732	6761	6791	6820	6850	6879	6909	6938	6967	6997
8	7026	7056	7085	7114	7143	7173	7202	7231	7260	7289
9	7319	7348	7377	7406	7435	7464	7493	7522	7551	7580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
1	7898	7926	7955	7984	8013	8041	8070	8099	8127	8156
2	8184	8213	8241	8270	8298	8327	8355	8384	8412	8441
3	8469	8498	8526	8554	8583	8611	8639	8667	8696	8724
4	8752	8780	8808	8837	8865	8893	8921	8949	8977	9005
5	9033	9061	9089	9117	9145	9173	9201	9229	9257	9285
6	9312	9340	9368	9396	9424	9451	9479	9507	9535	9562
7	9590	9618	9645	9673	9700	9728	9756	9783	9811	9838
8	9866	9893	9921	9948	9976	20003	20030	20058	20085	20112
9	20140	20167	20194	20222	20249	0276	0303	0330	0358	0385
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
1	0683	0710	0737	0763	0790	0817	0844	0871	0898	0925
2	0952	0978	1005	1032	1059	1085	1112	1139	1165	1192
3	1219	1245	1272	1299	1325	1352	1378	1405	1431	1458
4	1484	1511	1537	1564	1590	1617	1643	1669	1696	1722
5	1748	1775	1801	1827	1854	1880	1906	1932	1958	1985
6	2011	2037	2063	2089	2115	2141	2167	2194	2220	2246
7	2272	2298	2324	2350	2376	2401	2427	2453	2479	2505
8	2531	2557	2583	2608	2634	2660	2686	2712	2737	2763
9	2789	2814	2840	2866	2891	2917	2943	2968	2994	3019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
1	3300	3325	3350	3376	3401	3426	3452	3477	3502	3528
2	3553	3578	3603	3629	3654	3679	3704	3729	3754	3779
3	3805	3830	3855	3880	3905	3930	3955	3980	4005	4030
4	4055	4080	4105	4130	4155	4180	4204	4229	4254	4279
5	4304	4329	4353	4378	4403	4428	4452	4477	4502	4527
6	4551	4576	4601	4625	4650	4674	4699	4724	4748	4773
7	4797	4822	4846	4871	4895	4920	4944	4969	4993	5018
8	5042	5066	5091	5115	5139	5164	5188	5212	5237	5261
9	5285	5310	5334	5358	5382	5406	5431	5455	5479	5503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
1	5768	5792	5816	5840	5864	5888	5912	5935	5959	5983
2	6007	6031	6055	6079	6102	6126	6150	6174	6198	6221
3	6245	6269	6293	6316	6340	6364	6387	6411	6435	6458
4	6482	6505	6529	6553	6576	6600	6623	6647	6670	6694
5	6717	6741	6764	6788	6811	6834	6858	6881	6905	6928
6	6951	6975	6998	7021	7045	7068	7091	7114	7138	7161
7	7184	7207	7231	7254	7277	7300	7323	7346	7370	7393
8	7416	7439	7462	7485	7508	7531	7554	7577	7600	7623
9	7646	7669	7692	7715	7738	7761	7784	7807	7830	7852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
1	8103	8126	8149	8171	8194	8217	8240	8262	8285	8307
2	8330	8353	8375	8398	8421	8443	8466	8488	8511	8533
3	8556	8578	8601	8623	8646	8668	8691	8713	8735	8758
4	8780	8803	8825	8847	8870	8892	8914	8937	8959	8981
5	9003	9026	9048	9070	9092	9115	9137	9159	9181	9203
6	9226	9248	9270	9292	9314	9336	9358	9380	9403	9425
7	9447	9469	9491	9513	9535	9557	9579	9601	9623	9645
8	9667	9688	9710	9732	9754	9776	9798	9820	9842	9863
9	9885	9907	9929	9951	9973	9994	30016	30038	30060	30081
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298

Table XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298
1	0320	0341	0363	0384	0406	0428	0449	0471	0492	0514
2	0535	0557	0578	0600	0621	0643	0664	0685	0707	0728
3	0750	0771	0792	0814	0835	0856	0878	0899	0920	0942
4	0963	0984	1006	1027	1048	1069	1091	1112	1133	1154
5	1175	1197	1218	1239	1260	1281	1302	1323	1345	1366
6	1387	1408	1429	1450	1471	1492	1513	1534	1555	1576
7	1597	1618	1639	1660	1681	1702	1723	1744	1765	1785
8	1806	1827	1848	1869	1890	1911	1931	1952	1973	1994
9	2015	2035	2056	2077	2098	2118	2139	2160	2181	2201
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408
1	2428	2449	2469	2490	2510	2531	2552	2572	2593	2613
2	2634	2654	2675	2695	2715	2736	2756	2777	2797	2818
3	2838	2858	2879	2899	2919	2940	2960	2980	3001	3021
4	3041	3062	3082	3102	3122	3143	3163	3183	3203	3224
5	3244	3264	3284	3304	3325	3345	3365	3385	3405	3425
6	3445	3465	3486	3506	3526	3546	3566	3586	3606	3626
7	3646	3666	3686	3706	3726	3746	3766	3786	3806	3826
8	3846	3866	3885	3905	3925	3945	3965	3985	4005	4025
9	4044	4064	4084	4104	4124	4143	4163	4183	4203	4223
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420
1	4439	4459	4479	4498	4518	4537	4557	4577	4596	4616
2	4635	4655	4674	4694	4713	4733	4753	4772	4792	4811
3	4830	4850	4869	4889	4908	4928	4947	4967	4986	5005
4	5025	5044	5064	5083	5102	5122	5141	5160	5180	5199
5	5218	5238	5257	5276	5295	5315	5334	5353	5372	5392
6	5411	5430	5449	5468	5488	5507	5526	5545	5564	5583
7	5603	5622	5641	5660	5679	5698	5717	5736	5755	5774
8	5793	5813	5832	5851	5870	5889	5908	5927	5946	5965
9	5984	6003	6021	6040	6059	6078	6097	6116	6135	6154
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342
1	6361	6380	6399	6418	6436	6455	6474	6493	6511	6530
2	6549	6568	6586	6605	6624	6642	6661	6680	6698	6717
3	6736	6754	6773	6791	6810	6829	6847	6866	6884	6903
4	6922	6940	6959	6977	6996	7014	7033	7051	7070	7088
5	7107	7125	7144	7162	7181	7199	7218	7236	7254	7273
6	7291	7310	7328	7346	7365	7383	7401	7420	7438	7457
7	7475	7493	7511	7530	7548	7566	7585	7603	7621	7639
8	7658	7676	7694	7712	7731	7749	7767	7785	7803	7822
9	7840	7858	7876	7894	7912	7931	7949	7967	7985	8003
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184
1	8202	8220	8238	8256	8274	8292	8310	8328	8346	8364
2	8382	8399	8417	8435	8453	8471	8489	8507	8525	8543
3	8561	8578	8596	8614	8632	8650	8668	8686	8703	8721
4	8739	8757	8775	8792	8810	8828	8846	8863	8881	8899
5	8917	8034	8952	8970	8987	9005	9023	9041	9058	9076
6	9094	9111	9129	9146	9164	9182	9199	9217	9235	9252
7	9270	9287	9305	9322	9340	9358	9375	9393	9410	9428
8	9445	9463	9480	9498	9515	9533	9550	9568	9585	9602
9	9620	9637	9655	9672	9690	9707	9724	9742	9759	9777
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950

Table XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950
1	9967	9985	40002	40019	40037	40054	40071	40088	40106	40123
2	40140	40157	0175	0192	0209	0226	0243	0261	0278	0295
3	0312	0329	0346	0364	0381	0398	0415	0432	0449	0466
4	0483	0500	0518	0535	0552	0569	0586	0603	0620	0637
5	0654	0671	0688	0705	0722	0739	0756	0773	0790	0807
6	0824	0841	0858	0875	0892	0909	0926	0943	0960	0976
7	0993	1010	1027	1044	1061	1078	1095	1111	1128	1145
8	1162	1179	1196	1212	1229	1246	1263	1280	1296	1313
9	1330	1347	1363	1380	1397	1414	1430	1447	1464	1481
260	41497	41514	41531	41547	41564	41581	41597	41614	41631	41647
1	1664	1681	1697	1714	1731	1747	1764	1780	1797	1814
2	1830	1847	1863	1880	1896	1913	1929	1946	1963	1979
3	1996	2012	2029	2045	2062	2078	2095	2111	2127	2144
4	2160	2177	2193	2210	2226	2243	2259	2275	2292	2308
5	2325	2341	2357	2374	2390	2406	2423	2439	2455	2472
6	2488	2504	2521	2537	2553	2570	2586	2602	2619	2635
7	2651	2667	2684	2700	2716	2732	2749	2765	2781	2797
8	2813	2830	2846	2862	2878	2894	2911	2927	2943	2959
9	2975	2991	3008	3024	3040	3056	3072	3088	3104	3120
270	43136	43152	43169	43185	43201	43217	43233	43249	43265	43281
1	3297	3313	3329	3345	3361	3377	3393	3409	3425	3441
2	3457	3473	3489	3505	3521	3537	3553	3569	3584	3600
3	3616	3632	3648	3664	3680	3696	3712	3727	3743	3759
4	3775	3791	3807	3823	3838	3854	3870	3886	3902	3917
5	3933	3949	3965	3981	3996	4012	4028	4044	4059	4075
6	4091	4107	4122	4138	4154	4170	4185	4201	4217	4232
7	4248	4264	4279	4295	4311	4326	4342	4358	4373	4389
8	4404	4420	4436	4451	4467	4483	4498	4514	4529	4545
9	4560	4576	4592	4607	4623	4638	4654	4669	4685	4700
280	44716	44731	44747	44762	44778	44793	44809	44824	44840	44855
1	4871	4886	4902	4917	4932	4948	4963	4979	4994	5010
2	5025	5040	5056	5071	5086	5102	5117	5133	5148	5163
3	5179	5194	5209	5225	5240	5255	5271	5286	5301	5317
4	5332	5347	5362	5378	5393	5408	5423	5439	5454	5469
5	5484	5500	5515	5530	5545	5561	5576	5591	5606	5621
6	5637	5652	5667	5682	5697	5712	5728	5743	5758	5773
7	5788	5803	5818	5834	5849	5864	5879	5894	5909	5924
8	5939	5954	5969	5984	6000	6015	6030	6045	6060	6075
9	6090	6105	6120	6135	6150	6165	6180	6195	6210	6225
290	46240	46255	46270	46285	46300	46315	46330	46345	46359	46374
1	6389	6404	6419	6434	6449	6464	6479	6494	6509	6523
2	6538	6553	6568	6583	6598	6613	6627	6642	6657	6672
3	6687	6702	6716	6731	6746	6761	6776	6790	6805	6820
4	6835	6850	6864	6879	6894	6909	6923	6938	6953	6967
5	6982	6997	7012	7026	7041	7056	7070	7085	7100	7114
6	7129	7144	7159	7173	7188	7202	7217	7232	7246	7261
7	7276	7290	7305	7319	7334	7349	7363	7378	7392	7407
8	7422	7436	7451	7465	7480	7494	7509	7524	7538	7553
9	7567	7582	7596	7611	7625	7640	7654	7669	7683	7698
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842
1	7857	7871	7885	7900	7914	7929	7943	7958	7972	7986
2	8001	8015	8029	8044	8058	8073	8087	8101	8116	8130
3	8144	8159	8173	8187	8202	8216	8230	8244	8259	8273
4	8287	8302	8316	8330	8344	8359	8373	8387	8401	8416
5	8430	8444	8458	8473	8487	8501	8515	8530	8544	8558
6	8572	8586	8601	8615	8629	8643	8657	8671	8686	8700
7	8714	8728	8742	8756	8770	8785	8799	8813	8827	8841
8	8855	8869	8883	8897	8911	8926	8940	8954	8968	8982
9	8996	9010	9024	9038	9052	9066	9080	9094	9108	9122
310	49136	49150	49164	49178	49192	49206	49220	49234	49248	49262
1	9276	9290	9304	9318	9332	9346	9360	9374	9388	9402
2	9415	9429	9443	9457	9471	9485	9499	9513	9527	9541
3	9554	9568	9582	9596	9610	9624	9638	9651	9665	9679
4	9693	9707	9721	9734	9748	9762	9776	9790	9803	9817
5	9831	9845	9859	9872	9886	9900	9914	9927	9941	9955
6	9969	9982	9996	50010	50024	50037	50051	50065	50079	50092
7	50106	50120	50133	0147	0161	0174	0188	0202	0215	0229
8	0243	0256	0270	0284	0297	0311	0325	0338	0352	0365
9	0379	0393	0406	0420	0433	0447	0461	0474	0488	0501
320	50515	50529	50542	50556	50569	50583	50596	50610	50623	50637
1	0651	0664	0678	0691	0705	0718	0732	0745	0759	0772
2	0786	0799	0813	0826	0840	0853	0866	0880	0893	0907
3	0920	0934	0947	0961	0974	0987	1001	1014	1028	1041
4	1055	1068	1081	1095	1108	1121	1135	1148	1162	1175
5	1188	1202	1215	1228	1242	1255	1268	1282	1295	1308
6	1322	1335	1348	1362	1375	1388	1402	1415	1428	1441
7	1455	1468	1481	1495	1508	1521	1534	1548	1561	1574
8	1587	1601	1614	1627	1640	1654	1667	1680	1693	1706
9	1720	1733	1746	1759	1772	1786	1799	1812	1825	1838
330	51851	51865	51878	51891	51904	51917	51930	51943	51957	51970
1	1983	1996	2009	2022	2035	2048	2061	2075	2088	2101
2	2114	2127	2140	2153	2166	2179	2192	2205	2218	2231
3	2244	2257	2270	2284	2297	2310	2323	2336	2349	2362
4	2375	2388	2401	2414	2427	2440	2453	2466	2479	2492
5	2504	2517	2530	2543	2556	2569	2582	2595	2608	2621
6	2634	2647	2660	2673	2686	2699	2711	2724	2737	2750
7	2763	2776	2789	2802	2815	2827	2840	2853	2866	2879
8	2892	2905	2917	2930	2943	2956	2969	2982	2994	3007
9	3020	3033	3046	3058	3071	3084	3097	3110	3122	3135
340	53148	53161	53173	53186	53199	53212	53224	53237	53250	53263
1	3275	3288	3301	3314	3326	3339	3352	3364	3377	3390
2	3403	3415	3428	3441	3453	3466	3479	3491	3504	3517
3	3529	3542	3555	3567	3580	3593	3605	3618	3631	3643
4	3656	3668	3681	3694	3706	3719	3732	3744	3757	3769
5	3782	3794	3807	3820	3832	3845	3857	3870	3882	3895
6	3908	3920	3933	3945	3958	3970	3983	3995	4008	4020
7	4033	4045	4058	4070	4083	4095	4108	4120	4133	4145
8	4158	4170	4183	4195	4208	4220	4233	4245	4258	4270
9	4283	4295	4307	4320	4332	4345	4357	4370	4382	4394
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518
1	4531	4543	4555	4568	4580	4593	4605	4617	4630	4642
2	4654	4667	4679	4691	4704	4716	4728	4741	4753	4765
3	4777	4790	4802	4814	4827	4839	4851	4864	4876	4888
4	4900	4913	4925	4937	4949	4962	4974	4986	4998	5011
5	5023	5035	5047	5060	5072	5084	5096	5108	5121	5133
6	5145	5157	5169	5182	5194	5206	5218	5230	5242	5255
7	5267	5279	5291	5303	5315	5328	5340	5352	5364	5376
8	5388	5400	5413	5425	5437	5449	5461	5473	5485	5497
9	5509	5522	5534	5546	5558	5570	5582	5594	5606	5618
360	55630	55642	55654	55666	55678	55691	55703	55715	55727	55739
1	5751	5763	5775	5787	5799	5811	5823	5835	5847	5859
2	5871	5883	5895	5907	5919	5931	5943	5955	5967	5979
3	5991	6003	6015	6027	6038	6050	6062	6074	6086	6098
4	6110	6122	6134	6146	6158	6170	6182	6194	6205	6217
5	6229	6241	6253	6265	6277	6289	6301	6312	6324	6336
6	6348	6360	6372	6384	6396	6407	6419	6431	6443	6455
7	6467	6478	6490	6502	6514	6526	6538	6549	6561	6573
8	6585	6597	6608	6620	6632	6644	6656	6667	6679	6691
9	6703	6714	6726	6738	6750	6761	6773	6785	6797	6808
370	56820	56832	56844	56855	56867	56879	56891	56902	56914	56926
1	6937	6949	6961	6972	6984	6996	7008	7019	7031	7043
2	7054	7066	7078	7089	7101	7113	7124	7136	7148	7159
3	7171	7183	7194	7206	7217	7229	7241	7252	7264	7276
4	7287	7299	7310	7322	7334	7345	7357	7368	7380	7392
5	7403	7415	7426	7438	7449	7461	7473	7484	7496	7507
6	7519	7530	7542	7553	7565	7576	7588	7600	7611	7623
7	7634	7646	7657	7669	7680	7692	7703	7715	7726	7738
8	7749	7761	7772	7784	7795	7807	7818	7830	7841	7852
9	7864	7875	7887	7898	7910	7921	7933	7944	7955	7967
380	57978	57990	58001	58013	58024	58035	58047	58058	58070	58081
1	8092	8104	8115	8127	8138	8149	8161	8172	8184	8195
2	8206	8218	8229	8240	8252	8263	8274	8286	8297	8309
3	8320	8331	8343	8354	8365	8377	8388	8399	8410	8422
4	8433	8444	8456	8467	8478	8490	8501	8512	8524	8535
5	8546	8557	8569	8580	8591	8602	8614	8625	8636	8647
6	8659	8670	8681	8692	8704	8715	8726	8737	8749	8760
7	8771	8782	8794	8805	8816	8827	8838	8850	8861	8872
8	8883	8894	8906	8917	8928	8939	8950	8961	8973	8984
9	8995	9006	9017	9028	9040	9051	9062	9073	9084	9095
390	59106	59118	59129	59140	59151	59162	59173	59184	59195	59207
1	9218	9229	9240	9251	9262	9273	9284	9295	9306	9318
2	9329	9340	9351	9362	9373	9384	9395	9406	9417	9428
3	9439	9450	9461	9472	9483	9494	9506	9517	9528	9539
4	9550	9561	9572	9583	9594	9605	9616	9627	9638	9649
5	9660	9671	9682	9693	9704	9715	9726	9737	9748	9759
6	9770	9780	9791	9802	9813	9824	9835	9846	9857	9868
7	9879	9890	9901	9912	9923	9934	9945	9956	9966	9977
8	9988	9999	60010	60021	60032	60043	60054	60065	60076	60086
9	60097	60108	0119	0130	0141	0152	0163	0173	0184	0195
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304
1	0314	0325	0336	0347	0358	0369	0379	0390	0401	0412
2	0423	0433	0444	0455	0466	0477	0487	0498	0509	0520
3	0531	0541	0552	0563	0574	0584	0595	0606	0617	0627
4	0638	0649	0660	0670	0681	0692	0703	0713	0724	0735
5	0746	0756	0767	0778	0788	0799	0810	0821	0831	0842
6	0853	0863	0874	0885	0895	0906	0917	0927	0938	0949
7	0959	0970	0981	0991	1002	1013	1023	1034	1045	1055
8	1066	1077	1087	1098	1109	1119	1130	1140	1151	1162
9	1172	1183	1194	1204	1215	1225	1236	1247	1257	1268
410	61278	61289	61300	61310	61321	61331	61342	61352	61363	61374
1	1384	1395	1405	1416	1426	1437	1448	1458	1469	1479
2	1490	1500	1511	1521	1532	1542	1553	1563	1574	1584
3	1595	1606	1616	1627	1637	1648	1658	1669	1679	1690
4	1700	1711	1721	1731	1742	1752	1763	1773	1784	1794
5	1805	1815	1826	1836	1847	1857	1868	1878	1888	1899
6	1909	1920	1930	1941	1951	1962	1972	1982	1993	2003
7	2014	2024	2034	2045	2055	2066	2076	2086	2097	2107
8	2118	2128	2138	2149	2159	2170	2180	2190	2201	2211
9	2221	2232	2242	2252	2263	2273	2284	2294	2304	2315
420	62325	62335	62346	62356	62366	62377	62387	62397	62408	62418
1	2428	2439	2449	2459	2469	2480	2490	2500	2511	2521
2	2531	2542	2552	2562	2572	2583	2593	2603	2613	2624
3	2634	2644	2655	2665	2675	2685	2696	2706	2716	2726
4	2737	2747	2757	2767	2778	2788	2798	2808	2818	2829
5	2839	2849	2859	2870	2880	2890	2900	2910	2921	2931
6	2941	2951	2961	2972	2982	2992	3002	3012	3022	3033
7	3043	3053	3063	3073	3083	3094	3104	3114	3124	3134
8	3144	3155	3165	3175	3185	3195	3205	3215	3225	3236
9	3246	3256	3266	3276	3286	3296	3306	3317	3327	3337
430	63347	63357	63367	63377	63387	63397	63407	63417	63428	63438
1	3448	3458	3468	3478	3488	3498	3508	3518	3528	3538
2	3548	3558	3568	3579	3589	3599	3609	3619	3629	3639
3	3649	3659	3669	3679	3689	3699	3709	3719	3729	3739
4	3749	3759	3769	3779	3789	3799	3809	3819	3829	3839
5	3849	3859	3869	3879	3889	3899	3909	3919	3929	3939
6	3949	3959	3969	3979	3988	3998	4008	4018	4028	4038
7	4048	4058	4068	4078	4088	4098	4108	4118	4128	4137
8	4147	4157	4167	4177	4187	4197	4207	4217	4227	4237
9	4246	4256	4266	4276	4286	4296	4306	4316	4326	4335
440	64345	64355	64365	64375	64385	64395	64404	64414	64424	64434
1	4444	4454	4464	4473	4483	4493	4503	4513	4523	4532
2	4542	4552	4562	4572	4582	4591	4601	4611	4621	4631
3	4640	4650	4660	4670	4680	4689	4699	4709	4719	4729
4	4738	4748	4758	4768	4777	4787	4797	4807	4816	4826
5	4836	4846	4856	4865	4875	4885	4895	4904	4914	4924
6	4933	4943	4953	4963	4972	4982	4992	5002	5011	5021
7	5031	5040	5050	5060	5070	5079	5089	5099	5108	5118
8	5128	5137	5147	5157	5167	5176	5186	5196	5205	5215
9	5225	5234	5244	5254	5263	5273	5283	5292	5302	5312
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408
1	5418	5427	5437	5447	5456	5466	5475	5485	5495	5504
2	5514	5523	5533	5543	5552	5562	5571	5581	5591	5600
3	5610	5619	5629	5639	5648	5658	5667	5677	5686	5696
4	5706	5715	5725	5734	5744	5753	5763	5772	5782	5792
5	5801	5811	5820	5830	5839	5849	5858	5868	5877	5887
6	5896	5906	5916	5925	5935	5944	5954	5963	5973	5982
7	5992	6001	6011	6020	6030	6039	6049	6058	6068	6077
8	6087	6096	6106	6115	6124	6134	6143	6153	6162	6172
9	6181	6191	6200	6210	6219	6229	6238	6247	6257	6266
460	66276	66285	66295	66304	66314	66323	66332	66342	66351	66361
1	6370	6380	6389	6398	6408	6417	6427	6436	6445	6455
2	6464	6474	6483	6492	6502	6511	6521	6530	6539	6549
3	6558	6567	6577	6586	6596	6605	6614	6624	6633	6642
4	6652	6661	6671	6680	6689	6699	6708	6717	6727	6736
5	6745	6755	6764	6773	6783	6792	6801	6811	6820	6829
6	6839	6848	6857	6867	6876	6885	6894	6904	6913	6922
7	6932	6941	6950	6960	6969	6978	6987	6997	7006	7015
8	7025	7034	7043	7052	7062	7071	7080	7089	7099	7108
9	7117	7127	7136	7145	7154	7164	7173	7182	7191	7201
470	67210	67219	67228	67237	67247	67256	67265	67274	67284	67293
1	7302	7311	7321	7330	7339	7348	7357	7367	7376	7385
2	7394	7403	7413	7422	7431	7440	7449	7459	7468	7477
3	7486	7495	7504	7514	7523	7532	7541	7550	7560	7569
4	7578	7587	7596	7605	7614	7624	7633	7642	7651	7660
5	7669	7679	7688	7697	7706	7715	7724	7733	7742	7752
6	7761	7770	7779	7788	7797	7806	7815	7825	7834	7843
7	7852	7861	7870	7879	7888	7897	7906	7916	7925	7934
8	7943	7952	7961	7970	7979	7988	7997	8006	8015	8024
9	8034	8043	8052	8061	8070	8079	8088	8097	8106	8115
480	68124	68133	68142	68151	68160	68169	68178	68187	68196	68205
1	8215	8224	8233	8242	8251	8260	8269	8278	8287	8296
2	8305	8314	8323	8332	8341	8350	8359	8368	8377	8386
3	8395	8404	8413	8422	8431	8440	8449	8458	8467	8476
4	8485	8494	8502	8511	8520	8529	8538	8547	8556	8565
5	8574	8583	8592	8601	8610	8619	8628	8637	8646	8655
6	8664	8673	8681	8690	8699	8708	8717	8726	8735	8744
7	8753	8762	8771	8780	8789	8797	8806	8815	8824	8833
8	8842	8851	8860	8869	8878	8886	8895	8904	8913	8922
9	8931	8940	8949	8958	8966	8975	8984	8993	9002	9011
490	69020	69028	69037	69046	69055	69064	69073	69082	69090	69099
1	9108	9117	9126	9135	9144	9152	9161	9170	9179	9188
2	9197	9205	9214	9223	9232	9241	9249	9258	9267	9276
3	9285	9294	9302	9311	9320	9329	9338	9346	9355	9364
4	9373	9381	9390	9399	9408	9417	9425	9434	9443	9452
5	9461	9469	9478	9487	9496	9504	9513	9522	9531	9539
6	9548	9557	9566	9574	9583	9592	9601	9609	9618	9627
7	9636	9644	9653	9662	9671	9679	9688	9697	9705	9714
8	9723	9732	9740	9749	9758	9767	9775	9784	9793	9801
9	9810	9819	9827	9836	9845	9854	9862	9871	9880	9888
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975

Table XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975
1	9984	9992	70001	70010	70018	70027	70036	70044	70053	70062
2	70070	70079	0088	0096	0105	0114	0122	0131	0140	0148
3	0157	0165	0174	0183	0191	0200	0209	0217	0226	0234
4	0243	0252	0260	0269	0278	0286	0295	0303	0312	0321
5	0329	0338	0346	0355	0364	0372	0381	0389	0398	0406
6	0415	0424	0432	0441	0449	0458	0467	0475	0484	0492
7	0501	0509	0518	0526	0535	0544	0552	0561	0569	0578
8	0586	0595	0603	0612	0621	0629	0638	0646	0655	0663
9	0672	0680	0689	0697	0706	0714	0723	0731	0740	0749
510	70757	70766	70774	70783	70791	70800	70808	70817	70825	70834
1	0842	0851	0859	0868	0876	0885	0893	0902	0910	0919
2	0927	0935	0944	0952	0961	0969	0978	0986	0995	1003
3	1012	1020	1029	1037	1046	1054	1063	1071	1079	1088
4	1096	1105	1113	1122	1130	1139	1147	1155	1164	1172
5	1181	1189	1198	1206	1214	1223	1231	1240	1248	1257
6	1265	1273	1282	1290	1299	1307	1315	1324	1332	1341
7	1349	1357	1366	1374	1383	1391	1399	1408	1416	1425
8	1433	1441	1450	1458	1466	1475	1483	1492	1500	1508
9	1517	1525	1533	1542	1550	1559	1567	1575	1584	1592
520	71600	71609	71617	71625	71634	71642	71650	71659	71667	71675
1	1684	1692	1700	1709	1717	1725	1734	1742	1750	1759
2	1767	1775	1784	1792	1800	1809	1817	1825	1834	1842
3	1850	1858	1867	1875	1883	1892	1900	1908	1917	1925
4	1933	1941	1950	1958	1966	1975	1983	1991	1999	2008
5	2016	2024	2032	2041	2049	2057	2066	2074	2082	2090
6	2099	2107	2115	2123	2132	2140	2148	2156	2165	2173
7	2181	2189	2198	2206	2214	2222	2230	2239	2247	2255
8	2263	2272	2280	2288	2296	2304	2313	2321	2329	2337
9	2346	2354	2362	2370	2378	2387	2395	2403	2411	2419
530	72428	72436	72444	72452	72460	72469	72477	72485	72493	72501
1	2509	2518	2526	2534	2542	2550	2558	2567	2575	2583
2	2591	2599	2607	2616	2624	2632	2640	2648	2656	2665
3	2673	2681	2689	2697	2705	2713	2722	2730	2738	2746
4	2754	2762	2770	2779	2787	2795	2803	2811	2819	2827
5	2835	2843	2852	2860	2868	2876	2884	2892	2900	2908
6	2916	2925	2933	2941	2949	2957	2965	2973	2981	2989
7	2997	3006	3014	3022	3030	3038	3046	3054	3062	3070
8	3078	3086	3094	3102	3111	3119	3127	3135	3143	3151
9	3159	3167	3175	3183	3191	3199	3207	3215	3223	3231
540	73239	73247	73255	73263	73272	73280	73288	73296	73304	73312
1	3320	3328	3336	3344	3352	3360	3368	3376	3384	3392
2	3400	3408	3416	3424	3432	3440	3448	3456	3464	3472
3	3480	3488	3496	3504	3512	3520	3528	3536	3544	3552
4	3560	3568	3576	3584	3592	3600	3608	3616	3624	3632
5	3640	3648	3656	3664	3672	3679	3687	3695	3703	3711
6	3719	3727	3735	3743	3751	3759	3767	3775	3783	3791
7	3799	3807	3815	3823	3830	3838	3846	3854	3862	3870
8	3878	3886	3894	3902	3910	3918	3926	3933	3941	3949
9	3957	3965	3973	3981	3989	3997	4005	4013	4020	4028
550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
550	74036	74044	74052	74060	74068	74076	74084	74092	74099*	74107
1	4115	4123	4131	4139	4147	4155	4162	4170	4178	4186
2	4194	4202	4210	4218	4225	4233	4241	4249	4257	4265
3	4273	4280	4288	4296	4304	4312	4320	4327	4335	4343
4	4351	4359	4367	4374	4382	4390	4398	4406	4414	4421
5	4429	4437	4445	4453	4461	4468	4476	4484	4492	4500
6	4507	4515	4523	4531	4539	4547	4554	4562	4570	4578
7	4586	4593	4601	4609	4617	4624	4632	4640	4648	4656
8	4663	4671	4679	4687	4695	4702	4710	4718	4726	4733
9	4741	4749	4757	4764	4772	4780	4788	4796	4803	4811
560	74819	74827	74834	74842	74850	74858	74865	74873	74881	74889
1	4896	4904	4912	4920	4927	4935	4943	4950	4958	4966
2	4974	4981	4989	4997	5005	5012	5020	5028	5035	5043
3	5051	5059	5066	5074	5082	5089	5097	5105	5113	5120
4	5128	5136	5143	5151	5159	5166	5174	5182	5189	5197
5	5205	5213	5220	5228	5236	5243	5251	5259	5266	5274
6	5282	5289	5297	5305	5312	5320	5328	5335	5343	5351
7	5358	5366	5374	5381	5389	5397	5404	5412	5420	5427
8	5435	5442	5450	5458	5465	5473	5481	5488	5496	5504
9	5511	5519	5526	5534	5542	5549	5557	5565	5572	5580
570	75587	75595	75603	75610	75618	75626	75633	75641	75648	75656
1	5664	5671	5679	5686	5694	5702	5709	5717	5724	5732
2	5740	5747	5755	5762	5770	5778	5785	5793	5800	5808
3	5815	5823	5831	5838	5846	5853	5861	5868	5876	5884
4	5891	5899	5906	5914	5921	5929	5937	5944	5952	5959
5	5967	5974	5982	5989	5997	6005	6012	6020	6027	6035
6	6042	6050	6057	6065	6072	6080	6087	6095	6103	6110
7	6118	6125	6133	6140	6148	6155	6163	6170	6178	6185
8	6193	6200	6208	6215	6223	6230	6238	6245	6253	6260
9	6268	6275	6283	6290	6298	6305	6313	6320	6328	6335
580	76343	76350	76358	76365	76373	76380	76388	76395	76403	76410
1	6418	6425	6433	6440	6448	6455	6462	6470	6477	6485
2	6492	6500	6507	6515	6522	6530	6537	6545	6552	6559
3	6567	6574	6582	6589	6597	6604	6612	6619	6626	6634
4	6641	6649	6656	6664	6671	6678	6686	6693	6701	6708
5	6716	6723	6730	6738	6745	6753	6760	6768	6775	6782
6	6790	6797	6805	6812	6819	6827	6834	6842	6849	6856
7	6864	6871	6879	6886	6893	6901	6908	6916	6923	6930
8	6938	6945	6953	6960	6967	6975	6982	6989	6997	7004
9	7012	7019	7026	7034	7041	7048	7056	7063	7070	7078
590	77085	77093	77100	77107	77115	77122	77129	77137	77144	77151
1	7159	7166	7173	7181	7188	7195	7203	7210	7217	7225
2	7232	7240	7247	7254	7262	7269	7276	7283	7291	7298
3	7305	7313	7320	7327	7335	7342	7349	7357	7364	7371
4	7379	7386	7393	7401	7408	7415	7422	7430	7437	7444
5	7452	7459	7466	7474	7481	7488	7495	7503	7510	7517
6	7525	7532	7539	7546	7554	7561	7568	7576	7583	7590
7	7597	7605	7612	7619	7627	7634	7641	7648	7656	7663
8	7670	7677	7685	7692	7699	7706	7714	7721	7728	7735
9	7743	7750	7757	7764	7772	7779	7786	7793	7801	7808
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880

Table XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880
1	7887	7895	7902	7909	7916	7924	7931	7938	7945	7952
2	7960	7967	7974	7981	7988	7996	8003	8010	8017	8025
3	8032	8039	8046	8053	8061	8068	8075	8082	8089	8097
4	8104	8111	8118	8125	8132	8140	8147	8154	8161	8168
5	8176	8183	8190	8197	8204	8211	8219	8226	8233	8240
6	8247	8254	8262	8269	8276	8283	8290	8297	8305	8312
7	8319	8326	8333	8340	8347	8355	8362	8369	8376	8383
8	8390	8398	8405	8412	8419	8426	8433	8440	8447	8455
9	8462	8469	8476	8483	8490	8497	8504	8512	8519	8526
610	78533	78540	78547	78554	78561	78569	78576	78583	78590	78597
1	8604	8611	8618	8625	8633	8640	8647	8654	8661	8668
2	8675	8682	8689	8696	8704	8711	8718	8725	8732	8739
3	8746	8753	8760	8767	8774	8781	8789	8796	8803	8810
4	8817	8824	8831	8838	8845	8852	8859	8866	8873	8880
5	8888	8895	8902	8909	8916	8923	8930	8937	8944	8951
6	8958	8965	8972	8979	8986	8993	9000	9007	9014	9021
7	9029	9036	9043	9050	9057	9064	9071	9078	9085	9092
8	9099	9106	9113	9120	9127	9134	9141	9148	9155	9162
9	9169	9176	9183	9190	9197	9204	9211	9218	9225	9232
620	79239	79246	79253	79260	79267	79274	79281	79288	79295	79302
1	9309	9316	9323	9330	9337	9344	9351	9358	9365	9372
2	9379	9386	9393	9400	9407	9414	9421	9428	9435	9442
3	9449	9456	9463	9470	9477	9484	9491	9498	9505	9511
4	9518	9525	9532	9539	9546	9553	9560	9567	9574	9581
5	9588	9595	9602	9609	9616	9623	9630	9637	9644	9650
6	9657	9664	9671	9678	9685	9692	9699	9706	9713	9720
7	9727	9734	9741	9748	9754	9761	9768	9775	9782	9789
8	9796	9803	9810	9817	9824	9831	9837	9844	9851	9858
9	9865	9872	9879	9886	9893	9900	9906	9913	9920	9927
630	79934	79941	79948	79955	79962	79969	79975	79982	79989	79996
1	80003	80010	80017	80024	80030	80037	80044	80051	80058	80065
2	0072	0079	0085	0092	0099	0106	0113	0120	0127	0134
3	0140	0147	0154	0161	0168	0175	0182	0188	0195	0202
4	0209	0216	0223	0229	0236	0243	0250	0257	0264	0271
5	0277	0284	0291	0298	0305	0312	0318	0325	0332	0339
6	0346	0353	0359	0366	0373	0380	0387	0393	0400	0407
7	0414	0421	0428	0434	0441	0448	0455	0462	0468	0475
8	0482	0489	0496	0502	0509	0516	0523	0530	0536	0543
9	0550	0557	0564	0570	0577	0584	0591	0598	0604	0611
640	80618	80625	80632	80638	80645	80652	80659	80665	80672	80679
1	0686	0693	0699	0706	0713	0720	0726	0733	0740	0747
2	0754	0760	0767	0774	0781	0787	0794	0801	0808	0814
3	0821	0828	0835	0841	0848	0855	0862	0868	0875	0882
4	0889	0895	0902	0909	0916	0922	0929	0936	0943	0949
5	0956	0963	0969	0976	0983	0990	0996	1003	1010	1017
6	1023	1030	1037	1043	1050	1057	1064	1070	1077	1084
7	1090	1097	1104	1111	1117	1124	1131	1137	1144	1151
8	1158	1164	1171	1178	1184	1191	1198	1204	1211	1218
9	1224	1231	1238	1245	1251	1258	1265	1271	1278	1285
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351
1	1358	1365	1371	1378	1385	1391	1398	1405	1411	1418
2	1425	1431	1438	1445	1451	1458	1465	1471	1478	1485
3	1491	1498	1505	1511	1518	1525	1531	1538	1544	1551
4	1558	1564	1571	1578	1584	1591	1598	1604	1611	1617
5	1624	1631	1637	1644	1651	1657	1664	1671	1677	1684
6	1690	1697	1704	1710	1717	1723	1730	1737	1743	1750
7	1757	1763	1770	1776	1783	1790	1796	1803	1809	1816
8	1823	1829	1836	1842	1849	1856	1862	1869	1875	1882
9	1889	1895	1902	1908	1915	1921	1928	1935	1941	1948
660	81954	81961	81968	81974	81981	81987	81994	82000	82007	82014
1	2020	2027	2033	2040	2046	2053	2060	2066	2073	2079
2	2086	2092	2099	2105	2112	2119	2125	2132	2138	2145
3	2151	2158	2164	2171	2178	2184	2191	2197	2204	2210
4	2217	2223	2230	2236	2243	2249	2256	2263	2269	2276
5	2282	2289	2295	2302	2308	2315	2321	2328	2334	2341
6	2347	2354	2360	2367	2373	2380	2387	2393	2400	2406
7	2413	2419	2426	2432	2439	2445	2452	2458	2465	2471
8	2478	2484	2491	2497	2504	2510	2517	2523	2530	2536
9	2543	2549	2556	2562	2569	2575	2582	2588	2595	2601
670	82607	82614	82620	82627	82633	82640	82646	82653	82659	82666
1	2672	2679	2685	2692	2698	2705	2711	2718	2724	2730
2	2737	2743	2750	2756	2763	2769	2776	2782	2789	2795
3	2802	2808	2814	2821	2827	2834	2840	2847	2853	2860
4	2866	2872	2879	2885	2892	2898	2905	2911	2918	2924
5	2930	2937	2943	2950	2956	2963	2969	2975	2982	2988
6	2995	3001	3008	3014	3020	3027	3033	3040	3046	3052
7	3059	3065	3072	3078	3085	3091	3097	3104	3110	3117
8	3123	3129	3136	3142	3149	3155	3161	3168	3174	3181
9	3187	3193	3200	3206	3213	3219	3225	3232	3238	3245
680	83251	83257	83264	83270	83276	83283	83289	83296	83302	83308
1	3315	3321	3327	3334	3340	3347	3353	3359	3366	3372
2	3378	3385	3391	3398	3404	3410	3417	3423	3429	3436
3	3442	3448	3455	3461	3467	3474	3480	3487	3493	3499
4	3506	3512	3518	3525	3531	3537	3544	3550	3556	3563
5	3569	3575	3582	3588	3594	3601	3607	3613	3620	3626
6	3632	3639	3645	3651	3658	3664	3670	3677	3683	3689
7	3696	3702	3708	3715	3721	3727	3734	3740	3746	3753
8	3759	3765	3771	3778	3784	3790	3797	3803	3809	3816
9	3822	3828	3835	3841	3847	3853	3860	3866	3872	3879
690	83885	83891	83897	83904	83910	83916	83923	83929	83935	83942
1	3948	3954	3960	3967	3973	3979	3985	3992	3998	4004
2	4011	4017	4023	4029	4036	4042	4048	4055	4061	4067
3	4073	4080	4086	4092	4098	4105	4111	4117	4123	4130
4	4136	4142	4148	4155	4161	4167	4173	4180	4186	4192
5	4198	4205	4211	4217	4223	4230	4236	4242	4248	4255
6	4261	4267	4273	4280	4286	4292	4298	4305	4311	4317
7	4323	4330	4336	4342	4348	4354	4361	4367	4373	4379
8	4386	4392	4398	4404	4410	4417	4423	4429	4435	4442
9	4448	4454	4460	4466	4473	4479	4485	4491	4497	4504
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566
1	4572	4578	4584	4590	4597	4603	4609	4615	4621	4628
2	4634	4640	4646	4652	4658	4665	4671	4677	4683	4689
3	4696	4702	4708	4714	4720	4726	4733	4739	4745	4751
4	4757	4763	4770	4776	4782	4788	4794	4800	4807	4813
5	4819	4825	4831	4837	4844	4850	4856	4862	4868	4874
6	4880	4887	4893	4899	4905	4911	4917	4924	4930	4936
7	4942	4948	4954	4960	4967	4973	4979	4985	4991	4997
8	5003	5009	5016	5022	5028	5034	5040	5046	5052	5058
9	5065	5071	5077	5083	5089	5095	5101	5107	5114	5120
710	85126	85132	85138	85144	85150	85156	85163	85169	85175	85181
1	5187	5193	5199	5205	5211	5217	5224	5230	5236	5242
2	5248	5254	5260	5266	5272	5278	5285	5291	5297	5303
3	5309	5315	5321	5327	5333	5339	5345	5352	5358	5364
4	5370	5376	5382	5388	5394	5400	5406	5412	5418	5425
5	5431	5437	5443	5449	5455	5461	5467	5473	5479	5485
6	5491	5497	5503	5509	5516	5522	5528	5534	5540	5546
7	5552	5558	5564	5570	5576	5582	5588	5594	5600	5606
8	5612	5618	5625	5631	5637	5643	5649	5655	5661	5667
9	5673	5679	5685	5691	5697	5703	5709	5715	5721	5727
720	85733	85739	85745	85751	85757	85763	85769	85775	85781	85788
1	5794	5800	5806	5812	5818	5824	5830	5836	5842	5848
2	5854	5860	5866	5872	5878	5884	5890	5896	5902	5908
3	5914	5920	5926	5932	5938	5944	5950	5956	5962	5968
4	5974	5980	5986	5992	5998	6004	6010	6016	6022	6028
5	6034	6040	6046	6052	6058	6064	6070	6076	6082	6088
6	6094	6100	6106	6112	6118	6124	6130	6136	6141	6147
7	6153	6159	6165	6171	6177	6183	6189	6195	6201	6207
8	6213	6219	6225	6231	6237	6243	6249	6255	6261	6267
9	6273	6279	6285	6291	6297	6303	6308	6314	6320	6326
730	86332	86338	86344	86350	86356	86362	86368	86374	86380	86386
1	6392	6398	6404	6410	6415	6421	6427	6433	6439	6445
2	6451	6457	6463	6469	6475	6481	6487	6493	6499	6504
3	6510	6516	6522	6528	6534	6540	6546	6552	6558	6564
4	6570	6576	6581	6587	6593	6599	6605	6611	6617	6623
5	6629	6635	6641	6646	6652	6658	6664	6670	6676	6682
6	6688	6694	6700	6705	6711	6717	6723	6729	6735	6741
7	6747	6753	6759	6764	6770	6776	6782	6788	6794	6800
8	6806	6812	6817	6823	6829	6835	6841	6847	6853	6859
9	6864	6870	6876	6882	6888	6894	6900	6906	6911	6917
740	86923	86929	86935	86941	86947	86953	86958	86964	86970	86976
1	6982	6988	6994	6999	7005	7011	7017	7023	7029	7035
2	7040	7046	7052	7058	7064	7070	7075	7081	7087	7093
3	7099	7105	7111	7116	7122	7128	7134	7140	7146	7151
4	7157	7163	7169	7175	7181	7186	7192	7198	7204	7210
5	7216	7221	7227	7233	7239	7245	7251	7256	7262	7268
6	7274	7280	7286	7291	7297	7303	7309	7315	7320	7326
7	7332	7338	7344	7349	7355	7361	7367	7373	7379	7384
8	7390	7396	7402	7408	7413	7419	7425	7431	7437	7442
9	7448	7454	7460	7466	7471	7477	7483	7489	7495	7500
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558
1	7564	7570	7576	7581	7587	7593	7599	7604	7610	7616
2	7622	7628	7633	7639	7645	7651	7656	7662	7668	7674
3	7679	7685	7691	7697	7703	7708	7714	7720	7726	7731
4	7737	7743	7749	7754	7760	7766	7772	7777	7783	7789
5	7795	7800	7806	7812	7818	7823	7829	7835	7841	7846
6	7852	7858	7864	7869	7875	7881	7887	7892	7898	7904
7	7910	7915	7921	7927	7933	7938	7944	7950	7955	7961
8	7967	7973	7978	7984	7990	7996	8001	8007	8013	8018
9	8024	8030	8036	8041	8047	8053	8058	8064	8070	8076
760	88081	88087	88093	88098	88104	88110	88116	88121	88127	88133
1	8138	8144	8150	8156	8161	8167	8173	8178	8184	8190
2	8195	8201	8207	8213	8218	8224	8230	8235	8241	8247
3	8252	8258	8264	8270	8275	8281	8287	8292	8298	8304
4	8309	8315	8321	8326	8332	8338	8343	8349	8355	8360
5	8366	8372	8377	8383	8389	8395	8400	8406	8412	8417
6	8423	8429	8434	8440	8446	8451	8457	8463	8468	8474
7	8480	8485	8491	8497	8502	8508	8513	8519	8525	8530
8	8536	8542	8547	8553	8559	8564	8570	8576	8581	8587
9	8593	8598	8604	8610	8615	8621	8627	8632	8638	8643
770	88649	88655	88660	88666	88672	88677	88683	88689	88694	88700
1	8705	8711	8717	8722	8728	8734	8739	8745	8750	8756
2	8762	8767	8773	8779	8784	8790	8795	8801	8807	8812
3	8818	8824	8829	8835	8840	8846	8852	8857	8863	8868
4	8874	8880	8885	8891	8897	8902	8908	8913	8919	8925
5	8930	8936	8941	8947	8953	8958	8964	8969	8975	8981
6	8986	8992	8997	9003	9009	9014	9020	9025	9031	9037
7	9042	9048	9053	9059	9064	9070	9076	9081	9087	9092
8	9098	9104	9109	9115	9120	9126	9131	9137	9143	9148
9	9154	9159	9165	9170	9176	9182	9187	9193	9198	9204
780	89209	89215	89221	89226	89232	89237	89243	89248	89254	89260
1	9265	9271	9276	9282	9287	9293	9298	9304	9310	9315
2	9321	9326	9332	9337	9343	9348	9354	9360	9365	9371
3	9376	9382	9387	9393	9398	9404	9409	9415	9421	9426
4	9432	9437	9443	9448	9454	9459	9465	9470	9476	9481
5	9487	9492	9498	9504	9509	9515	9520	9526	9531	9537
6	9542	9548	9553	9559	9564	9570	9575	9581	9586	9592
7	9597	9603	9609	9614	9620	9625	9631	9636	9642	9647
8	9653	9658	9664	9669	9675	9680	9686	9691	9697	9702
9	9708	9713	9719	9724	9730	9735	9741	9746	9752	9757
790	89763	89768	89774	89779	89785	89790	89796	89801	89807	89812
1	9818	9823	9829	9834	9840	9845	9851	9856	9862	9867
2	9873	9878	9883	9889	9894	9900	9905	9911	9916	9922
3	9927	9933	9938	9944	9949	9955	9960	9966	9971	9977
4	9982	9988	9993	9998	90004	90009	90015	90020	90026	90031
5	90037	90042	90048	90053	0059	0064	0069	0075	0080	0086
6	0091	0097	0102	0108	0113	0119	0124	0129	0135	0140
7	0146	0151	0157	0162	0168	0173	0179	0184	0189	0195
8	0200	0206	0211	0217	0222	0227	0233	0238	0244	0249
9	0255	0260	0266	0271	0276	0282	0287	0293	0298	0304
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358
1	0363	0369	0374	0380	0385	0390	0396	0401	0407	0412
2	0417	0423	0428	0434	0439	0445	0450	0455	0461	0466
3	0472	0477	0482	0488	0493	0499	0504	0509	0515	0520
4	0526	0531	0536	0542	0547	0553	0558	0563	0569	0574
5	0580	0585	0590	0596	0601	0607	0612	0617	0623	0628
6	0634	0639	0644	0650	0655	0660	0666	0671	0677	0682
7	0687	0693	0698	0703	0709	0714	0720	0725	0730	0736
8	0741	0747	0752	0757	0763	0768	0773	0779	0784	0789
9	0795	0800	0806	0811	0816	0822	0827	0832	0838	0843
810	90849	90854	90859	90865	90870	90875	90881	90886	90891	90897
1	0902	0907	0913	0918	0924	0929	0934	0940	0945	0950
2	0956	0961	0966	0972	0977	0982	0988	0993	0998	1004
3	1009	1014	1020	1025	1030	1036	1041	1046	1052	1057
4	1062	1068	1073	1078	1084	1089	1094	1100	1105	1110
5	1116	1121	1126	1132	1137	1142	1148	1153	1158	1164
6	1169	1174	1180	1185	1190	1196	1201	1206	1212	1217
7	1222	1228	1233	1238	1243	1249	1254	1259	1265	1270
8	1275	1281	1286	1291	1297	1302	1307	1312	1318	1323
9	1328	1334	1339	1344	1350	1355	1360	1365	1371	1376
820	91381	91387	91392	91397	91403	91408	91413	91418	91424	91429
1	1434	1440	1445	1450	1455	1461	1466	1471	1477	1482
2	1487	1492	1498	1503	1508	1514	1519	1524	1529	1535
3	1540	1545	1551	1556	1561	1566	1572	1577	1582	1587
4	1593	1598	1603	1609	1614	1619	1624	1630	1635	1640
5	1645	1651	1656	1661	1666	1672	1677	1682	1687	1693
6	1698	1703	1709	1714	1719	1724	1730	1735	1740	1745
7	1751	1756	1761	1766	1772	1777	1782	1787	1793	1798
8	1803	1808	1814	1819	1824	1829	1834	1840	1845	1850
9	1855	1861	1866	1871	1876	1882	1887	1892	1897	1903
830	91908	91913	91918	91924	91929	91934	91939	91944	91950	91955
1	1960	1965	1971	1976	1981	1986	1991	1997	2002	2007
2	2012	2018	2023	2028	2033	2038	2044	2049	2054	2059
3	2065	2070	2075	2080	2085	2091	2096	2101	2106	2111
4	2117	2122	2127	2132	2137	2143	2148	2153	2158	2163
5	2169	2174	2179	2184	2189	2195	2200	2205	2210	2215
6	2221	2226	2231	2236	2241	2247	2252	2257	2262	2267
7	2273	2278	2283	2288	2293	2298	2304	2309	2314	2319
8	2324	2330	2335	2340	2345	2350	2355	2361	2366	2371
9	2376	2381	2387	2392	2397	2402	2407	2412	2418	2423
840	92428	92433	92438	92443	92449	92454	92459	92464	92469	92474
1	2480	2485	2490	2495	2500	2505	2511	2516	2521	2526
2	2531	2536	2542	2547	2552	2557	2562	2567	2572	2578
3	2583	2588	2593	2598	2603	2609	2614	2619	2624	2629
4	2634	2639	2645	2650	2655	2660	2665	2670	2675	2681
5	2686	2691	2696	2701	2706	2711	2716	2722	2727	2732
6	2737	2742	2747	2752	2758	2763	2768	2773	2778	2783
7	2788	2793	2799	2804	2809	2814	2819	2824	2829	2834
8	2840	2845	2850	2855	2860	2865	2870	2875	2881	2886
9	2891	2896	2901	2906	2911	2916	2921	2927	2932	2937
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988

Table XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988
1	2993	2998	3003	3008	3013	3018	3024	3029	3034	3039
2	3044	3049	3054	3059	3064	3069	3075	3080	3085	3090
3	3095	3100	3105	3110	3115	3120	3125	3131	3136	3141
4	3146	3151	3156	3161	3166	3171	3176	3181	3186	3192
5	3197	3202	3207	3212	3217	3222	3227	3232	3237	3242
6	3247	3252	3258	3263	3268	3273	3278	3283	3288	3293
7	3298	3303	3308	3313	3318	3323	3328	3334	3339	3344
8	3349	3354	3359	3364	3369	3374	3379	3384	3389	3394
9	3399	3404	3409	3414	3420	3425	3430	3435	3440	3445
860	93450	93455	93460	93465	93470	93475	93480	93485	93490	93495
1	3500	3505	3510	3515	3520	3526	3531	3536	3541	3546
2	3551	3556	3561	3566	3571	3576	3581	3586	3591	3596
3	3601	3606	3611	3616	3621	3626	3631	3636	3641	3646
4	3651	3656	3661	3666	3671	3676	3682	3687	3692	3697
5	3702	3707	3712	3717	3722	3727	3732	3737	3742	3747
6	3752	3757	3762	3767	3772	3777	3782	3787	3792	3797
7	3802	3807	3812	3817	3822	3827	3832	3837	3842	3847
8	3852	3857	3862	3867	3872	3877	3882	3887	3892	3897
9	3902	3907	3912	3917	3922	3927	3932	3937	3942	3947
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997
1	4002	4007	4012	4017	4022	4027	4032	4037	4042	4047
2	4052	4057	4062	4067	4072	4077	4082	4086	4091	4096
3	4101	4106	4111	4116	4121	4126	4131	4136	4141	4146
4	4151	4156	4161	4166	4171	4176	4181	4186	4191	4196
5	4201	4206	4211	4216	4221	4226	4231	4236	4240	4245
6	4250	4255	4260	4265	4270	4275	4280	4285	4290	4295
7	4300	4305	4310	4315	4320	4325	4330	4335	4340	4345
8	4349	4354	4359	4364	4369	4374	4379	4384	4389	4394
9	4399	4404	4409	4414	4419	4424	4429	4433	4438	4443
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493
1	4498	4503	4507	4512	4517	4522	4527	4532	4537	4542
2	4547	4552	4557	4562	4567	4571	4576	4581	4586	4591
3	4596	4601	4606	4611	4616	4621	4626	4630	4635	4640
4	4645	4650	4655	4660	4665	4670	4675	4680	4685	4689
5	4694	4699	4704	4709	4714	4719	4724	4729	4734	4738
6	4743	4748	4753	4758	4763	4768	4773	4778	4783	4787
7	4792	4797	4802	4807	4812	4817	4822	4827	4832	4836
8	4841	4846	4851	4856	4861	4866	4871	4876	4880	4885
9	4890	4895	4900	4905	4910	4915	4919	4924	4929	4934
890	94939	94944	94949	94954	94959	94963	94968	94973	94978	94983
1	4988	4993	4998	5002	5007	5012	5017	5022	5027	5032
2	5036	5041	5046	5051	5056	5061	5066	5071	5075	5080
3	5085	5090	5095	5100	5105	5109	5114	5119	5124	5129
4	5134	5139	5143	5148	5153	5158	5163	5168	5173	5177
5	5182	5187	5192	5197	5202	5207	5211	5216	5221	5226
6	5231	5236	5240	5245	5250	5255	5260	5265	5270	5274
7	5279	5284	5289	5294	5299	5303	5308	5313	5318	5323
8	5328	5332	5337	5342	5347	5352	5357	5361	5366	5371
9	5376	5381	5386	5390	5395	5400	5405	5410	5415	5419
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468

TABLE XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468
1	5472	5477	5482	5487	5492	5497	5501	5506	5511	5516
2	5521	5525	5530	5535	5540	5545	5550	5554	5559	5564
3	5569	5574	5578	5583	5588	5593	5598	5602	5607	5612
4	5617	5622	5626	5631	5636	5641	5646	5650	5655	5660
5	5665	5670	5674	5679	5684	5689	5694	5698	5703	5708
6	5713	5718	5722	5727	5732	5737	5742	5746	5751	5756
7	5761	5766	5770	5775	5780	5785	5789	5794	5799	5804
8	5809	5813	5818	5823	5828	5832	5837	5842	5847	5852
9	5856	5861	5866	5871	5875	5880	5885	5890	5895	5899
910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947
1	5952	5957	5961	5966	5971	5976	5980	5985	5990	5995
2	5999	6004	6009	6014	6019	6023	6028	6033	6038	6042
3	6047	6052	6057	6061	6066	6071	6076	6080	6085	6090
4	6095	6099	6104	6109	6114	6118	6123	6128	6133	6137
5	6142	6147	6152	6156	6161	6166	6171	6175	6180	6185
6	6190	6194	6199	6204	6209	6213	6218	6223	6227	6232
7	6237	6242	6246	6251	6256	6261	6265	6270	6275	6280
8	6284	6289	6294	6298	6303	6308	6313	6317	6322	6327
9	6332	6336	6341	6346	6350	6355	6360	6365	6369	6374
920	96379	96384	96388	96393	96398	96402	96407	96412	96417	96421
1	6426	6431	6435	6440	6445	6450	6454	6459	6464	6468
2	6473	6478	6483	6487	6492	6497	6501	6506	6511	6515
3	6520	6525	6530	6534	6539	6544	6548	6553	6558	6562
4	6567	6572	6577	6581	6586	6591	6595	6600	6605	6609
5	6614	6619	6624	6628	6633	6638	6642	6647	6652	6656
6	6661	6666	6670	6675	6680	6685	6689	6694	6699	6703
7	6708	6713	6717	6722	6727	6731	6736	6741	6745	6750
8	6755	6759	6764	6769	6774	6778	6783	6788	6792	6797
9	6802	6806	6811	6816	6820	6825	6830	6834	6839	6844
930	96848	96853	96858	96862	96867	96872	96876	96881	96886	96890
1	6895	6900	6904	6909	6914	6918	6923	6928	6932	6937
2	6942	6946	6951	6956	6960	6965	6970	6974	6979	6984
3	6988	6993	6997	7002	7007	7011	7016	7021	7025	7030
4	7035	7039	7044	7049	7053	7058	7063	7067	7072	7077
5	7081	7086	7090	7095	7100	7104	7109	7114	7118	7123
6	7128	7132	7137	7142	7146	7151	7155	7160	7165	7169
7	7174	7179	7183	7188	7192	7197	7202	7206	7211	7216
8	7220	7225	7230	7234	7239	7243	7248	7253	7257	7262
9	7267	7271	7276	7280	7285	7290	7294	7299	7304	7308
940	97313	97317	97322	97327	97331	97336	97340	97345	97350	97354
1	7359	7364	7368	7373	7377	7382	7387	7391	7396	7400
2	7405	7410	7414	7419	7424	7428	7433	7437	7442	7447
3	7451	7456	7460	7465	7470	7474	7479	7483	7488	7493
4	7497	7502	7506	7511	7516	7520	7525	7529	7534	7539
5	7543	7548	7552	7557	7562	7566	7571	7575	7580	7585
6	7589	7594	7598	7603	7607	7612	7617	7621	7626	7630
7	7635	7640	7644	7649	7653	7658	7663	7667	7672	7676
8	7681	7685	7690	7695	7699	7704	7708	7713	7717	7722
9	7727	7731	7736	7740	7745	7749	7754	7759	7763	7768
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813

Table XIII.—*Continued.*

LOGARITHMS OF NUMBERS.

N	0	1	2	3	4	5	6	7	8	9
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813
1	7818	7823	7827	7832	7836	7841	7845	7850	7855	7859
2	7864	7868	7873	7877	7882	7886	7891	7896	7900	7905
3	7909	7914	7918	7923	7928	7932	7937	7941	7946	7950
4	7955	7959	7964	7968	7973	7978	7982	7987	7991	7996
5	8000	8005	8009	8014	8019	8023	8028	8032	8037	8041
6	8046	8050	8055	8059	8064	8068	8073	8078	8082	8087
7	8091	8096	8100	8105	8109	8114	8118	8123	8127	8132
8	8137	8141	8146	8150	8155	8159	8164	8168	8173	8177
9	8182	8186	8191	8195	8200	8204	8209	8214	8218	8223
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268
1	8272	8277	8281	8286	8290	8295	8299	8304	8308	8313
2	8318	8322	8327	8331	8336	8340	8345	8349	8354	8358
3	8363	8367	8372	8376	8381	8385	8390	8394	8399	8403
4	8408	8412	8417	8421	8426	8430	8435	8439	8444	8448
5	8453	8457	8462	8466	8471	8475	8480	8484	8489	8493
6	8498	8502	8507	8511	8516	8520	8525	8529	8534	8538
7	8543	8547	8552	8556	8561	8565	8570	8574	8579	8583
8	8588	8592	8507	8601	8605	8610	8614	8619	8623	8628
9	8632	8637	8641	8646	8650	8655	8659	8664	8668	8673
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717
1	8722	8726	8731	8735	8740	8744	8749	8753	8758	8762
2	8767	8771	8776	8780	8784	8789	8793	8798	8802	8807
3	8811	8816	8820	8825	8829	8834	8838	8843	8847	8851
4	8856	8860	8865	8869	8874	8878	8883	8887	8892	8896
5	8900	8905	8909	8914	8918	8923	8927	8932	8936	8941
6	8945	8949	8954	8958	8963	8967	8972	8976	8981	8985
7	8989	8994	8998	9003	9007	9012	9016	9021	9025	9029
8	9034	9038	9043	9047	9052	9056	9061	9065	9069	9074
9	9078	9083	9087	9092	9096	9100	9105	9109	9114	9118
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162
1	9167	9171	9176	9180	9185	9189	9193	9198	9202	9207
2	9211	9216	9220	9224	9229	9233	9238	9242	9247	9251
3	9255	9260	9264	9269	9273	9277	9282	9286	9291	9295
4	9300	9304	9308	9313	9317	9322	9326	9330	9335	9339
5	9344	9348	9352	9357	9361	9366	9370	9374	9379	9383
6	9388	9392	9396	9401	9405	9410	9414	9419	9423	9427
7	9432	9436	9441	9445	9449	9454	9458	9463	9467	9471
8	9476	9480	9484	9489	9493	9498	9502	9506	9511	9515
9	9520	9524	9528	9533	9537	9542	9546	9550	9555	9559
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603
1	9607	9612	9616	9621	9625	9629	9634	9638	9642	9647
2	9651	9656	9660	9664	9669	9673	9677	9682	9686	9691
3	9695	9699	9704	9708	9712	9717	9721	9726	9730	9734
4	9739	9743	9747	9752	9756	9760	9765	9769	9774	9778
5	9782	9787	9791	9795	9800	9804	9808	9813	9817	9822
6	9826	9830	9835	9839	9843	9848	9852	9856	9861	9865
7	9870	9874	9878	9883	9887	9891	9896	9900	9904	9909
8	9913	9917	9922	9926	9930	9935	9939	9944	9948	9952
9	9957	9961	9965	9970	9974	9978	9983	9987	9991	9996
1000	00000	00004	00009	00013	00017	00022	00026	00030	00035	00039

TABLE XIV.

LOGARITHMIC SINES AND COSINES.

′	0°		1°		2°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	—∞	10.00000	8.24186	9.99993	8.54282	9.99974	60
1	6.46373	00000	24903	99993	54642	99973	59
2	76476	00000	25609	99993	54999	99973	58
3	94085	00000	26304	99993	55354	99972	57
4	7.06579	00000	26988	99992	55705	99972	56
5	16270	00000	27661	99992	56054	99971	55
6	24188	00000	28324	99992	56400	99971	54
7	30882	00000	28977	99992	56743	99970	53
8	36682	00000	29621	99992	57084	99970	52
9	41797	00000	30255	99991	57421	99969	51
10	7.46373	10.00000	8.30879	9.99991	8.57757	9.99969	50
11	50512	00000	31495	99991	58089	99968	49
12	54291	00000	32108	99990	58419	99968	48
13	57767	00000	32702	99990	58747	99967	47
14	60985	00000	33292	99990	59072	99967	46
15	63982	00000	33875	99990	59395	99967	45
16	66784	00000	34450	99989	59715	99966	44
17	69417	9.99999	35018	99989	60033	99966	43
18	71900	99999	35578	99989	60349	99965	42
19	74248	99999	36131	99989	60662	99964	41
20	7.76475	9.99999	8.36678	9.99988	8.60973	9.99964	40
21	78594	99999	37217	99988	61282	99963	39
22	80615	99999	37750	99988	61589	99963	38
23	82545	99999	38276	99987	61894	99962	37
24	84393	99999	38796	99987	62196	99962	36
25	86166	99999	39310	99987	62497	99961	35
26	87870	99999	39818	99986	62795	99961	34
27	89509	99999	40320	99986	63091	99960	33
28	91088	99999	40816	99986	63385	99960	32
29	92612	99998	41307	99985	63678	99959	31
30	7.94084	9.99998	8.41792	9.99985	8.63968	9.99959	30
31	95508	99998	42272	99985	64256	99958	29
32	96887	99998	42746	99984	64543	99958	28
33	98223	99998	43216	99984	64827	99957	27
34	99520	99998	43680	99984	65110	99956	26
35	8.00779	99998	44139	99983	65391	99956	25
36	02002	99998	44594	99983	65670	99955	24
37	03192	99997	45044	99983	65947	99955	23
38	04350	99997	45489	99982	66223	99954	22
39	05478	99997	45930	99982	66497	99954	21
40	8.06578	9.99997	8.46366	9.99982	8.66769	9.99953	20
41	07650	99997	46799	99981	67039	99952	19
42	08696	99997	47226	99981	67308	99952	18
43	09718	99997	47650	99981	67575	99951	17
44	10717	99996	48069	99980	67841	99951	16
45	11693	99996	48485	99980	68104	99950	15
46	12647	99996	48896	99979	68367	99949	14
47	13581	99996	49304	99979	68627	99949	13
48	14495	99996	49708	99979	68886	99948	12
49	15391	99996	50108	99978	69144	99948	11
50	8.16268	9.99995	8.50504	9.99978	8.69400	9.99947	10
51	17128	99995	50897	99977	69654	99946	9
52	17971	99995	51287	99977	69907	99946	8
53	18798	99995	51673	99977	70159	99945	7
54	19610	99995	52055	99976	70409	99944	6
55	20407	99994	52434	99976	70658	99944	5
56	21189	99994	52810	99975	70905	99943	4
57	21958	99994	53183	99975	71151	99942	3
58	22713	99994	53552	99974	71395	99942	2
59	23456	99994	53919	99974	71638	99941	1
60	24186	99993	54282	99974	71880	99940	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	3°		4°		5°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	8.71880	9.99940	8.84358	9.99894	8.94030	9.99834	60
1	72120	99940	84539	99893	94174	99833	59
2	72359	99939	84718	99892	94317	99832	58
3	72597	99988	84897	99891	94461	99831	57
4	72834	99988	85075	99891	94603	99830	56
5	73069	99937	85252	99890	94746	99829	55
6	73303	99936	85429	99889	94887	99828	54
7	73535	99936	85605	99888	95029	99827	53
8	73767	99935	85780	99887	95170	99825	52
9	73997	99934	85955	99886	95310	99824	51
10	8.74226	9.99934	8.86128	9.99885	8.95450	9.99823	50
11	74454	99933	86301	99884	95589	99822	49
12	74680	99932	86474	99883	95728	99821	48
13	74906	99932	86645	99882	95867	99820	47
14	75130	99931	86816	99881	96005	99819	46
15	75353	99930	86987	99880	96143	99817	45
16	75575	99929	87156	99879	96280	99816	44
17	75795	99929	87325	99879	96417	99815	43
18	76015	99928	87494	99878	96553	99814	42
19	76234	99927	87661	99877	96689	99813	41
20	8.76451	9.99926	8.87829	9.99876	8.96825	9.99812	40
21	76667	99926	87995	99875	96960	99810	39
22	76883	99925	88161	99874	97095	99809	38
23	77097	99924	88326	99873	97229	99808	37
24	77310	99923	88490	99872	97363	99807	36
25	77522	99923	88654	99871	97496	99806	35
26	77733	99922	88817	99870	97629	99804	34
27	77943	99921	88980	99869	97762	99803	33
28	78152	99920	89142	99868	97894	99802	32
29	78360	99920	89304	99867	98026	99801	31
30	8.78568	9.99919	8.89464	9.99866	8.98157	9.99800	30
31	78774	99918	89625	99865	98288	99798	29
32	78979	99917	89784	99864	98419	99797	28
33	79183	99917	89943	99863	98549	99796	27
34	79386	99916	90102	99862	98679	99795	26
35	79588	99915	90260	99861	98808	99793	25
36	79789	99914	90417	99860	98937	99792	24
37	79990	99913	90574	99859	99066	99791	23
38	80189	99913	90730	99858	99194	99790	22
39	80388	99912	90885	99857	99322	99788	21
40	8.80585	9.99911	8.91040	9.99856	8.99450	9.99787	20
41	80782	99910	91195	99855	99577	99786	19
42	80978	99909	91349	99854	99704	99785	18
43	81173	99909	91502	99853	99830	99783	17
44	81367	99908	91655	99852	99956	99782	16
45	81560	99907	91807	99851	9.00082	99781	15
46	81752	99906	91959	99850	00207	99780	14
47	81944	99905	92110	99848	00332	99778	13
48	82134	99904	92261	99847	00456	99777	12
49	82324	99904	92411	99846	00581	99776	11
50	8.82513	9.99903	8.92561	9.99845	9.00704	9.99775	10
51	82701	99902	92710	99844	00828	99773	9
52	82888	99901	92859	99843	00951	99772	8
53	83075	99900	93007	99842	01074	99771	7
54	83261	99899	93154	99841	01196	99769	6
55	83446	99898	93301	99840	01318	99768	5
56	83630	99898	93448	99839	01440	99767	4
57	83813	99897	93594	99838	01561	99765	3
58	83996	99896	93740	99837	01682	99764	2
59	84177	99895	93885	99836	01803	99763	1
60	84358	99894	94030	99834	01923	99761	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	86°		85°		84°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	6° Sine	6° Cosine	7° Sine	7° Cosine	8° Sine	8° Cosine	′
0	9.01923	9.99761	9.08589	9.99675	9.14356	9.99575	60
1	02043	99760	08692	99674	14445	99574	59
2	02163	99759	08795	99672	14535	99572	58
3	02283	99757	08897	99670	14624	99570	57
4	02402	99756	08999	99669	14714	99568	56
5	02520	99755	09101	99667	14803	99566	55
6	02639	99753	09202	99666	14891	99565	54
7	02757	99752	09304	99664	14980	99563	53
8	02874	99751	09405	99663	15069	99561	52
9	02992	99749	09506	99661	15157	99559	51
10	9.03109	9.99748	9.09606	9.99659	9.15245	9.99557	50
11	03226	99747	09707	99658	15333	99556	49
12	03342	99745	09807	99656	15421	99554	48
13	03458	99744	09907	99655	15508	99552	47
14	03574	99742	10006	99653	15596	99550	46
15	03690	99741	10106	99651	15683	99548	45
16	03805	99740	10205	99650	15770	99546	44
17	03920	99738	10304	99648	15857	99545	43
18	04034	99737	10402	99647	15944	99543	42
19	04149	99736	10501	99645	16030	99541	41
20	9.04262	9.99734	9.10599	9.99643	9.16116	9.99539	40
21	04376	99733	10697	99642	16203	99537	39
22	04490	99731	10795	99640	16289	99535	38
23	04603	99730	10893	99638	16374	99533	37
24	04715	99728	10990	99637	16460	99532	36
25	04828	99727	11087	99635	16545	99530	35
26	04940	99726	11184	99633	16631	99528	34
27	05052	99724	11281	99632	16716	99526	33
28	05164	99723	11377	99630	16801	99524	32
29	05275	99721	11474	99629	16886	99522	31
30	9.05386	9.99720	9.11570	9.99627	9.16970	9.99520	30
31	05497	99718	11666	99625	17055	99518	29
32	05607	99717	11761	99624	17139	99517	28
33	05717	99716	11857	99622	17223	99515	27
34	05827	99714	11952	99620	17307	99513	26
35	05937	99713	12047	99618	17391	99511	25
36	06046	99711	12142	99617	17474	99509	24
37	06155	99710	12236	99615	17558	99507	23
38	06264	99708	12331	99613	17641	99505	22
39	06372	99707	12425	99612	17724	99503	21
40	9.06481	9.99705	9.12519	9.99610	9.17807	9.99501	20
41	06589	99704	12612	99608	17890	99499	19
42	06696	99702	12706	99607	17973	99497	18
43	06804	99701	12799	99605	18055	99495	17
44	06911	99699	12892	99603	18137	99494	16
45	07018	99698	12985	99601	18220	99492	15
46	07124	99696	13078	99600	18302	99490	14
47	07231	99695	13171	99598	18383	99488	13
48	07337	99693	13263	99596	18465	99486	12
49	07442	99692	13355	99595	18547	99484	11
50	9.07548	9.99690	9.13447	9.99593	9.18628	9.99482	10
51	07653	99689	13539	99591	18709	99480	9
52	07758	99687	13630	99589	18790	99478	8
53	07863	99686	13722	99588	18871	99476	7
54	07968	99684	13813	99586	18952	99474	6
55	08072	99683	13904	99584	19033	99472	5
56	08176	99681	13994	99582	19113	99470	4
57	08280	99680	14085	99581	19193	99468	3
58	08383	99678	14175	99579	19273	99466	2
59	08486	99677	14266	99577	19353	99464	1
60	08589	99675	14356	99575	19433	99462	0
′	83° Cosine	83° Sine	82° Cosine	82° Sine	81° Cosine	81° Sine	′

Table XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	9° Sine	9° Cosine	10° Sine	10° Cosine	11° Sine	11° Cosine	′
0	9.19433	9.99462	9.23967	9.99335	9.28060	9.99195	60
1	19513	99460	24039	99333	28125	99192	59
2	19592	99458	24110	99331	28190	99190	58
3	19672	99456	24181	99328	28254	99187	57
4	19751	99454	24253	99326	28319	99185	56
5	19830	99452	24324	99324	28384	99182	55
6	19909	99450	24395	99322	28448	99180	54
7	19988	99448	24466	99319	28512	99177	53
8	20067	99446	24536	99317	28577	99175	52
9	20145	99444	24607	99315	28641	99172	51
10	9.20223	9.99442	9.24677	9.99313	9.28705	9.99170	50
11	20302	99440	24748	99310	28769	99167	49
12	20380	99438	24818	99308	28833	99165	48
13	20458	99436	24888	99306	28896	99162	47
14	20535	99434	24958	99304	28960	99160	46
15	20613	99432	25028	99301	29024	99157	45
16	20691	99429	25098	99299	29087	99155	44
17	20768	99427	25168	99297	29150	99152	43
18	20845	99425	25237	99294	29214	99150	42
19	20922	99423	25307	99292	29277	99147	41
20	9.20999	9.99421	9.25376	9.99290	9.29340	9.99145	40
21	21076	99419	25445	99288	29403	99142	39
22	21153	99417	25514	99285	29466	99140	38
23	21229	99415	25583	99283	29529	99137	37
24	21306	99413	25652	99281	29591	99135	36
25	21382	99411	25721	99278	29654	99132	35
26	21458	99409	25790	99276	29716	99130	34
27	21534	99407	25858	99274	29779	99127	33
28	21610	99404	25927	99271	29841	99124	32
29	21685	99402	25995	99269	29903	99122	31
30	9.21761	9.99400	9.26063	9.99267	9.29966	9.99119	30
31	21836	99398	26131	99264	30028	99117	29
32	21912	99396	26199	99262	30090	99114	28
33	21987	99394	26267	99260	30151	99112	27
34	22062	99392	26335	99257	30213	99109	26
35	22137	99390	26403	99255	30275	99106	25
36	22211	99388	26470	99252	30336	99104	24
37	22286	99385	26538	99250	30398	99001	23
38	22361	99383	26605	99248	30459	99099	22
39	22435	99381	26672	99245	30521	99096	21
40	9.22509	9.99379	9.26739	9.99243	9.30582	9.99093	20
41	22583	99377	26806	99241	30643	99091	19
42	22657	99375	26873	99238	30704	99088	18
43	22731	99372	26940	99236	30765	99086	17
44	22805	99370	27007	99233	30826	99083	16
45	22878	99368	27073	99231	30887	99080	15
46	22952	99366	27140	99229	30947	99078	14
47	23025	99364	27206	99226	31008	99075	13
48	23098	99362	27273	99224	31068	99072	12
49	23171	99359	27339	99221	31129	99070	11
50	9.23244	9.99357	9.27405	9.99219	9.31189	9.99067	10
51	23317	99355	27471	99217	31250	99064	9
52	23390	99353	27537	99214	31310	99062	8
53	23462	99351	27602	99212	31370	99059	7
54	23535	99348	27668	99209	31430	99056	6
55	23607	99346	27734	99207	31490	99054	5
56	23679	99344	27799	99204	31549	99051	4
57	23752	99342	27864	99202	31609	99048	3
58	23823	99340	27930	99200	31669	99046	2
59	23895	99337	27995	99197	31728	99043	1
60	23967	99335	28060	99195	31788	99040	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	80°		79°		78°		

Table XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	12° Sine	12° Cosine	13° Sine	13° Cosine	14° Sine	14° Cosine	′
0	9.31788	9.99040	9.35209	9.98872	9.38368	9.98690	60
1	31847	99038	35263	98869	38418	98687	59
2	31907	99035	35318	98867	38469	98684	58
3	31966	99032	35373	98864	38519	98681	57
4	32025	99030	35427	98861	38570	98678	56
5	32084	99027	35481	98858	38620	98675	55
6	32143	99024	35536	98855	38670	98671	54
7	32202	99022	35590	98852	38721	98668	53
8	32261	99019	35644	98849	38771	98665	52
9	32319	99016	35698	98846	38821	98662	51
10	9.32378	9.99013	9.35752	9.98843	9.38871	9.98659	50
11	32437	99011	35806	98840	38921	98656	49
12	32495	99008	35860	98837	38971	98652	48
13	32553	99005	35914	98834	39021	98649	47
14	32612	99002	35968	98831	39071	98646	46
15	32670	99000	36022	98828	39121	98643	45
16	32728	98997	36075	98825	39170	98640	44
17	32786	98994	36129	98822	39220	98636	43
18	32844	98991	36182	98819	39270	98633	42
19	32902	98989	36236	98816	39319	98630	41
20	9.32960	9.98986	9.36289	9.98813	9.39369	9.98627	40
21	33018	98983	36342	98810	39418	98623	39
22	33075	98980	36395	98807	39467	98620	38
23	33133	98978	36449	98804	39517	98617	37
24	33190	98975	36502	98801	39566	98614	36
25	33248	98972	36555	98798	39615	98610	35
26	33305	98969	36608	98795	39664	98607	34
27	33362	98967	36660	98792	39713	98604	33
28	33420	98964	36713	98789	39762	98601	32
29	33477	98961	36766	98786	39811	98597	31
30	9.33534	9.98958	9.36819	9.98783	9.39860	9.98594	30
31	33591	98955	36871	98780	39909	98591	29
32	33647	98953	36924	98777	39958	98588	28
33	33704	98950	36976	98774	40006	98584	27
34	33761	98947	37028	98771	40055	98581	26
35	33818	98944	37081	98768	40103	98578	25
36	33874	98941	37133	98765	40152	98574	24
37	33931	98938	37185	98762	40200	98571	23
38	33987	98936	37237	98759	40249	98568	22
39	34043	98933	37289	98756	40297	98565	21
40	9.34100	9.98930	9.37341	9.98753	9.40346	9.98561	20
41	34156	98927	37393	98750	40394	98558	19
42	34212	98924	37445	98746	40442	98555	18
43	34268	98921	37497	98743	40490	98551	17
44	34324	98919	37549	98740	40538	98548	16
45	34380	98916	37600	98737	40586	98545	15
46	34436	98913	37652	98734	40634	98541	14
47	34491	98910	37703	98731	40682	98538	13
48	34547	98907	37755	98728	40730	98535	12
49	34602	98904	37806	98725	40778	98531	11
50	9.34658	9.98901	9.37858	9.98722	9.40825	9.98528	10
51	34713	98898	37909	98719	40873	98525	9
52	34769	98896	37960	98715	40921	98521	8
53	34824	98893	38011	98712	40968	98518	7
54	34879	98890	38062	98709	41016	98515	6
55	34934	98887	38113	98706	41063	98511	5
56	34989	98884	38164	98703	41111	98508	4
57	35044	98881	38215	98700	41158	98505	3
58	35099	98878	38266	98697	41205	98501	2
59	35154	98875	38317	98694	41252	98498	1
60	35209	98872	38368	98690	41300	98494	0
′	77° Cosine	77° Sine	76° Cosine	76° Sine	75° Cosine	75° Sine	′

Table XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	15° Sine	15° Cosine	16° Sine	16° Cosine	17° Sine	17° Cosine	′
0	9.41300	9.98494	9.44034	9.98284	9.46594	9.98060	60
1	41347	98491	44078	98281	46635	98056	59
2	41394	98488	44122	98277	46676	98052	58
3	41441	98484	44166	98273	46717	98048	57
4	41488	98481	44210	98270	46758	98044	56
5	41535	98477	44253	98266	46800	98040	55
6	41582	98474	44297	98262	46841	98036	54
7	41628	98471	44341	98259	46882	98032	53
8	41675	98467	44385	98255	46923	98029	52
9	41722	98464	44428	98251	46964	98025	51
10	9.41768	9.98460	9.44472	9.98248	9.47005	9.98021	50
11	41815	98457	44516	98244	47045	98017	49
12	41861	98453	44559	98240	47086	98013	48
13	41908	98450	44602	98237	47127	98009	47
14	41954	98447	44646	98233	47168	98005	46
15	42001	98443	44689	98229	47209	98001	45
16	42047	98440	44733	98226	47249	97997	44
17	42093	98436	44776	98222	47290	97993	43
18	42140	98433	44819	98218	47330	97989	42
19	42186	98429	44862	98215	47371	97986	41
20	9.42232	9.98426	9.44905	9.98211	9.47411	9.97982	40
21	42278	98422	44948	98207	47452	97978	39
22	42324	98419	44992	98204	47492	97974	38
23	42370	98415	45035	98200	47533	97970	37
24	42416	98412	45077	98196	47573	97966	36
25	42461	98409	45120	98192	47613	97962	35
26	42507	98405	45163	98189	47654	97958	34
27	42553	98402	45206	98185	47694	97954	33
28	42599	98398	45249	98181	47734	97950	32
29	42644	98395	45292	98177	47774	97946	31
30	9.42690	9.98391	9.45334	9.98174	9.47814	9.97942	30
31	42735	98388	45377	98170	47854	97938	29
32	42781	98384	45419	98166	47894	97934	28
33	42826	98381	45462	98162	47934	97930	27
34	42872	98377	45504	98159	47974	97926	26
35	42917	98373	45547	98155	48014	97922	25
36	42962	98370	45589	98151	48054	97918	24
37	43008	98366	45632	98147	48094	97914	23
38	43053	98363	45674	98144	48133	97910	22
39	43098	98359	45716	98140	48173	97906	21
40	9.43143	9.98356	9.45758	9.98136	9.48213	9.97902	20
41	43188	98352	45801	98132	48252	97898	19
42	43233	98349	45843	98129	48292	97894	18
43	43278	98345	45885	98125	48332	97890	17
44	43323	98342	45927	98121	48371	97886	16
45	43367	98338	45969	98117	48411	97882	15
46	43412	98334	46011	98113	48450	97878	14
47	43457	98331	46053	98110	48490	97874	13
48	43502	98327	46095	98106	48529	97870	12
49	43546	98324	46136	98102	48568	97866	11
50	9.43591	9.98320	9.46178	9.98098	9.48607	9.97861	10
51	43635	98317	46220	98094	48647	97857	9
52	43680	98313	46262	98090	48686	97853	8
53	43724	98309	46303	98087	48725	97849	7
54	43769	98306	46345	98083	48764	97845	6
55	43813	98302	46386	98079	48803	97841	5
56	43857	98299	46428	98075	48842	97837	4
57	43901	98295	46469	98071	48881	97833	3
58	43946	98291	46511	98067	48920	97829	2
59	43990	98288	46552	98063	48959	97825	1
60	44034	98284	46594	98060	48998	97821	0
′	74° Cosine	74° Sine	73° Cosine	73° Sine	72° Cosine	72° Sine	′

Table XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	18°		19°		20°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.48998	9.97821	9.51264	9.97567	9.53405	9.97299	60
1	49037	97817	51301	97563	53440	97294	59
2	49076	97812	51338	97558	53475	97289	58
3	49115	97808	51374	97554	53509	97285	57
4	49153	97804	51411	97550	53544	97280	56
5	49192	97800	51447	97545	53578	97276	55
6	49231	97796	51484	97541	53613	97271	54
7	49269	97792	51520	97536	53647	97266	53
8	49308	97788	51557	97532	53682	97262	52
9	49347	97784	51593	97528	53716	97257	51
10	9.49385	9.97779	9.51629	9.97523	9.53751	9.97252	50
11	49424	97775	51666	97519	53785	97248	49
12	49462	97771	51702	97515	53819	97243	48
13	49500	97767	51738	97510	53854	97238	47
14	49539	97763	51774	97506	53888	97234	46
15	49577	97759	51811	97501	53922	97229	45
16	49615	97754	51847	97497	53957	97224	44
17	49654	97750	51883	97492	53991	97220	43
18	49692	97746	51919	97488	54025	97215	42
19	49730	97742	51955	97484	54059	97210	41
20	9.49768	9.97738	9.51991	9.97479	9.54093	9.97206	40
21	49806	97734	52027	97475	54127	97201	39
22	49844	97729	52063	97470	54161	97196	38
23	49882	97725	52099	97466	54195	97192	37
24	49920	97721	52135	97461	54229	97187	36
25	49958	97717	52171	97457	54263	97182	35
26	49996	97713	52207	97453	54297	97178	34
27	50034	97708	52242	97448	54331	97173	33
28	50072	97704	52278	97444	54365	97168	32
29	50110	97700	52314	97439	54399	97163	31
30	9.50148	9.97696	9.52350	9.97435	9.54433	9.97159	30
31	50185	97691	52385	97430	54466	97154	29
32	50223	97687	52421	97426	54500	97149	28
33	50261	97683	52456	97421	54534	97145	27
34	50298	97679	52492	97417	54567	97140	26
35	50336	97674	52527	97412	54601	97135	25
36	50374	97670	52563	97408	54635	97130	24
37	50411	97666	52598	97403	54668	97126	23
38	50449	97662	52634	97399	54702	97121	22
39	50486	97657	52669	97394	54735	97116	21
40	9.50523	9.97653	9.52705	9.97390	9.54769	9.97111	20
41	50561	97649	52740	97385	54802	97107	19
42	50598	97645	52775	97381	54836	97102	18
43	50635	97640	52811	97376	54869	97097	17
44	50673	97636	52846	97372	54903	97092	16
45	50710	97632	52881	97367	54936	97087	15
46	50747	97628	52916	97363	54969	97083	14
47	50784	97623	52951	97358	55003	97078	13
48	50821	97619	52986	97353	55036	97073	12
49	50858	97615	53021	97349	55069	97068	11
50	9.50896	9.97610	9.53056	9.97344	9.55102	9.97063	10
51	50933	97606	53092	97340	55136	97059	9
52	50970	97602	53126	97335	55169	97054	8
53	51007	97597	53161	97331	55202	97049	7
54	51043	97593	53196	97326	55235	97044	6
55	51080	97589	53231	97322	55268	97039	5
56	51117	97584	53266	97317	55301	97035	4
57	51154	97580	53301	97312	55334	97030	3
58	51191	97576	53336	97308	55367	97025	2
59	51227	97571	53370	97303	55400	97020	1
60	51264	97567	53405	97299	55433	97015	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	71°		70°		69°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	21°		22°		23°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	9.55433	9.97015	9.57358	9.96717	9.59188	9.96403	60
1	55466	97010	57389	96711	59218	96397	59
2	55499	97005	57420	96706	59247	96392	58
3	55532	97001	57451	96701	59277	96387	57
4	55564	96996	57482	96696	59307	96381	56
5	55597	96991	57514	96691	59336	96376	55
6	55630	96986	57545	96686	59366	96370	54
7	55663	96981	57576	96681	59396	96365	53
8	55695	96976	57607	96676	59425	96360	52
9	55728	96971	57638	96670	59455	96354	51
10	9.55761	9.96966	9.57669	9.96665	9.59484	9.96349	50
11	55793	96962	57700	96660	59514	96343	49
12	55826	96957	57731	96655	59543	96338	48
13	55858	96952	57762	96650	59573	96333	47
14	55891	96947	57793	96645	59602	96327	46
15	55923	96942	57824	96640	59632	96322	45
16	55956	96937	57855	96634	59661	96316	44
17	55988	96932	57885	96629	59690	96311	43
18	56021	96927	57916	96624	59720	96305	42
19	56053	96922	57947	96619	59749	96300	41
20	9.56085	9.96917	9.57978	9.96614	9.59778	9.96294	40
21	56118	96912	58008	96608	59808	96289	39
22	56150	96907	58039	96603	59837	96284	38
23	56182	96903	58070	96598	59866	96278	37
24	56215	96898	58101	96593	59895	96273	36
25	56247	96893	58131	96588	59924	96267	35
26	56279	96888	58162	96582	59954	96262	34
27	56311	96883	58192	96577	59983	96256	33
28	56343	96878	58223	96572	60012	96251	32
29	56375	96873	58253	96567	60041	96245	31
30	9.56408	9.96868	9.58284	9.96562	9.60070	9.96240	30
31	56440	96863	58314	96556	60099	96234	29
32	56472	96858	58345	96551	60128	96229	28
33	56504	96853	58375	96546	60157	96223	27
34	56536	96848	58406	96541	60186	96218	26
35	56568	96843	58436	96535	60215	96212	25
36	56599	96838	58467	96530	60244	96207	24
37	56631	96833	58497	96525	60273	96201	23
38	56663	96828	58527	96520	60302	96196	22
39	56695	96823	58557	96514	60331	96190	21
40	9.56727	9.96818	9.58588	9.96509	9.60359	9.96185	20
41	56759	96813	58618	96504	60388	96179	19
42	56790	96808	58648	96498	60417	96174	18
43	56822	96803	58678	96493	60446	96168	17
44	56854	96798	58709	96488	60474	96162	16
45	56886	96793	58739	96483	60503	96157	15
46	56917	96788	58769	96477	60532	96151	14
47	56949	96783	58799	96472	60561	96146	13
48	56980	96778	58829	96467	60589	96140	12
49	57012	96772	58859	96461	60618	96135	11
50	9.57044	9.96767	9.58889	9.96456	9.60646	9.96129	10
51	57075	96762	58919	96451	60675	96123	9
52	57107	96757	58949	96445	60704	96118	8
53	57138	96752	58979	96440	60732	96112	7
54	57169	96747	59009	96435	60761	96107	6
55	57201	96742	59039	96429	60789	96101	5
56	57232	96737	59069	96424	60818	96095	4
57	57264	96732	59098	96419	60846	96090	3
58	57295	96727	59128	96413	60875	96084	2
59	57326	96722	59158	96408	60903	96079	1
60	57358	96717	59188	96403	60931	96073	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	68°		67°		66°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	24° Sine	24° Cosine	25° Sine	25° Cosine	26° Sine	26° Cosine	′
0	9.60931	9.96073	9.62595	9.95728	9.64184	9.95366	60
1	60960	96067	62622	95722	64210	95360	59
2	60988	96062	62649	95716	64236	95354	58
3	61016	96056	62676	95710	64262	95348	57
4	61045	96050	62703	95704	64288	95341	56
5	61073	96045	62730	95698	64313	95335	55
6	61101	96039	62757	95692	64339	95329	54
7	61129	96034	62784	95686	64365	95323	53
8	61158	96028	62811	95680	64391	95317	52
9	61186	96022	62838	95674	64417	95310	51
10	9.61214	9.96017	9.62865	9.95668	9.64442	9.95304	50
11	61242	96011	62892	95663	64468	95298	49
12	61270	96005	62918	95657	64494	95292	48
13	61298	96000	62945	95651	64519	95286	47
14	61326	95994	62972	95645	64545	95279	46
15	61354	95988	62999	95639	64571	95273	45
16	61382	95982	63026	95633	64596	95267	44
17	61411	95977	63052	95627	64622	95261	43
18	61438	95971	63079	95621	64647	95254	42
19	61466	95965	63106	95615	64673	95248	41
20	9.61494	9.95960	9.63133	9.95609	9.64698	9.95242	40
21	61522	95954	63159	95603	64724	95236	39
22	61550	95948	63186	95597	64749	95229	38
23	61578	95942	63213	95591	64775	95223	37
24	61606	95937	63239	95585	64800	95217	36
25	61634	95931	63266	95579	64826	95211	35
26	61662	95925	63292	95573	64851	95204	34
27	61689	95920	63319	95567	64877	95198	33
28	61717	95914	63345	95561	64902	95192	32
29	61745	95908	63372	95555	64927	95185	31
30	9.61773	9.95902	9.63398	9.95549	9.64953	9.95179	30
31	61800	95897	63425	95543	64978	95173	29
32	61828	95891	63451	95537	65003	95167	28
33	61856	95885	63478	95531	65029	95160	27
34	61883	95879	63504	95525	65054	95154	26
35	61911	95873	63531	95519	65079	95148	25
36	61939	95868	63557	95513	65104	95141	24
37	61966	95862	63583	95507	65130	95135	23
38	61994	95856	63610	95500	65155	95129	22
39	62021	95850	63636	95494	65180	95122	21
40	9.62049	9.95844	9.63662	9.95488	9.65205	9.95116	20
41	62076	95839	63689	95482	65230	95110	19
42	62104	95833	63715	95476	65255	95103	18
43	62131	95827	63741	95470	65281	95097	17
44	62159	95821	63767	95464	65306	95090	16
45	62186	95815	63794	95458	65331	95084	15
46	62214	95810	63820	95452	65356	95078	14
47	62241	95804	63846	95446	65381	95071	13
48	62268	95798	63872	95440	65406	95065	12
49	62296	95792	63898	95434	65431	95059	11
50	9.62328	9.95786	9.63924	9.95427	9.65456	9.95052	10
51	62350	95780	63950	95421	65481	95046	9
52	62377	95775	63976	95415	65506	95039	8
53	62405	95769	64002	95409	65531	95033	7
54	62432	95763	64028	95403	65565	95027	6
55	62459	95757	64054	95397	65580	95020	5
56	62486	95751	64080	95391	65605	95014	4
57	62513	95745	64106	95384	65630	95007	3
58	62541	95739	64132	95378	65655	95001	2
59	62568	95733	64158	95372	65680	94995	1
60	62595	95728	64184	95366	65705	94988	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	65°		64°		63°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	27° Sine	27° Cosine	28° Sine	28° Cosine	29° Sine	29° Cosine	′
0	9.65705	9.94988	9.67161	9.94593	9.68557	9.94182	60
1	65729	94982	67185	94587	68580	94175	59
2	65754	94975	67208	94580	68603	94168	58
3	65779	94969	67232	94573	68625	94161	57
4	65804	94962	67256	94567	68648	94154	56
5	65828	94956	67280	94560	68671	94147	55
6	65853	94949	67303	94553	68694	94140	54
7	65878	94943	67327	94546	68716	94133	53
8	65902	94936	67350	94540	68739	94126	52
9	65927	94930	67374	94533	68762	94119	51
10	9.65952	9.94923	9.67398	9.94526	9.68784	9.94112	50
11	65976	94917	67421	94519	68807	94105	49
12	66001	94911	67445	94513	68830	94098	48
13	66025	94904	67468	94506	68852	94090	47
14	66050	94898	67492	94499	68875	94083	46
15	66075	94891	67515	94492	68897	94076	45
16	66099	94885	67539	94485	68920	94069	44
17	66124	94878	67562	94479	68942	94062	43
18	66148	94871	67586	94472	68965	94055	42
19	66173	94865	67609	94465	68987	94048	41
20	9.66197	9.94858	9.67633	9.94458	9.69010	9.94041	40
21	66221	94852	67656	94451	69032	94034	39
22	66246	94845	67680	94445	69055	94027	38
23	66270	94839	67703	94438	69077	94020	37
24	66295	94832	67726	94431	69100	94012	36
25	66319	94826	67750	94424	69122	94005	35
26	66343	94819	67773	94417	69144	93998	34
27	66368	94813	67796	94410	69167	93991	33
28	66392	94806	67820	94404	69189	93984	32
29	66416	94799	67843	94397	69212	93977	31
30	9.66441	9.94793	9.67866	9.94390	9.69234	9.93970	30
31	66465	94786	67890	94383	69256	93963	29
32	66489	94780	67913	94376	69279	93955	28
33	66513	94773	67936	94369	69301	93948	27
34	66537	94767	67959	94362	69323	93941	26
35	66562	94760	67982	94355	69345	93934	25
36	66586	94753	68006	94349	69368	93927	24
37	66610	94747	68029	94342	69390	93920	23
38	66634	94740	68052	94335	69412	93912	22
39	66658	94734	68075	94328	69434	93905	21
40	9.66682	9.94727	9.68098	9.94321	9.69456	9.93898	20
41	66706	94720	68121	94314	69479	93891	19
42	66731	94714	68144	94307	69501	93884	18
43	66755	94707	68167	94300	69523	93876	17
44	66779	94700	68190	94293	69545	93869	16
45	66803	94694	68213	94286	69567	93862	15
46	66827	94687	68237	94279	69589	93855	14
47	66851	94680	68260	94273	69611	93847	13
48	66875	94674	68283	94266	69633	93840	12
49	66899	94667	68305	94259	69655	93833	11
50	9.66922	9.94660	9.68328	9.94252	9.69677	9.93826	10
51	66946	94654	68351	94245	69699	93819	9
52	66970	94647	68374	94238	69721	93811	8
53	66994	94640	68397	94231	69743	93804	7
54	67018	94634	68420	94224	69765	93797	6
55	67042	94627	68443	94217	69787	93789	5
56	67066	94620	68466	94210	69809	93782	4
57	67090	94614	68489	94203	69831	93775	3
58	67113	94607	68512	94196	69853	93768	2
59	67137	94600	68534	94189	69875	93760	1
60	67161	94593	68557	94182	69897	93753	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	62°		61°		60°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	30° Sine	30° Cosine	31° Sine	31° Cosine	32° Sine	32° Cosine	′
0	9.69897	9.93753	9.71184	9.93307	9.72421	9.92842	60
1	69919	93746	71205	93299	72441	92834	59
2	69941	93738	71226	93291	72461	92826	58
3	69963	93731	71247	93284	72482	92818	57
4	69984	93724	71268	93276	72502	92810	56
5	70006	93717	71289	93269	72522	92803	55
6	70028	93709	71310	93261	72542	92795	54
7	70050	93702	71331	93253	72562	92787	53
8	70072	93695	71352	93246	72582	92779	52
9	70093	93687	71373	93238	72602	92771	51
10	9.70115	9.93680	9.71393	9.93230	9.72622	9.92763	50
11	70137	93673	71414	93223	72643	92755	49
12	70159	93665	71435	93215	72663	92747	48
13	70180	93658	71456	93207	72683	92739	47
14	70202	93650	71477	93200	72703	92731	46
15	70224	93643	71498	93192	72723	92723	45
16	70245	93636	71519	93184	72743	92715	44
17	70267	93628	71539	93177	72763	92707	43
18	70288	93621	71560	93169	72783	92699	42
19	70310	93614	71581	93161	72803	92691	41
20	9.70332	9.93606	9.71602	9.93154	9.72823	9.92683	40
21	70353	93599	71622	93146	72843	92675	39
22	70375	93591	71643	93138	72863	92667	38
23	70396	93584	71664	93131	72883	92659	37
24	70418	93577	71685	93123	72902	92651	36
25	70439	93569	71705	93115	72922	92643	35
26	70461	93562	71726	93108	72942	92635	34
27	70482	93554	71747	93100	72962	92627	33
28	70504	93547	71767	93092	72982	92619	32
29	70525	93539	71788	93084	73002	92611	31
30	9.70547	9.93532	9.71809	9.93077	9.73022	9.92603	30
31	70568	93525	71829	93069	73041	92595	29
32	70590	93517	71850	93061	73061	92587	28
33	70611	93510	71870	93053	73081	92579	27
34	70633	93502	71891	93046	73101	92571	26
35	70654	93495	71911	93038	73121	92563	25
36	70675	93487	71932	93030	73140	92555	24
37	70697	93480	71952	93022	73160	92546	23
38	70718	93472	71973	93014	73180	92538	22
39	70739	93465	71994	93007	73200	92530	21
40	9.70761	9.93457	9.72014	9.92999	9.73219	9.92522	20
41	70782	93450	72034	92991	73239	92514	19
42	70803	93442	72055	92983	73259	92506	18
43	70824	93435	72075	92976	73278	92498	17
44	70846	93427	72096	92968	73298	92490	16
45	70867	93420	72116	92960	73318	92482	15
46	70888	93412	72137	92952	73337	92473	14
47	70909	93405	72157	92944	73357	92465	13
48	70931	93397	72177	92936	73377	92457	12
49	70952	93390	72198	92929	73396	92449	11
50	9.70973	9.93382	9.72218	9.92921	9.73416	9.92441	10
51	70994	93375	72238	92913	73435	92433	9
52	71015	93367	72259	92905	73455	92425	8
53	71036	93360	72279	92897	73474	92416	7
54	71058	93352	72299	92889	73494	92408	6
55	71079	93344	72320	92881	73513	92400	5
56	71100	93337	72340	92874	73533	92392	4
57	71121	93329	72360	92866	73552	92384	3
58	71142	93322	72381	92858	73572	92376	2
59	71163	93314	72401	92850	73591	92367	1
60	71184	93307	72421	92842	73611	92359	0
′	59° Cosine	59° Sine	58° Cosine	58° Sine	57° Cosine	57° Sine	′

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	33° Sine	33° Cosine	34° Sine	34° Cosine	35° Sine	35° Cosine	′
0	9.73611	9.92359	9.74756	9.91857	9.75859	9.91336	60
1	73630	92351	74775	91849	75877	91328	59
2	73650	92343	74794	91840	75895	91319	58
3	73669	92335	74812	91832	75913	91310	57
4	73689	92326	74831	91823	75931	91301	56
5	73708	92318	74850	91815	75949	91292	55
6	73727	92310	74868	91806	75967	91283	54
7	73747	92302	74887	91798	75985	91274	53
8	73766	92293	74906	91789	76003	91266	52
9	73785	92285	74924	91781	76021	91257	51
10	9.73805	9.92277	9.74943	9.91772	9.76039	9.91248	50
11	73824	92269	74961	91763	76057	91239	49
12	73843	92260	74980	91755	76075	91230	48
13	73863	92252	74999	91746	76093	91221	47
14	73882	92244	75017	91738	76111	91212	46
15	73901	92235	75036	91729	76129	91203	45
16	73921	92227	75054	91720	76146	91194	44
17	73940	92219	75073	91712	76164	91185	43
18	73959	92211	75091	91703	76182	91176	42
19	73978	92202	75110	91695	76200	91167	41
20	9.73997	9.92194	9.75128	9.91686	9.76218	9.91158	40
21	74017	92186	75147	91677	76236	91149	39
22	74036	92177	75165	91669	76253	91141	38
23	74055	92169	75184	91660	76271	91132	37
24	74074	92161	75202	91651	76289	91123	36
25	74093	92152	75221	91643	76307	91114	35
26	74113	92144	75239	91634	76324	91105	34
27	74132	92136	75258	91625	76342	91096	33
28	74151	92127	75276	91617	76360	91087	32
29	74170	92119	75294	91608	76378	91078	31
30	9.74189	9.92111	9.75313	9.91599	9.76395	9.91069	30
31	74208	92102	75331	91591	76413	91060	29
32	74227	92094	75350	91582	76431	91051	28
33	74246	92086	75368	91573	76448	91042	27
34	74265	92077	75386	91565	76466	91033	26
35	74284	92069	75405	91556	76484	91023	25
36	74303	92060	75423	91547	76501	91014	24
37	74322	92052	75441	91538	76519	91005	23
38	74341	92044	75459	91530	76537	90996	22
39	74360	92035	75478	91521	76554	90987	21
40	9.74379	9.92027	9.75496	9.91512	9.76572	9.90978	20
41	74398	92018	75514	91504	76590	90969	19
42	74417	92010	75533	91495	76607	90960	18
43	74436	92002	75551	91486	76625	90951	17
44	74455	91993	75569	91477	76642	90942	16
45	74474	91985	75587	91469	76660	90933	15
46	74493	91976	75605	91460	76677	90924	14
47	74512	91968	75624	91451	76695	90915	13
48	74531	91959	75642	91442	76712	90906	12
49	74549	91951	75660	91433	76730	90896	11
50	9.74568	9.91942	9.75678	9.91425	9.76747	9.90887	10
51	74587	91934	75696	91416	76765	90878	9
52	74606	91925	75714	91407	76782	90869	8
53	74625	91917	75733	91398	76800	90860	7
54	74644	91908	75751	91389	76817	90851	6
55	74662	91900	75769	91381	76835	90842	5
56	74681	91891	75787	91372	76852	90832	4
57	74700	91883	75805	91363	76870	90823	3
58	74719	91874	75823	91354	76887	90814	2
59	74737	91866	75841	91345	76904	90805	1
60	74756	91857	75859	91336	76922	90796	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	56°		55°		54°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	36° Sine	36° Cosine	37° Sine	37° Cosine	38° Sine	38° Cosine	′
0	9.76922	9.90796	9.77946	9.90235	9.78934	9.89653	60
1	76939	90787	77963	90225	78950	89643	59
2	76957	90777	77980	90216	78967	89633	58
3	76974	90768	77997	90206	78983	89624	57
4	76991	90759	78013	90197	78999	89614	56
5	77009	90750	78030	90187	79015	89604	55
6	77026	90741	78047	90178	79031	89594	54
7	77043	90731	78063	90168	79047	89584	53
8	77061	90722	78080	90159	79063	89574	52
9	77078	90713	78097	90149	79079	89564	51
10	9.77095	9.90704	9.78113	9.90139	9.79095	9.89554	50
11	77112	90694	78130	90130	79111	89544	49
12	77130	90685	78147	90120	79128	89534	48
13	77147	90676	78163	90111	79144	89524	47
14	77164	90667	78180	90101	79160	89514	46
15	77181	90657	78197	90091	79176	89504	45
16	77199	90648	78213	90082	79192	89495	44
17	77216	90639	78230	90072	79208	89485	43
18	77233	90630	78246	90063	79224	89475	42
19	77250	90620	78263	90053	79240	89465	41
20	9.77268	9.90611	9.78280	9.90043	9.79256	9.89455	40
21	77285	90602	78296	90034	79272	89445	39
22	77302	90592	78313	90024	79288	89435	38
23	77319	90583	78329	90014	79304	89425	37
24	77336	90574	78346	90005	79319	89415	36
25	77353	90565	78362	89995	79335	89405	35
26	77370	90555	78379	89985	79351	89395	34
27	77387	90546	78395	89976	79367	89385	33
28	77405	90537	78412	89966	79383	89375	32
29	77422	90527	78428	89956	79399	89364	31
30	9.77439	9.90518	9.78445	9.89947	9.79415	9.89354	30
31	77456	90509	78461	89937	79431	89344	29
32	77473	90499	78478	89927	79447	89334	28
33	77490	90490	78494	89918	79463	89324	27
34	77507	90480	78510	89908	79478	89314	26
35	77524	90471	78527	89898	79494	89304	25
36	77541	90462	78543	89888	79510	89294	24
37	77558	90452	78560	89879	79526	89284	23
38	77575	90443	78576	89869	79542	89274	22
39	77592	90434	78592	89859	79558	89264	21
40	9.77609	9.90424	9.78609	9.89849	9.79573	9.89254	20
41	77626	90415	78625	89840	79589	89244	19
42	77643	90405	78642	89830	79605	89233	18
43	77660	90396	78658	89820	79621	89223	17
44	77677	90386	78674	89810	79636	89213	16
45	77694	90377	78691	89801	79652	89203	15
46	77711	90368	78707	89791	79668	89193	14
47	77728	90358	78723	89781	79684	89183	13
48	77744	90349	78739	89771	79699	89173	12
49	77761	90339	78756	89761	79715	89162	11
50	9.77778	9.90330	9.78772	9.89752	9.79731	9.89152	10
51	77795	90320	78788	89742	79746	89142	9
52	77812	90311	78805	89732	79762	89132	8
53	77829	90301	78821	89722	79778	89122	7
54	77846	90292	78837	89712	79793	89112	6
55	77862	90282	78853	89702	79809	89101	5
56	77879	90273	78869	89693	79825	89091	4
57	77896	90263	78886	89683	79840	89081	3
58	77913	90254	78902	89673	79856	89071	2
59	77930	90244	78918	89663	79872	89060	1
60	77946	90235	78934	89653	79887	89050	0
′	Cosine 53°	Sine 53°	Cosine 52°	Sine 52°	Cosine 51°	Sine 51°	′

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	39° Sine	39° Cosine	40° Sine	40° Cosine	41° Sine	41° Cosine	′
0	9.79887	9.89050	9.80807	9.88425	9.81694	9.87778	60
1	79903	89040	80822	88415	81709	87767	59
2	79918	89030	80837	88404	81723	87756	58
3	79934	89020	80852	88394	81738	87745	57
4	79950	89009	80867	88383	81752	87734	56
5	79965	88999	80882	88372	81767	87723	55
6	79981	88989	80897	88362	81781	87712	54
7	79996	88978	80912	88351	81796	87701	53
8	80012	88968	80927	88340	81810	87690	52
9	80027	88958	80942	88330	81825	87679	51
10	9.80043	9.88948	9.80957	9.88319	9.81839	9.87668	50
11	80058	88937	80972	88308	81854	87657	49
12	80074	88927	80987	88298	81868	87646	48
13	80089	88917	81002	88287	81882	87635	47
14	80105	88906	81017	88276	81897	87624	46
15	80120	88896	81032	88266	81911	87613	45
16	80136	88886	81047	88255	81926	87601	44
17	80151	88875	81061	88244	81940	87590	43
18	80166	88865	81076	88234	81955	87579	42
19	80182	88855	81091	88223	81969	87568	41
20	9.80197	9.88844	9.81106	9.88212	9.81983	9.87557	40
21	80213	88834	81121	88201	81998	87546	39
22	80228	88824	81136	88191	82012	87535	38
23	80244	88813	81151	88180	82026	87524	37
24	80259	88803	81166	88169	82041	87513	36
25	80274	88793	81180	88158	82055	87501	35
26	80290	88782	81195	88148	82069	87490	34
27	80305	88772	81210	88137	82084	87479	33
28	80320	88761	81225	88126	82098	87468	32
29	80336	88751	81240	88115	82112	87457	31
30	9.80351	9.88741	9.81254	9.88105	9.82126	9.87446	30
31	80366	88730	81269	88094	82141	87434	29
32	80382	88720	81284	88083	82155	87423	28
33	80397	88709	81299	88072	82169	87412	27
34	80412	88699	81314	88061	82184	87401	26
35	80428	88688	81328	88051	82198	87390	25
36	80443	88678	81343	88040	82212	87378	24
37	80458	88668	81358	88029	82226	87367	23
38	80473	88657	81372	88018	82240	87356	22
39	80489	88647	81387	88007	82255	87345	21
40	9.80504	9.88636	9.81402	9.87996	9.82269	9.87334	20
41	80519	88626	81417	87985	82283	87322	19
42	80534	88615	81431	87975	82297	87311	18
43	80550	88605	81446	87964	82311	87300	17
44	80565	88594	81461	87953	82326	87288	16
45	80580	88584	81475	87942	82340	87277	15
46	80595	88573	81490	87931	82354	87266	14
47	80610	88563	81505	87920	82368	87255	13
48	80625	88552	81519	87909	82382	87243	12
49	80641	88542	81534	87898	82396	87232	11
50	9.80656	9.88531	9.81549	9.87887	9.82410	9.87221	10
51	80671	88521	81563	87877	82424	87209	9
52	80686	88510	81578	87866	82439	87198	8
53	80701	88499	81592	87855	82453	87187	7
54	80716	88489	81607	87844	82467	87175	6
55	80731	88478	81622	87833	82481	87164	5
56	80746	88468	81636	87822	82495	87153	4
57	80762	88457	81651	87811	82509	87141	3
58	80777	88447	81665	87800	82523	87130	2
59	80792	88436	81680	87789	82537	87119	1
60	80807	88425	81694	87778	82551	87107	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	50°		49°		48°		

TABLE XIV.—*Continued.*

LOGARITHMIC SINES AND COSINES.

′	42° Sine	42° Cosine	43° Sine	43° Cosine	44° Sine	44° Cosine	′
0	9.82551	9.87107	9.83378	9.86413	9.84177	9.85693	60
1	82565	87096	83392	86401	84190	85681	59
2	82579	87085	83405	86389	84203	85669	58
3	82593	87073	83419	86377	84216	85657	57
4	82607	87062	83432	86366	84229	85645	56
5	82621	87050	83446	86354	84242	85632	55
6	82635	87039	83459	86342	84255	85620	54
7	82649	87028	83473	86330	84269	85608	53
8	82663	87016	83486	86318	84282	85596	52
9	82677	87005	83500	86306	84295	85583	51
10	9.82691	9.86993	9.83513	9.86295	9.84308	9.85571	50
11	82705	86982	83527	86283	84321	85559	49
12	82719	86970	83540	86271	84334	85547	48
13	82733	86959	83554	86259	84347	85534	47
14	82747	86947	83567	86247	84360	85522	46
15	82761	86936	83581	86235	84373	85510	45
16	82775	86924	83594	86223	84385	85497	44
17	82788	86913	83608	86211	84398	85485	43
18	82802	86902	83621	86200	84411	85473	42
19	82816	86890	83634	86188	84424	85460	41
20	9.82830	9.86879	9.83648	9.86176	9.84437	9.85448	40
21	82844	86867	83661	86164	84450	85436	39
22	82858	86855	83674	86152	84463	85423	38
23	82872	86844	83688	86140	84476	85411	37
24	82885	86832	83701	86128	84489	85399	36
25	82899	86821	83715	86116	84502	85386	35
26	82913	86809	83728	86104	84515	85374	34
27	82927	86798	83741	86092	84528	85361	33
28	82941	86786	83755	86080	84540	85349	32
29	82955	86775	83768	86068	84553	85337	31
30	9.82968	9.86763	9.83781	9.86056	9.84566	9.85324	30
31	82982	86752	83795	86044	84579	85312	29
32	82996	86740	83808	86032	84592	85299	28
33	83010	86728	83821	86020	84605	85287	27
34	83023	86717	83834	86008	84618	85274	26
35	83037	86705	83848	85996	84630	85262	25
36	83051	86694	83861	85984	84643	85250	24
37	83065	86682	83874	85972	84656	85237	23
38	83078	86670	83887	85960	84669	85225	22
39	83092	86659	83901	85948	84682	85212	21
40	9.83106	9.86647	9.83914	9.85936	9.84694	9.85200	20
41	83120	86635	83927	85924	84707	85187	19
42	83133	86624	83940	85912	84720	85175	18
43	83147	86612	83954	85900	84733	85162	17
44	83161	86600	83967	85888	84745	85150	16
45	83174	86589	83980	85876	84758	85137	15
46	83188	86577	83993	85864	84771	85125	14
47	83202	86565	84006	85851	84784	85112	13
48	83215	86554	84020	85839	84796	85100	12
49	83229	86542	84033	85827	84809	85087	11
50	9.83242	9.86530	9.84046	9.85815	9.84822	9.85074	10
51	83256	86518	84059	85803	84835	85062	9
52	83270	86507	84072	85791	84847	85049	8
53	83283	86495	84085	85779	84860	85037	7
54	83297	86483	84098	85766	84873	85024	6
55	83310	86472	84112	85754	84885	85012	5
56	83324	86460	84125	85742	84898	84999	4
57	83338	86448	84138	85730	84911	84986	3
58	83351	86436	84151	85718	84923	84974	2
59	83365	86425	84164	85706	84936	84961	1
60	83378	86413	84177	85693	84949	84949	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	47°		46°		45°		

TABLE XV.

LOG. TANGENTS AND COTANGENTS.

′	0° Tan	0° Cotan	1° Tan	1° Cotan	2° Tan	2° Cotan	′
0	−∞	∞	8.24192	11.75808	8.54308	11.45692	60
1	6.46373	13.53627	24910	75090	54669	45331	59
2	76476	23524	25616	74384	55027	44973	58
3	94085	05915	26312	73688	55382	44618	57
4	7.06579	12.93421	26996	73004	55734	44266	56
5	16270	83730	27669	72331	56083	43917	55
6	24188	75812	28332	71668	56429	43571	54
7	30882	69118	28986	71014	56773	43227	53
8	36682	63318	29629	70371	57114	42886	52
9	41797	58203	30263	69737	57452	42548	51
10	7.46373	12.53627	8.30888	11.69112	8.57788	11.42212	50
11	50512	49488	31505	68495	58121	41879	49
12	54291	45709	32112	67888	58451	41549	48
13	57767	42233	32711	67289	58779	41221	47
14	60986	39014	33302	66698	59105	40895	46
15	63982	36018	33886	66114	59428	40572	45
16	66785	33215	34461	65539	59749	40251	44
17	69418	30582	35029	64971	60068	39932	43
18	71900	28100	35590	64410	60384	39616	42
19	74248	25752	36143	63857	60698	39302	41
20	7.76476	12.23524	8.36689	11.63311	8.61009	11.38991	40
21	78595	21405	37229	62771	61319	38681	39
22	80615	19385	37762	62238	61626	38374	38
23	82546	17454	38289	61711	61931	38069	37
24	84394	15606	38809	61191	62234	37766	36
25	86167	13833	39323	60677	62535	37465	35
26	87871	12129	39832	60168	62834	37166	34
27	89510	10490	40334	59666	63131	36869	33
28	91089	08911	40830	59170	63426	36574	32
29	92613	07387	41321	58679	63718	36282	31
30	7.94086	12.05914	8.41807	11.58193	8.64009	11.35991	30
31	95510	04490	42287	57713	64298	35702	29
32	96889	03111	42762	57238	64585	35415	28
33	98225	01775	43232	56768	64870	35130	27
34	99522	00478	43696	56304	65154	34846	26
35	8.00781	11.99219	44156	55844	65435	34565	25
36	02004	97996	44611	55389	65715	34285	24
37	03194	96806	45061	54939	65993	34007	23
38	04353	95647	45507	54493	66269	33731	22
39	05481	94519	45948	54052	66543	33457	21
40	8.06581	11.93419	8.46385	11.53615	8.66816	11.33184	20
41	07653	92347	46817	53183	67087	32913	19
42	08700	91300	47245	52755	67356	32644	18
43	09722	90278	47669	52331	67624	32376	17
44	10720	89280	48089	51911	67890	32110	16
45	11696	88304	48505	51495	68154	31846	15
46	12651	87349	48917	51083	68417	31583	14
47	13585	86415	49325	50675	68678	31322	13
48	14500	85500	49729	50271	68938	31062	12
49	15395	84605	50130	49870	69196	30804	11
50	8.16273	11.83727	8.50527	11.49473	8.69453	11.30547	10
51	17133	82867	50920	49080	69708	30292	9
52	17976	82024	51310	48690	69962	30038	8
53	18804	81196	51696	48304	70214	29786	7
54	19616	80384	52079	47921	70465	29535	6
55	20413	79587	52459	47541	70714	29286	5
56	21195	78805	52835	47165	70962	29038	4
57	21964	78036	53208	46792	71208	28792	3
58	22720	77280	53578	46422	71453	28547	2
59	23462	76538	53945	46055	71697	28303	1
60	24192	75808	54308	45692	71940	28060	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	89°		88°		87°		

Table XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	3° Tan	3° Cotan	4° Tan	4° Cotan	5° Tan	5° Cotan	′
0	8.71940	11.28060	8.84464	11.15536	8.94195	11.05805	60
1	72181	27819	84646	15354	94340	05660	59
2	72420	27580	84826	15174	94485	05515	58
3	72659	27341	85006	14994	94630	05370	57
4	72896	27104	85185	14815	94773	05227	56
5	73132	26868	85363	14637	94917	05083	55
6	73366	26634	85540	14460	95060	04940	54
7	73600	26400	85717	14283	95202	04798	53
8	73832	26168	85893	14107	95344	04656	52
9	74063	25937	86069	13931	95486	04514	51
10	8.74292	11.25708	8.86243	11.13757	8.95627	11.04373	50
11	74521	25479	86417	13583	95767	04233	49
12	74748	25252	86591	13409	95908	04092	48
13	74974	25026	86763	13237	96047	03953	47
14	75199	24801	86935	13065	96187	03813	46
15	75423	24577	87106	12894	96325	03675	45
16	75645	24355	87277	12723	96464	03536	44
17	75867	24133	87447	12553	96602	03398	43
18	76087	23913	87616	12384	96739	03261	42
19	76306	23694	87785	12215	96877	02123	41
20	8.76525	11.23475	8.87953	11.12047	8.97013	11.02987	40
21	76742	23258	88120	11880	97150	02850	39
22	76958	23042	88287	11713	97285	02715	38
23	77173	22827	88453	11547	97421	02579	37
24	77387	22613	88618	11382	97556	02444	36
25	77600	22400	88783	11217	97691	02309	35
26	77811	22189	88948	11052	97825	02175	34
27	78022	21978	89111	10889	97959	02041	33
28	78232	21768	89274	10726	98092	01908	32
29	78441	21559	89437	10563	98225	01775	31
30	8.78649	11.21351	8.89598	11.10402	8.98358	11.01642	30
31	78855	21145	89760	10240	98490	01510	29
32	79061	20939	89920	10080	98622	01378	28
33	79266	20734	90080	09920	98753	01247	27
34	79470	20530	90240	09760	98884	01116	26
35	79673	20327	90399	09601	99015	00985	25
36	79875	20125	90557	09443	99145	00855	24
37	80076	19924	90715	09285	99275	00725	23
38	80277	19723	90872	09128	99405	00595	22
39	80476	19524	91029	08971	99534	00466	21
40	8.80674	11.19326	8.91185	11.08815	8.99662	11.00338	20
41	80872	19128	91340	08660	99791	00209	19
42	81068	18932	91495	08505	99919	00081	18
43	81264	18736	91650	08350	9.00046	10.99954	17
44	81459	18541	91803	08197	00174	99826	16
45	81653	18347	91957	08043	00301	99699	15
46	81846	18154	92110	07890	00427	99573	14
47	82038	17962	92262	07738	00553	99447	13
48	82230	17770	92414	07586	00679	99321	12
49	82420	17580	92565	07435	00805	99195	11
50	8.82610	11.17390	8.92716	11.07284	9.00930	10.99070	10
51	82799	17201	92866	07134	01055	98945	9
52	82987	17013	93016	06984	01179	98821	8
53	83175	16825	93165	06835	01303	98697	7
54	83361	16639	93313	06687	01427	98573	6
55	83547	16453	93462	06538	01550	98450	5
56	83732	16268	93609	06391	01673	98327	4
57	83916	16084	93756	06244	01796	98204	3
58	84100	15900	93903	06097	01918	98082	2
59	84282	15718	94049	05951	02040	97960	1
60	84464	15536	94195	05805	02162	97838	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	86°		85°		84°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	6° Tan	6° Cotan	7° Tan	7° Cotan	8° Tan	8° Cotan	′
0	9.02162	10.97838	9.08914	10.91086	9.14780	10.85220	60
1	02283	97717	09019	90981	14872	85128	59
2	02404	97596	09123	90877	14963	85037	58
3	02525	97475	09227	90773	15054	84946	57
4	02645	97355	09330	90670	15145	84855	56
5	02766	97234	09434	90566	15236	84764	55
6	02885	97115	09537	90463	15327	84673	54
7	03005	96995	09640	90360	15417	84583	53
8	03124	96876	09742	90258	15508	84492	52
9	03242	96758	09845	90155	15598	84402	51
10	9.03361	10.96639	9.09947	10.90053	9.15688	10.84312	50
11	03479	96521	10049	89951	15777	84223	49
12	03597	96403	10150	89850	15867	84133	48
13	03714	96286	10252	89748	15956	84044	47
14	03832	96168	10353	89647	16046	83954	46
15	03948	96052	10454	89546	16135	83865	45
16	04065	95935	10555	89445	16224	83776	44
17	04181	95819	10656	89344	16312	83688	43
18	04297	95703	10756	89244	16401	83599	42
19	04413	95587	10856	89144	16489	83511	41
20	9.04528	10.95472	9.10956	10.89044	9.16577	10.83423	40
21	04643	95357	11056	88944	16665	83335	39
22	04758	95242	11155	88845	16753	83247	38
23	04873	95127	11254	88746	16841	83159	37
24	04987	95013	11353	88647	16928	83072	36
25	05101	94899	11452	88548	17016	82984	35
26	05214	94786	11551	88449	17103	82897	34
27	05328	94672	11649	88351	17190	82810	33
28	05441	94559	11747	88253	17277	82723	32
29	05553	94447	11845	88155	17363	82637	31
30	9.05666	10.94334	9.11943	10.88057	9.17450	10.82550	30
31	05778	94222	12040	87960	17536	82464	29
32	05890	94110	12138	87862	17622	82378	28
33	06002	93998	12235	87765	17708	82292	27
34	06113	93887	12332	87668	17794	82206	26
35	06224	93776	12428	87572	17880	82120	25
36	06335	93665	12525	87475	17965	82035	24
37	06445	93555	12621	87379	18051	81949	23
38	06556	93444	12717	87283	18136	81864	22
39	06666	93334	12813	87187	18221	81779	21
40	9.06775	10.93225	9.12909	10.87091	9.18306	10.81694	20
41	06885	93115	13004	86996	18391	81609	19
42	06994	93006	13099	86901	18475	81525	18
43	07103	92897	13194	86806	18560	81440	17
44	07211	92789	13289	86711	18644	81356	16
45	07320	92680	13384	86616	18728	81272	15
46	07428	92572	13478	86522	18812	81188	14
47	07536	92464	13573	86427	18896	81104	13
48	07643	92357	13667	86333	18979	81021	12
49	07751	92249	13761	86239	19063	80937	11
50	9.07858	10.92142	9.13854	10.86146	9.19146	10.80854	10
51	07964	92036	13948	86052	19229	80771	9
52	08071	91929	14041	85959	19312	80688	8
53	08177	91823	14134	85866	19395	80605	7
54	08283	91717	14227	85773	19478	80522	6
55	08389	91611	14320	85680	19561	80439	5
56	08495	91505	14412	85588	19643	80357	4
57	08600	91400	14504	85496	19725	80275	3
58	08705	91295	14597	85403	19807	80193	2
59	08810	91190	14688	85312	19889	80111	1
60	08914	91086	14780	85220	19971	80029	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	83°		82°		81°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	9° Tan	9° Cotan	10° Tan	10° Cotan	11° Tan	11° Cotan	′
0	9.19971	10.80029	9.24632	10.75368	9.28865	10.71135	60
1	20053	79947	24706	75294	28933	71067	59
2	20134	79866	24779	75221	29000	71000	58
3	20216	79784	24853	75147	29067	70933	57
4	20297	79703	24926	75074	29134	70866	56
5	20378	79622	25000	75000	29201	70799	55
6	20459	79541	25073	74927	29268	70732	54
7	20540	79460	25146	74854	29335	70665	53
8	20621	79379	25219	74781	29402	70598	52
9	20701	79299	25292	74708	29468	70532	51
10	9.20782	10.79218	9.25365	10.74635	9.29535	10.70465	50
11	20862	79138	25437	74563	29601	70399	49
12	20942	79058	25510	74490	29668	70332	48
13	21022	78978	25582	74418	29734	70266	47
14	21102	78898	25655	74345	29800	70200	46
15	21182	78818	25727	74273	29866	70134	45
16	21261	78739	25799	74201	29932	70068	44
17	21341	78659	25871	74129	29998	70002	43
18	21420	78580	25943	74057	30064	69936	42
19	21499	78501	26015	73985	30130	69870	41
20	9.21578	10.78422	9.26086	10.73914	9.30195	10.69805	40
21	21657	78343	26158	73842	30261	69739	39
22	21736	78264	26229	73771	30326	69674	38
23	21814	78186	26301	73699	30391	69609	37
24	21893	78107	26372	73628	30457	69543	36
25	21971	78029	26443	73557	30522	69478	35
26	22049	77951	26514	73486	30587	69413	34
27	22127	77873	26585	73415	30652	69348	33
28	22205	77795	26655	73345	30717	69283	32
29	22283	77717	26726	73274	30782	69218	31
30	9.22361	10.77639	9.26797	10.73203	9.30846	10.69154	30
31	22438	77562	26867	73133	30911	69089	29
32	22516	77484	26937	73063	30975	69025	28
33	22593	77407	27008	72992	31040	68960	27
34	22670	77330	27078	72922	31104	68896	26
35	22747	77253	27148	72852	31168	68832	25
36	22824	77176	27218	72782	31233	68767	24
37	22901	77099	27288	72712	31297	68703	23
38	22977	77023	27357	72643	31361	68639	22
39	23054	76946	27427	72573	31425	68575	21
40	9.23130	10.76870	9.27496	10.72504	9.31489	10.68511	20
41	23206	76794	27566	72434	31552	68448	19
42	23283	76717	27635	72365	31616	68384	18
43	23359	76641	27704	72296	31679	68321	17
44	23435	76565	27773	72227	31743	68257	16
45	23510	76490	27842	72158	31806	68194	15
46	23586	76414	27911	72089	31870	68130	14
47	23661	76339	27980	72020	31933	68067	13
48	23737	76263	28049	71951	31996	68004	12
49	23812	76188	28117	71883	32059	67941	11
50	9.23887	10.76113	9.28186	10.71814	9.32122	10.67878	10
51	23962	76038	28254	71746	32185	67815	9
52	24037	75963	28323	71677	32248	67752	8
53	24112	75888	28391	71609	32311	67689	7
54	24186	75814	28459	71541	32373	67627	6
55	24261	75739	28527	71473	32436	67564	5
56	24335	75665	28595	71405	32498	67502	4
57	24410	75590	28662	71338	32561	67439	3
58	24484	75516	28730	71270	32623	67377	2
59	24558	75442	28798	71202	32685	67315	1
60	24632	75368	28865	71135	32747	67253	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	80°		79°		78°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	12°		13°		14°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.32747	10.67253	9.36336	10.63664	9.39677	10.60323	60
1	32810	67190	36394	63606	39731	60269	59
2	32872	67128	36452	63548	39785	60215	58
3	32933	67067	36509	63491	39838	60162	57
4	32995	67005	36566	63434	39892	60108	56
5	33057	66943	36624	63376	39945	60055	55
6	33119	66881	36681	63319	39999	60001	54
7	33180	66820	36738	63262	40052	59948	53
8	33242	66758	36795	63205	40106	59894	52
9	33303	66697	36852	63148	40159	59841	51
10	9.33365	10.66635	9.36909	10.63091	9.40212	10.59788	50
11	33426	66574	36966	63034	40266	59734	49
12	33487	66513	37023	62977	40319	59681	48
13	33548	66452	37080	62920	40372	59628	47
14	33609	66391	37137	62863	40425	59575	46
15	33670	66330	37193	62807	40478	59522	45
16	33731	66269	37250	62750	40531	59469	44
17	33792	66208	37306	62694	40584	59416	43
18	33853	66147	37363	62637	40636	59364	42
19	33913	66087	37419	62581	40689	59311	41
20	9.33974	10.66026	9.37476	10.62524	9.40742	10.59258	40
21	34034	65966	37532	62468	40795	59205	39
22	34095	65905	37588	62412	40847	59153	38
23	34155	65845	37644	62356	40900	59100	37
24	34215	65785	37700	62300	40952	59048	36
25	34276	65724	37756	62244	41005	58995	35
26	34336	65664	37812	62188	41057	58943	34
27	34396	65604	37868	62132	41109	58891	33
28	34456	65544	37924	62076	41161	58839	32
29	34516	65484	37980	62020	41214	58786	31
30	9.34576	10.65424	9.38035	10.61965	9.41266	10.58734	30
31	34635	65365	38091	61909	41318	58682	29
32	34695	65305	38147	61853	41370	58630	28
33	34755	65245	38202	61798	41422	58578	27
34	34814	65186	38257	61743	41474	58526	26
35	34874	65126	38313	61687	41526	58474	25
36	34933	65067	38368	61632	41578	58422	24
37	34992	65008	38423	61577	41629	58371	23
38	35051	64949	38479	61521	41681	58319	22
39	35111	64889	38534	61466	41733	58267	21
40	9.35170	10.64830	9.38589	10.61411	9.41784	10.58216	20
41	35229	64771	38644	61356	41836	58164	19
42	35288	64712	38699	61301	41887	58113	18
43	35347	64653	38754	61246	41939	58061	17
44	35405	64595	38808	61192	41990	58010	16
45	35464	64536	38863	61137	42041	57959	15
46	35523	64477	38918	61082	42093	57907	14
47	35581	64419	38972	61028	42144	57856	13
48	35640	64360	39027	60973	42195	57805	12
49	35698	64302	39082	60918	42246	57754	11
50	9.35757	10.64243	9.39136	10.60864	9.42297	10.57703	10
51	35815	64185	39190	60810	42348	57652	9
52	35873	64127	39245	60755	42399	57601	8
53	35931	64069	39299	60701	42450	57550	7
54	35989	64011	39353	60647	42501	57499	6
55	36047	63953	39407	60593	42552	57448	5
56	36105	63895	39461	60539	42603	57397	4
57	36163	63837	39515	60485	42653	57347	3
58	36221	63779	39569	60431	42704	57296	2
59	36279	63721	39623	60377	42755	57245	1
60	36336	63664	39677	60323	42805	57195	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	77°		76°		75°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	15° Tan	15° Cotan	16° Tan	16° Cotan	17° Tan	17° Cotan	′
0	9.42805	10.57195	9.45750	10.54250	9.48534	10.51466	60
1	42856	57144	45797	54203	48579	51421	59
2	42906	57094	45845	54155	48624	51376	58
3	42957	57043	45892	54108	48669	51331	57
4	43007	56993	45940	54060	48714	51286	56
5	43057	56943	45987	54013	48759	51241	55
6	43108	56892	46035	53965	48804	51196	54
7	43158	56842	46082	53918	48849	51151	53
8	43208	56792	46130	53870	48894	51106	52
9	43258	56742	46177	53823	48939	51061	51
10	9.43308	10.56692	9.46224	10.53776	9.48984	10.51016	50
11	43358	56642	46271	53729	49029	50971	49
12	43408	56592	46319	53681	49073	50927	48
13	43458	56542	46366	53634	49118	50882	47
14	43508	56492	46413	53587	49163	50837	46
15	43558	56442	46460	53540	49207	50793	45
16	43607	56393	46507	53493	49252	50748	44
17	43657	56343	46554	53446	49296	50704	43
18	43707	56293	46601	53399	49341	50659	42
19	43756	56244	46648	53352	49385	50615	41
20	9.43806	10.56194	9.46694	10.53306	9.49430	10.50570	40
21	43855	56145	46741	53259	49474	50526	39
22	43905	56095	46788	53212	49519	50481	38
23	43954	56046	46835	53165	49563	50437	37
24	44004	55996	46881	53119	49607	50393	36
25	44053	55947	46928	53072	49652	50348	35
26	44102	55898	46975	53025	49696	50304	34
27	44151	55849	47021	52979	49740	50260	33
28	44201	55799	47068	52932	49784	50216	32
29	44250	55750	47114	52886	49828	50172	31
30	9.44299	10.55701	9.47160	10.52840	9.49872	10.50128	30
31	44348	55652	47207	52793	49916	50084	29
32	44397	55603	47253	52747	49960	50040	28
33	44446	55554	47299	52701	50004	49996	27
34	44495	55505	47346	52654	50048	49952	26
35	44544	55456	47392	52608	50092	49908	25
36	44592	55408	47438	52562	50136	49864	24
37	44641	55359	47484	52516	50180	49820	23
38	44690	55310	47530	52470	50223	49777	22
39	44738	55262	47576	52424	50267	49733	21
40	9.44787	10.55213	9.47622	10.52378	9.50311	10.49689	20
41	44836	55164	47668	52332	50355	49645	19
42	44884	55116	47714	52286	50398	49602	18
43	44933	55067	47760	52240	50442	49558	17
44	44981	55019	47806	52194	50485	49515	16
45	45029	54971	47852	52148	50529	49471	15
46	45078	54922	47897	52103	50572	49428	14
47	45126	54874	47943	52057	50616	49384	13
48	45174	54826	47989	52011	50659	49341	12
49	45222	54778	48035	51965	50703	49297	11
50	9.45271	10.54729	9.48080	10.51920	9.50746	10.49254	10
51	45319	54681	48126	51874	50789	49211	9
52	45367	54633	48171	51829	50833	49167	8
53	45415	54585	48217	51783	50876	49124	7
54	45463	54537	48262	51738	50919	49081	6
55	45511	54489	48307	51693	50962	49038	5
56	45559	54441	48353	51647	51005	48995	4
57	45606	54394	48398	51602	51048	48952	3
58	45654	54346	48443	51557	51092	48908	2
59	45702	54298	48489	51511	51135	48865	1
60	45750	54250	48534	51466	51178	48822	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	74°		73°		72°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	18°		19°		20°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.51178	10.48822	9.53697	10.46303	9.56107	10.43893	60
1	51221	48779	53738	46262	56146	43854	59
2	51264	48736	53779	46221	56185	43815	58
3	51306	48694	53820	46180	56224	43776	57
4	51349	48651	53861	46139	56264	43736	56
5	51392	48608	53902	46098	56303	43697	55
6	51435	48565	53943	46057	56342	43658	54
7	51478	48522	53984	46016	56381	43619	53
8	51520	48480	54025	45975	56420	43580	52
9	51563	48437	54065	45935	56459	43541	51
10	9.51606	10.48394	9.54106	10.45894	9.56498	10.43502	50
11	51648	48352	54147	45853	56537	43463	49
12	51691	48309	54187	45813	56576	43424	48
13	51734	48266	54228	45772	56615	43385	47
14	51776	48224	54269	45731	56654	43346	46
15	51819	48181	54309	45691	56693	43307	45
16	51861	48139	54350	45650	56732	43268	44
17	51903	48097	54390	45610	56771	43229	43
18	51946	48054	54431	45569	56810	43190	42
19	51988	48012	54471	45529	56849	43151	41
20	9.52031	10.47969	9.54512	10.45488	9.56887	10.43113	40
21	52073	47927	54552	45448	56926	43074	39
22	52115	47885	54593	45407	56965	43035	38
23	52157	47843	54633	45367	57004	42996	37
24	52200	47800	54673	45327	57042	42958	36
25	52242	47758	54714	45286	57081	42919	35
26	52284	47716	54754	45246	57120	42880	34
27	52326	47674	54794	45206	57158	42842	33
28	52368	47632	54835	45165	57197	42803	32
29	52410	47590	54875	45125	57235	42765	31
30	9.52452	10.47548	9.54915	10.45085	9.57274	10.42726	30
31	52494	47506	54955	45045	57312	42688	29
32	52536	47464	54995	45005	57351	42649	28
33	52578	47422	55035	44965	57389	42611	27
34	52620	47380	55075	44925	57428	42572	26
35	52661	47339	55115	44885	57466	42534	25
36	52703	47297	55155	44845	57504	42496	24
37	52745	47255	55195	44805	57543	42457	23
38	52787	47213	55235	44765	57581	42419	22
39	52829	47171	55275	44725	57619	42381	21
40	9.52870	10.47130	9.55315	10.44685	9.57658	10.42342	20
41	52912	47088	55355	44645	57696	42304	19
42	52953	47047	55395	44605	57734	42266	18
43	52995	47005	55434	44566	57772	42228	17
44	53037	46963	55474	44526	57810	42190	16
45	53078	46922	55514	44486	57849	42151	15
46	53120	46880	55554	44446	57887	42113	14
47	53161	46839	55593	44407	57925	42075	13
48	53202	46798	55633	44367	57963	42037	12
49	53244	46756	55673	44327	58001	41999	11
50	9.53285	10.46715	9.55712	10.44288	9.58039	10.41961	10
51	53327	46673	55752	44248	58077	41923	9
52	53368	46632	55791	44209	58115	41885	8
53	53409	46591	55831	44169	58153	41847	7
54	53450	46550	55870	44130	58191	41809	6
55	53492	46508	55910	44090	58229	41771	5
56	53533	46467	55949	44051	58267	41733	4
57	53574	46426	55989	44011	58304	41696	3
58	53615	46385	56028	43972	58342	41658	2
59	53656	46344	56067	43933	58380	41620	1
60	53697	46303	56107	43893	58418	41582	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	71°		70°		69°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	21°		22°		23°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.58418	10.41582	9.60641	10.39359	9.62785	10.37215	60
1	58455	41545	60677	39323	62820	37180	59
2	58493	41507	60714	39286	62855	37145	58
3	58531	41469	60750	39250	62890	37110	57
4	58569	41431	60786	39214	62926	37074	56
5	58606	41394	60823	39177	62961	37039	55
6	58644	41356	60859	39141	62996	37004	54
7	58681	41319	60895	39105	63031	36969	53
8	58719	41281	60931	39069	63066	36934	52
9	58757	41243	60967	39033	63101	36899	51
10	9.58794	10.41206	9.61004	10.38996	9.63135	10.36865	50
11	58832	41168	61040	38960	63170	36830	49
12	58869	41131	61076	38924	63205	36795	48
13	58907	41093	61112	38888	63240	36760	47
14	58944	41056	61148	38852	63275	36725	46
15	58981	41019	61184	38816	63310	36690	45
16	59019	40981	61220	38780	63345	36655	44
17	59056	40944	61256	38744	63379	36621	43
18	59094	40906	61292	38708	63414	36586	42
19	59131	40869	61328	38672	63449	36551	41
20	9.59168	10.40832	9.61364	10.38636	9.63484	10.36516	40
21	59205	40795	61400	38600	63519	36481	39
22	59243	40757	61436	38564	63553	36447	38
23	59280	40720	61472	38528	63588	36412	37
24	59317	40683	61508	38492	63623	36377	36
25	59354	40646	61544	38456	63657	36343	35
26	59391	40609	61579	38421	63692	36308	34
27	59429	40571	61615	38385	63726	36274	33
28	59466	40534	61651	38349	63761	36239	32
29	59503	40497	61687	38313	63796	36204	31
30	9.59540	10.40460	9.61722	10.38278	9.63830	10.36170	30
31	59577	40423	61758	38242	63865	36135	29
32	59614	40386	61794	38206	63899	36101	28
33	59651	40349	61830	38170	63934	36066	27
34	59688	40312	61865	38135	63968	36032	26
35	59725	40275	61901	38099	64003	35997	25
36	59762	40238	61936	38064	64037	35963	24
37	59799	40201	61972	38028	64072	35928	23
38	59835	40165	62008	37992	64106	35894	22
39	59872	40128	62043	37957	64140	35860	21
40	9.59909	10.40091	9.62079	10.37921	9.64175	10.35825	20
41	59946	40054	62114	37886	64209	35791	19
42	59983	40017	62150	37850	64243	35757	18
43	60019	39981	62185	37815	64278	35722	17
44	60056	39944	62221	37779	64312	35688	16
45	60093	39907	62256	37744	64346	35654	15
46	60130	39870	62292	37708	64381	35619	14
47	60166	39834	62327	37673	64415	35585	13
48	60203	39797	62362	37638	64449	35551	12
49	60240	39760	62398	37602	64483	35517	11
50	9.60276	10.39724	9.62433	10.37567	9.64517	10.35483	10
51	60313	39687	62468	37532	64552	35448	9
52	60349	39651	62504	37496	64586	35414	8
53	60386	39614	62539	37461	64620	35380	7
54	60422	39578	62574	37426	64654	35346	6
55	60459	39541	62609	37391	64688	35312	5
56	60495	39505	62645	37355	64722	35278	4
57	60532	39468	62680	37320	64756	35244	3
58	60568	39432	62715	37285	64790	35210	2
59	60605	39395	62750	37250	64824	35176	1
60	60641	39359	62785	37215	64858	35142	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	68°		67°		66°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	24° Tan	24° Cotan	25° Tan	25° Cotan	26° Tan	26° Cotan	′
0	9.64858	10.35142	9.66867	10.33133	9.68818	10.31182	60
1	64892	35108	66900	33100	68850	31150	59
2	64926	35074	66933	33067	68882	31118	58
3	64960	35040	66966	33034	68914	31086	57
4	64994	35006	66999	33001	68946	31054	56
5	65028	34972	67032	32968	68978	31022	55
6	65062	34938	67065	32935	69010	30990	54
7	65096	34904	67098	32902	69042	30958	53
8	65130	34870	67131	32869	69074	30926	52
9	65164	34836	67163	32837	69106	30894	51
10	9.65197	10.34803	9.67196	10.32804	9.69138	10.30862	50
11	65231	34769	67229	32771	69170	30830	49
12	65265	34735	67262	32738	69202	30798	48
13	65299	34701	67295	32705	69234	30766	47
14	65333	34667	67327	32673	69266	30734	46
15	65366	34634	67360	32640	69298	30702	45
16	65400	34600	67393	32607	69329	30671	44
17	65434	34566	67426	32574	69361	30639	43
18	65467	34533	67458	32542	69393	30607	42
19	65501	34499	67491	32509	69425	30575	41
20	9.65535	10.34465	9.67524	10.32476	9.69457	10.30543	40
21	65568	34432	67556	32444	69488	30512	39
22	65602	34398	67589	32411	69520	30480	38
23	65636	34364	67622	32378	69552	30448	37
24	65669	34331	67654	32346	69584	30416	36
25	65703	34297	67687	32313	69615	30385	35
26	65736	34264	67719	32281	69647	30353	34
27	65770	34230	67752	32248	69679	30321	33
28	65803	34197	67785	32215	69710	30290	32
29	65837	34163	67817	32183	69742	30258	31
30	9.65870	10.34130	9.67850	10.32150	9.69774	10.30226	30
31	65904	34096	67882	32118	69805	30195	29
32	65937	34063	67915	32085	69837	30163	28
33	65971	34029	67947	32053	69868	30132	27
34	66004	33996	67980	32020	69900	30100	26
35	66038	33962	68012	31988	69932	30068	25
36	66071	33929	68044	31956	69963	30037	24
37	66104	33896	68077	31923	69995	30005	23
38	66138	33862	68109	31891	70026	29974	22
39	66171	33829	68142	31858	70058	29942	21
40	9.66204	10.33796	9.68174	10.31826	9.70089	10.29911	20
41	66238	33762	68206	31794	70121	29879	19
42	66271	33729	68239	31761	70152	29848	18
43	66304	33696	68271	31729	70184	29816	17
44	66337	33663	68303	31697	70215	29785	16
45	66371	33629	68336	31664	70247	29753	15
46	66404	33596	68368	31632	70278	29722	14
47	66437	33563	68400	31600	70309	29691	13
48	66470	33530	68432	31568	70341	29659	12
49	66503	33497	68465	31535	70372	29628	11
50	9.66537	10.33463	9.68497	10.31503	9.70404	10.29596	10
51	66570	33430	68529	31471	70435	29565	9
52	66603	33397	68561	31439	70466	29534	8
53	66636	33364	68593	31407	70498	29502	7
54	66669	33331	68626	31374	70529	29471	6
55	66702	33298	68658	31342	70560	29440	5
56	66735	33265	68690	31310	70592	29408	4
57	66768	33232	68722	31278	70623	29377	3
58	66801	33199	68754	31246	70654	29346	2
59	66834	33166	68786	31214	70685	29315	1
60	66867	33133	68818	31182	70717	29283	0
′	Cotan 65°	Tan 65°	Cotan 64°	Tan 64°	Cotan 63°	Tan 63°	′

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	27° Tan	27° Cotan	28° Tan	28° Cotan	29° Tan	29° Cotan	′
0	9.70717	10.29283	9.72567	10.27433	9.74375	10.25625	60
1	70748	29252	72598	27402	74405	25595	59
2	70779	29221	72628	27372	74435	25565	58
3	70810	29190	72659	27341	74465	25535	57
4	70841	29159	72689	27311	74494	25506	56
5	70873	29127	72720	27280	74524	25476	55
6	70904	29096	72750	27250	74554	25446	54
7	70935	29065	72780	27220	74583	25417	53
8	70966	29034	72811	27189	74613	25387	52
9	70997	29003	72841	27159	74643	25357	51
10	9.71028	10.28972	9.72872	10.27128	9.74673	10.25327	50
11	71059	28941	72902	27098	74702	25298	49
12	71090	28910	72932	27068	74732	25268	48
13	71121	28879	72963	27037	74762	25238	47
14	71153	28847	72993	27007	74791	25209	46
15	71184	28816	73023	26977	74821	25179	45
16	71215	28785	73054	26946	74851	25149	44
17	71246	28754	73084	26916	74880	25120	43
18	71277	28723	73114	26886	74910	25090	42
19	71308	28692	73144	26856	74939	25061	41
20	9.71339	10.28661	9.73175	10.26825	9.74969	10.25031	40
21	71370	28630	73205	26795	74998	25002	39
22	71401	28599	73235	26765	75028	24972	38
23	71431	28569	73265	26735	75058	24942	37
24	71462	28538	73295	26705	75087	24913	36
25	71493	28507	73326	26674	75117	24883	35
26	71524	28476	73356	26644	75146	24854	34
27	71555	28445	73386	26614	75176	24824	33
28	71586	28414	73416	26584	75205	24795	32
29	71617	28383	73446	26554	75235	24765	31
30	9.71648	10.28352	9.73476	10.26524	9.75264	10.24736	30
31	71679	28321	73507	26493	75294	24706	29
32	71709	28291	73537	26463	75323	24677	28
33	71740	28260	73567	26433	75353	24647	27
34	71771	28229	73597	26403	75382	24618	26
35	71802	28198	73627	26373	75411	24589	25
36	71833	28167	73657	26343	75441	24559	24
37	71863	28137	73687	26313	75470	24530	23
38	71894	28106	73717	26283	75500	24500	22
39	71925	28075	73747	26253	75529	24471	21
40	9.71955	10.28045	9.73777	10.26223	9.75558	10.24442	20
41	71986	28014	73807	26193	75588	24412	19
42	72017	27983	73837	26163	75617	24383	18
43	72048	27952	73867	26133	75647	24353	17
44	72078	27922	73897	26103	75676	24324	16
45	72109	27891	73927	26073	75705	24295	15
46	72140	27860	73957	26043	75735	24265	14
47	72170	27830	73987	26013	75764	24236	13
48	72201	27799	74017	25983	75793	24207	12
49	72231	27769	74047	25953	75822	24178	11
50	9.72262	10.27738	9.74077	10.25923	9.75852	10.24148	10
51	72293	27707	74107	25893	75881	24119	9
52	72323	27677	74137	25863	75910	24090	8
53	72354	27646	74166	25834	75939	24061	7
54	72384	27616	74196	25804	75969	24031	6
55	72415	27585	74226	25774	75998	24002	5
56	72445	27555	74256	25744	76027	23973	4
57	72476	27524	74286	25714	76056	23944	3
58	72506	27494	74316	25684	76086	23914	2
59	72537	27463	74345	25655	76115	23885	1
60	72567	27433	74375	25625	76144	23856	0
′	62° Cotan	62° Tan	61° Cotan	61° Tan	60° Cotan	60° Tan	′

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	30° Tan	30° Cotan	31° Tan	31° Cotan	32° Tan	32° Cotan	′
0	9.76144	10.23856	9.77877	10.22123	9.79579	10.20421	60
1	76173	23827	77906	22094	79607	20393	59
2	76202	23798	77935	22065	79635	20365	58
3	76231	23769	77963	22037	79663	20337	57
4	76261	23739	77992	22008	79691	20309	56
5	76290	23710	78020	21980	79719	20281	55
6	76319	23681	78049	21951	79747	20253	54
7	76348	23652	78077	21923	79776	20224	53
8	76377	23623	78106	21894	79804	20196	52
9	76406	23594	78135	21865	79832	20168	51
10	9.76435	10.23565	9.78163	10.21837	9.79860	10.20140	50
11	76464	23536	78192	21808	79888	20112	49
12	76493	23507	78220	21780	79916	20084	48
13	76522	23478	78249	21751	79944	20056	47
14	76551	23449	78277	21723	79972	20028	46
15	76580	23420	78306	21694	80000	20000	45
16	76609	23391	78334	21666	80028	19972	44
17	76639	23361	78363	21637	80056	19944	43
18	76668	23332	78391	21609	80084	19916	42
19	76697	23303	78419	21581	80112	19888	41
20	9.76725	10.23275	9.78448	10.21552	9.80140	10.19860	40
21	76754	23246	78476	21524	80168	19832	39
22	76783	23217	78505	21495	80195	19805	38
23	76812	23188	78533	21467	80223	19777	37
24	76841	23159	78562	21438	80251	19749	36
25	76870	23130	78590	21410	80279	19721	35
26	76899	23101	78618	21382	80307	19693	34
27	76928	23072	78647	21353	80335	19665	33
28	76957	23043	78675	21325	80363	19637	32
29	76986	23014	78704	21296	80391	19609	31
30	9.77015	10.22985	9.78732	10.21268	9.80419	10.19581	30
31	77044	22956	78760	21240	80447	19553	29
32	77073	22927	78789	21211	80474	19526	28
33	77101	22899	78817	21183	80502	19498	27
34	77130	22870	78845	21155	80530	19470	26
35	77159	22841	78874	21126	80558	19442	25
36	77188	22812	78902	21098	80586	19414	24
37	77217	22783	78930	21070	80614	19386	23
38	77246	22754	78959	21041	80642	19358	22
39	77274	22726	78987	21013	80669	19331	21
40	9.77303	10.22697	9.79015	10.20985	9.80697	10.19303	20
41	77332	22668	79043	20957	80725	19275	19
42	77361	22639	79072	20928	80753	19247	18
43	77390	22610	79100	20900	80781	19219	17
44	77418	22582	79128	20872	80808	19192	16
45	77447	22553	79156	20844	80836	19164	15
46	77476	22524	79185	20815	80864	19136	14
47	77505	22495	79213	20787	80892	19108	13
48	77533	22467	79241	20759	80919	19081	12
49	77562	22438	79269	20731	80947	19053	11
50	9.77591	10.22409	9.79297	10.20703	9.80975	10.19025	10
51	77619	22381	79326	20674	81003	18997	9
52	77648	22352	79354	20646	81030	18970	8
53	77677	22323	79382	20618	81058	18942	7
54	77706	22294	79410	20590	81086	18914	6
55	77734	22266	79438	20562	81113	18887	5
56	77763	22237	79466	20534	81141	18859	4
57	77791	22209	79495	20505	81169	18831	3
58	77820	22180	79523	20477	81196	18804	2
59	77849	22151	79551	20449	81224	18776	1
60	77877	22123	79579	20421	81252	18748	0
′	59° Cotan	59° Tan	58° Cotan	58° Tan	57° Cotan	57° Tan	′

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	33°		34°		35°		′
	Tan	Cotan	Tan	Cotan	Tan	Cotan	
0	9.81252	10.18748	9.82899	10.17101	9.84523	10.15477	60
1	81279	18721	82926	17074	84550	15450	59
2	81307	18693	82953	17047	84576	15424	58
3	81335	18665	82980	17020	84603	15397	57
4	81362	18638	83008	16992	84630	15370	56
5	81390	18610	83035	16965	84657	15343	55
6	81418	18582	83062	16938	84684	15316	54
7	81445	18555	83089	16911	84711	15289	53
8	81473	18527	83117	16883	84738	15262	52
9	81500	18500	83144	16856	84764	15236	51
10	9.81528	10.18472	9.83171	10.16829	9.84791	10.15209	50
11	81556	18444	83198	16802	84818	15182	49
12	81583	18417	83225	16775	84845	15155	48
13	81611	18389	83252	16748	84872	15128	47
14	81638	18362	83280	16720	84899	15101	46
15	81666	18334	83307	16693	84925	15075	45
16	81693	18307	83334	16666	84952	15048	44
17	81721	18279	83361	16639	84979	15021	43
18	81748	18252	83388	16612	85006	14994	42
19	81776	18224	83415	16585	85033	14967	41
20	9.81803	10.18197	9.83442	10.16558	9.85059	10.14941	40
21	81831	18169	83470	16530	85086	14914	39
22	81858	18142	83497	16503	85113	14887	38
23	81886	18114	83524	16476	85140	14860	37
24	81913	18087	83551	16449	85166	14834	36
25	81941	18059	83578	16422	85193	14807	35
26	81968	18032	83605	16395	85220	14780	34
27	81996	18004	83632	16368	85247	14753	33
28	82023	17977	83659	16341	85273	14727	32
29	82051	17949	83686	16314	85300	14700	31
30	9.82078	10.17922	9.83713	10.16287	9.85327	10.14673	30
31	82106	17894	83740	16260	85354	14646	29
32	82133	17867	83768	16232	85380	14620	28
33	82161	17839	83795	16205	85407	14593	27
34	82188	17812	83822	16178	85434	14566	26
35	82215	17785	83849	16151	85460	14540	25
36	82243	17757	83876	16124	85487	14513	24
37	82270	17730	83903	16097	85514	14486	23
38	82298	17702	83930	16070	85540	14460	22
39	82325	17675	83957	16043	85567	14433	21
40	9.82352	10.17648	9.83984	10.16016	9.85594	10.14406	20
41	82380	17620	84011	15989	85620	14380	19
42	82407	17593	84038	15962	85647	14353	18
43	82435	17565	84065	15935	85674	14326	17
44	82462	17538	84092	15908	85700	14300	16
45	82489	17511	84119	15881	85727	14273	15
46	82517	17483	84146	15854	85754	14246	14
47	82544	17456	84173	15827	85780	14220	13
48	82571	17429	84200	15800	85807	14193	12
49	82599	17401	84227	15773	85834	14166	11
50	9.82626	10.17374	9.84254	10.15746	9.85860	10.14140	10
51	82653	17347	84280	15720	85887	14113	9
52	82681	17319	84307	15693	85913	14087	8
53	82708	17292	84334	15666	85940	14060	7
54	82735	17265	84361	15639	85967	14033	6
55	82762	17238	84388	15612	85993	14007	5
56	82790	17210	84415	15585	86020	13980	4
57	82817	17183	84442	15558	86046	13954	3
58	82844	17156	84469	15531	86073	13927	2
59	82871	17129	84496	15504	86100	13900	1
60	82899	17101	84523	15477	86126	13874	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	56°		55°		54°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	36° Tan	36° Cotan	37° Tan	37° Cotan	38° Tan	38° Cotan	′
0	9.86126	10.13874	9.87711	10.12289	9.89281	10.10719	60
1	86153	13847	87738	12262	89307	10693	59
2	86179	13821	87764	12236	89333	10667	58
3	86206	13794	87790	12210	89359	10641	57
4	86232	13768	87817	12183	89385	10615	56
5	86259	13741	87843	12157	89411	10589	55
6	86285	13715	87869	12131	89437	10563	54
7	86312	13688	87895	12105	89463	10537	53
8	86338	13662	87922	12078	89489	10511	52
9	86365	13635	87948	12052	89515	10485	51
10	9.86392	10.13608	9.87974	10.12026	9.89541	10.10459	50
11	86418	13582	88000	12000	89567	10433	49
12	86445	13555	88027	11973	89593	10407	48
13	86471	13529	88053	11947	89619	10381	47
14	86498	13502	88079	11921	89645	10355	46
15	86524	13476	88105	11895	89671	10329	45
16	86551	13449	88131	11869	89697	10303	44
17	86577	13423	88158	11842	89723	10277	43
18	86603	13397	88184	11816	89749	10251	42
19	86630	13370	88210	11790	89775	10225	41
20	9.86656	10.13344	9.88236	10.11764	9.89801	10.10199	40
21	86683	13317	88262	11738	89827	10173	39
22	86709	13291	88289	11711	89853	10147	38
23	86736	13264	88315	11685	89879	10121	37
24	86762	13238	88341	11659	89905	10095	36
25	86789	13211	88367	11633	89931	10069	35
26	86815	13185	88393	11607	89957	10043	34
27	86842	13158	88420	11580	89983	10017	33
28	86868	13132	88446	11554	90009	09991	32
29	86894	13106	88472	11528	90035	09965	31
30	9.86921	10.13079	9.88498	10.11502	9.90061	10.09939	30
31	86947	13053	88524	11476	90086	09914	29
32	86974	13026	88550	11450	90112	09888	28
33	87000	13000	88577	11423	90138	09862	27
34	87027	12973	88603	11397	90164	09836	26
35	87053	12947	88629	11371	90190	09810	25
36	87079	12921	88655	11345	90216	09784	24
37	87106	12894	88681	11319	90242	09758	23
38	87132	12868	88707	11293	90268	09732	22
39	87158	12842	88733	11267	90294	09706	21
40	9.87185	10.12815	9.88759	10.11241	9.90320	10.09680	20
41	87211	12789	88786	11214	90346	09654	19
42	87238	12762	88812	11188	90371	09629	18
43	87264	12736	88838	11162	90397	09603	17
44	87290	12710	88864	11136	90423	09577	16
45	87317	12683	88890	11110	90449	09551	15
46	87343	12657	88916	11084	90475	09525	14
47	87369	12631	88942	11058	90501	09499	13
48	87396	12604	88968	11032	90527	09473	12
49	87422	12578	88994	11006	90553	09447	11
50	9.87448	10.12552	9.89020	10.10980	9.90578	10.09422	10
51	87475	12525	89046	10954	90604	09396	9
52	87501	12499	89073	10927	90630	09370	8
53	87527	12473	89099	10901	90656	09344	7
54	87554	12446	89125	10875	90682	09318	6
55	87580	12420	89151	10849	90708	09292	5
56	87606	12394	89177	10823	90734	09266	4
57	87633	12367	89203	10797	90759	09241	3
58	87659	12341	89229	10771	90785	09215	2
59	87685	12315	89255	10745	90811	09189	1
60	87711	12289	89281	10719	90837	09163	0
′	53° Cotan	53° Tan	52° Cotan	52° Tan	51° Cotan	51° Tan	′

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	39° Tan	39° Cotan	40° Tan	40° Cotan	41° Tan	41° Cotan	′
0	9.90837	10.09163	9.92381	10.07619	9.93916	10.06084	60
1	90863	09137	92407	07593	93942	06058	59
2	90889	09111	92433	07567	93967	06033	58
3	90914	09086	92458	07542	93993	06007	57
4	90940	09060	92484	07516	94018	05982	56
5	90966	09034	92510	07490	94044	05956	55
6	90992	09008	92535	07465	94069	05931	54
7	91018	08982	92561	07439	94095	05905	53
8	91043	08957	92587	07413	94120	05880	52
9	91069	08931	92612	07388	94146	05854	51
10	9.91095	10.08905	9.92638	10.07362	9.94171	10.05829	50
11	91121	08879	92663	07337	94197	05803	49
12	91147	08853	92689	07311	94222	05778	48
13	91172	08828	92715	07285	94248	05752	47
14	91198	08802	92740	07260	94273	05727	46
15	91224	08776	92766	07234	94299	05701	45
16	91250	08750	92792	07208	94324	05676	44
17	91276	08724	92817	07183	94350	05650	43
18	91301	08699	92843	07157	94375	05625	42
19	91327	08673	92868	07132	94401	05599	41
20	9.91353	10.08647	9.92894	10.07106	9.94426	10.05574	40
21	91379	08621	92920	07080	94452	05548	39
22	91404	08596	92945	07055	94477	05523	38
23	91430	08570	92971	07029	94503	05497	37
24	91456	08544	92996	07004	94528	05472	36
25	91482	08518	93022	06978	94554	05446	35
26	91507	08493	93048	06952	94579	05421	34
27	91533	08467	93073	06927	94604	05396	33
28	91559	08441	93099	06901	94630	05370	32
29	91585	08415	93124	06876	94655	05345	31
30	9.91610	10.08390	9.93150	10.06850	9.94681	10.05319	30
31	91636	08364	93175	06825	94706	05294	29
32	91662	08338	93201	06799	94732	05268	28
33	91688	08312	93227	06773	94757	05243	27
34	91713	08287	93252	06748	94783	05217	26
35	91739	08261	93278	06722	94808	05192	25
36	91765	08235	93303	06697	94834	05166	24
37	91791	08209	93329	06671	94859	05141	23
38	91816	08184	93354	06646	94884	05116	22
39	91842	08158	93380	06620	94910	05090	21
40	9.91868	10.08132	9.93406	10.06594	9.94935	10.05065	20
41	91893	08107	93431	06569	94961	05039	19
42	91919	08081	93457	06543	94986	05014	18
43	91945	08055	93482	06518	95012	04988	17
44	91971	08029	93508	06492	95037	04963	16
45	91996	08004	93533	06467	95062	04938	15
46	92022	07978	93559	06441	95088	04912	14
47	92048	07952	93584	06416	95113	04887	13
48	92073	07927	93610	06390	95139	04861	12
49	92099	07901	93636	06364	95164	04836	11
50	9.92125	10.07875	9.93661	10.06339	9.95190	10.04810	10
51	92150	07850	93687	06313	95215	04785	9
52	92176	07824	93712	06288	95240	04760	8
53	92202	07798	93738	06262	95266	04734	7
54	92227	07773	93763	06237	95291	04709	6
55	92253	07747	93789	06211	95317	04683	5
56	92279	07721	93814	06186	95342	04658	4
57	92304	07696	93840	06160	95368	04632	3
58	92330	07670	93865	06135	95393	04607	2
59	92356	07644	93891	06109	95418	04582	1
60	92381	07619	93916	06084	95444	04556	0
′	Cotan	Tan	Cotan	Tan	Cotan	Tan	′
	50°		49°		48°		

TABLE XV.—*Continued.*

LOG. TANGENTS AND COTANGENTS.

′	42° Tan	42° Cotan	43° Tan	43° Cotan	44° Tan	44° Cotan	′
0	9.95444	10.04556	9.96966	10.03034	9.98484	10.01516	60
1	95469	04531	96991	03009	98509	01491	59
2	95495	04505	97016	02984	98534	01466	58
3	95520	04480	97042	02958	98560	01440	57
4	95545	04455	97067	02933	98585	01415	56
5	95571	04429	97092	02908	98610	01390	55
6	95596	04404	97118	02882	98635	01365	54
7	95622	04378	97143	02857	98661	01339	53
8	95647	04353	97168	02832	98686	01314	52
9	95672	04328	97193	02807	98711	01289	51
10	9.95698	10.04302	9.97219	10.02781	9.98737	10.01263	50
11	95723	04277	97244	02756	98762	01238	49
12	95748	04252	97269	02731	98787	01213	48
13	95774	04226	97295	02705	98812	01188	47
14	95799	04201	97320	02680	98838	01162	46
15	95825	04175	97345	02655	98863	01137	45
16	95850	04150	97371	02629	98888	01112	44
17	95875	04125	97396	02604	98913	01087	43
18	95901	04099	97421	02579	98939	01061	42
19	95926	04074	97447	02553	98964	01036	41
20	9.95952	10.04048	9.97472	10.02528	9.98989	10.01011	40
21	95977	04023	97497	02503	99015	00985	39
22	96002	03998	97523	02477	99040	00960	38
23	96028	03972	97548	02452	99065	00935	37
24	96053	03947	97573	02427	99090	00910	36
25	96078	03922	97598	02402	99116	00884	35
26	96104	03896	97624	02376	99141	00859	34
27	96129	03871	97649	02351	99166	00834	33
28	96155	03845	97674	02326	99191	00809	32
29	96180	03820	97700	02300	99217	00783	31
30	9.96205	10.03795	9.97725	10.02275	9.99242	10.00758	30
31	96231	03769	97750	02250	99267	00733	29
32	96256	03744	97776	02224	99293	00707	28
33	96281	03719	97801	02199	99318	00682	27
34	96307	03693	97826	02174	99343	00657	26
35	96332	03668	97851	02149	99368	00632	25
36	96357	03643	97877	02123	99394	00606	24
37	96383	03617	97902	02098	99419	00581	23
38	96408	03592	97927	02073	99444	00556	22
39	96433	03567	97953	02047	99469	00531	21
40	9.96459	10.03541	9.97978	10.02022	9.99495	10.00505	20
41	96484	03516	98003	01997	99520	00480	19
42	96510	03490	98029	01971	99545	00455	18
43	96535	03465	98054	01946	99570	00430	17
44	96560	03440	98079	01921	99596	00404	16
45	96586	03414	98104	01896	99621	00379	15
46	96611	03389	98130	01870	99646	00354	14
47	96636	03364	98155	01845	99672	00328	13
48	96662	03338	98180	01820	99697	00303	12
49	96687	03313	98206	01794	99722	00278	11
50	9.96712	10.03288	9.98231	10.01769	9.99747	10.00253	10
51	96738	03262	98256	01744	99773	00227	9
52	96763	03237	98281	01719	99798	00202	8
53	96788	03212	98307	01693	99823	00177	7
54	96814	03186	98332	01668	99848	00152	6
55	96839	03161	98357	01643	99874	00126	5
56	96864	03136	98383	01617	99899	00101	4
57	96890	03110	98408	01592	99924	00076	3
58	96915	03085	98433	01567	99949	00051	2
59	96940	03060	98458	01542	99975	00025	1
60	96966	03034	98484	01516	10.00000	00000	0
′	Cotan 47°	Tan 47°	Cotan 46°	Tan 46°	Cotan 45°	Tan 45°	′

TABLE XVI.

SINES AND COSINES.

′	0°		1°		2°		3°		4°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

TABLE XVI.—*Continued.*

SINES AND COSINES.

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
′	84°		83°		82°		81°		80°		′

TABLE XVI.—*Continued.*

SINES AND COSINES.

′	10° Sine	10° Cosin	11° Sine	11° Cosin	12° Sine	12° Cosin	13° Sine	13° Cosin	14° Sine	14° Cosin	′
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

Table XVI.—*Continued.*

SINES AND COSINES.

′	15° Sine	15° Cosin	16° Sine	16° Cosin	17° Sine	17° Cosin	18° Sine	18° Cosin	19° Sine	19° Cosin	′
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

Table XVI.—*Continued.*

SINES AND COSINES.

′	20° Sine	20° Cosin	21° Sine	21° Cosin	22° Sine	22° Cosin	23° Sine	23° Cosin	24° Sine	24° Cosin	′
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

TABLE XVI.—*Continued.*

SINES AND COSINES.

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
′	64°		63°		62°		61°		60°		′

TABLE XVI.—*Continued.*

SINES AND COSINES.

′	30° Sine	30° Cosin	31° Sine	31° Cosin	32° Sine	32° Cosin	33° Sine	33° Cosin	34° Sine	34° Cosin	′
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

TABLE XVI.—*Continued.*

SINES AND COSINES.

′	35° Sine	35° Cosin	36° Sine	36° Cosin	37° Sine	37° Cosin	38° Sine	38° Cosin	39° Sine	39° Cosin	′
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	40
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

Table XVI.—*Continued.*

SINES AND COSINES.

′	40° Sine	40° Cosin	41° Sine	41° Cosin	42° Sine	42° Cosin	43° Sine	43° Cosin	44° Sine	44° Cosin	′
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74352	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

Table XVII.

TANGENTS AND COTANGENTS.

′	0° Tang	0° Cotang	1° Tang	1° Cotang	2° Tang	2° Cotang	3° Tang	3° Cotang	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4918	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	89° Cotang	89° Tang	88° Cotang	88° Tang	87° Cotang	87° Tang	86° Cotang	86° Tang	′

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	4° Tang	4° Cotang	5° Tang	5° Cotang	6° Tang	6° Cotang	7° Tang	7° Cotang	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.12639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	8.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	8° Tang	8° Cotang	9° Tang	9° Cotang	10° Tang	10° Cotang	11° Tang	11° Cotang	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	12° Tang	12° Cotang	13° Tang	13° Cotang	14° Tang	14° Cotang	15° Tang	15° Cotang	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	16° Tang	16° Cotang	17° Tang	17° Cotang	18° Tang	18° Cotang	19° Tang	19° Cotang	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

Table XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	20°		21°		22°		23°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	24° Tang	24° Cotang	25° Tang	25° Cotang	26° Tang	26° Cotang	27° Tang	27° Cotang	′
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	28° Tang	28° Cotang	29° Tang	29° Cotang	30° Tang	30° Cotang	31° Tang	31° Cotang	′
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	61° Cotang	61° Tang	60° Cotang	60° Tang	59° Cotang	59° Tang	58° Cotang	58° Tang	′

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	32° Tang	32° Cotang	33° Tang	33° Cotang	34° Tang	34° Cotang	35° Tang	35° Cotang	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62953	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	57° Cotang	57° Tang	56° Cotang	56° Tang	55° Cotang	55° Tang	54° Cotang	54° Tang	′

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	53°		52°		51°		50°		

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	40° Tang	40° Cotang	41° Tang	41° Cotang	42° Tang	42° Cotang	43° Tang	43° Cotang	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13223	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang 49°	Tang 49°	Cotang 48°	Tang 48°	Cotang 47°	Tang 47°	Cotang 46°	Tang 46°	′

TABLE XVII.—*Continued.*

TANGENTS AND COTANGENTS.

′	44° Tang	44° Cotang	′	′	44° Tang	44° Cotang	′	′	44° Tang	44° Cotang	′
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	45° Cotang	45° Tang	′	′	45° Cotang	45° Tang	′	′	45° Cotang	45° Tang	′

TABLE XVIII.

TRIGONOMETRIC AND MISCELLANEOUS FORMULAS.

TRIGONOMETRIC FORMULAS.

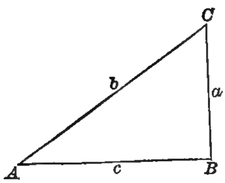


FIG. 98.

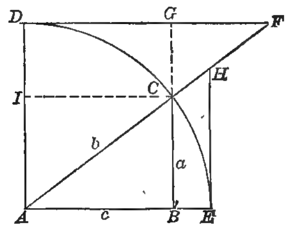


FIG. 99.

In Fig 99, let DCE be the arc of a quadrant, ABC a right triangle, the angle BAC subtended by the arc $CE = A$, and consider the radius $AC =$ unity. Then

$BC = \sin A.$	$AF = \operatorname{cosec} A.$
$AB = \cos A.$	$BE = \operatorname{versin} A.$
$HE = \tan A.$	$DI = \operatorname{coversin} A.$
$DF = \cot A.$	$CH = \operatorname{exsec} A.$
$AH = \sec A.$	$CF = \operatorname{coexsec} A.$

Using the small letters a, b, c, to represent the sides of a right triangle in Fig. 98 or 99, we may write

$$\sin A = \frac{a}{b}; \quad \operatorname{cosec} A = \frac{b}{a}; \quad \therefore \sin A = \frac{1}{\operatorname{cosec} A}.$$

$$\cos A = \frac{c}{b}; \quad \sec A = \frac{b}{c}; \quad \therefore \cos A = \frac{1}{\sec A}.$$

$$\tan A = \frac{a}{c}; \quad \cot A = \frac{c}{a}; \quad \therefore \tan A = \frac{1}{\cot A}.$$

TABLE XVIII.—*Continued.*

TRIGONOMETRIC AND MISCELLANEOUS FORMULAS.

SOLUTION OF RIGHT TRIANGLES.

Required.	Given.	Formulas.
A, C, c	a, b	$\sin A = \cos C = \frac{a}{b}$; $c = \sqrt{(b+a)(b-a)}$.
A, C, b	a, c	$\tan A = \cot B = \frac{a}{c}$; $b = \sqrt{a^2 + c^2}$.
C, b, c	A, a	$C = 90^\circ - A$; $c = a \cot A$; $b = a \operatorname{cosec} A$.
C, a, c	A, b	$C = 90^\circ - A$; $a = b \sin A$; $c = b \operatorname{cosin} A$.
C, a, b	A, c	$C = 90^\circ - A$; $a = c \tan A$; $b = c \sec A$.

SOLUTION OF OBLIQUE TRIANGLES.

Required.	Given.	Formulas.
b	A, B, a	$b = \frac{a \sin B}{\sin A}$
B	A, a, b	$\sin B = \frac{b \sin A}{a}$
$\frac{1}{2}(A+B)$	a, b, C	$\frac{1}{2}(A+B) = \frac{1}{2}(180 - C)$
$\frac{1}{2}(A-B)$		$\tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A+B)$
A		$A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B)$
B		$B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B)$
A	a, b, c	If $s = \frac{1}{2}(a+b+c)$, $\sin \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}}$
		$\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$,
		$\tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
		$\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$
Area		Area $= \sqrt{s(s-a)(s-b)(s-c)}$
Area	A, b, c	Area $= \frac{1}{2} bc \sin A$
Area	A, B, c	Area $= \frac{c^2 \sin A \sin B}{2 \sin(A+B)}$

TABLE XVIII.—*Continued.*

TRIGONOMETRIC AND MISCELLANEOUS FORMULAS.

GENERAL FORMULAS.

$$\sin A = \sqrt{1 - \cos^2 A} = \tan A \cos A.$$

$$\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A.$$

$$\sin A = \frac{1}{\operatorname{cosec} A} = \sqrt{\tfrac{1}{2}(1 - \cos 2A)}.$$

$$\cos A = \frac{1}{\sec A} = \sqrt{1 - \sin^2 A} = \cot A \sin A.$$

$$\cos A = 1 - 2 \sin^2 \tfrac{1}{2} A = 1 - \operatorname{vers} A.$$

$$\cos A = \sqrt{\tfrac{1}{2} + \tfrac{1}{2} \cos 2A} = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A.$$

$$\tan A = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1}.$$

$$\tan A = \frac{\sqrt{1 - \cos^2 A}}{\cos A} = \frac{\sin 2A}{1 + \cos 2A}.$$

$$\tan A = \frac{1}{\cot A} = \frac{1 - \cos 2A}{\sin 2A}.$$

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \sqrt{\operatorname{cosec}^2 A - 1}.$$

$$\cot A = \frac{\sin 2A}{1 - \cos 2A} = \frac{1 + \cos 2A}{\sin 2A}.$$

$$\sec A = \frac{1}{\cos A} = \text{the reciprocal of any expression for } \cos A.$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \text{the reciprocal of any expression for } \sin A.$$

$$\operatorname{vers} A = 1 - \cos A = 2 \sin^2 \tfrac{1}{2} A.$$

$$\operatorname{exsec} A = \sec A - 1 = \frac{\operatorname{vers} A}{\cos A}.$$

$$\sin \tfrac{1}{2} A = \sqrt{\frac{1 - \cos A}{2}} = \sqrt{\frac{\operatorname{vers} A}{2}}.$$

TABLE XVIII.—*Continued.*

TRIGONOMETRIC AND MISCELLANEOUS FORMULAS.

$$\cos \tfrac{1}{2} A = \sqrt{\frac{1 + \cos A}{2}}.$$

$$\tan \tfrac{1}{2} A = \frac{\tan A}{1 + \sec A} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}.$$

$$\cot \tfrac{1}{2} A = \frac{1 + \cos A}{\sin A} = \frac{\sin A}{1 - \cos A}.$$

$$\sin 2A = 2 \sin A \cos A.$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1.$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.$$

$$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A}.$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2} (A + B) \cos \tfrac{1}{2} (A - B).$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2} (A + B) \sin \tfrac{1}{2} (A - B).$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2} (A + B) \cos \tfrac{1}{2} (A - B).$$

$$\cos B - \cos A = 2 \sin \tfrac{1}{2} (A + B) \sin \tfrac{1}{2} (A - B).$$

$$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin (A + B) \sin (A - B).$$

$$\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B).$$

$$\tan A \pm \tan B = \frac{\sin (A \pm B)}{\cos A \cos B}.$$

$$\cot A \pm \cot B = \frac{\pm \sin (A \pm B)}{\sin A \sin B}.$$

Table XVIII.—*Continued.*

TRIGONOMETRIC AND MISCELLANEOUS FORMULAS.

MISCELLANEOUS FORMULAS.

Required.	Given.	Formulas.
Area of		
Trapezoid	Parallel sides $= m$ and n Perp. dist. bet. them $= p$	$\frac{p}{2}(m+n)$
Regular Polygon	Length of side $= l$ Number of sides $= n$	$\frac{nl^2}{4}\cot\frac{180^\circ}{n}$
Circle	Radius $= r$	πr^2 $[\pi = 3.1416]$
Ellipse	Semi-axes $= a$ and b	πab
Parabola	Base $= b$, height $= h$	$\frac{2}{3}bh$
Surface of		
Cone	Radius of base $= r$ Slant height $= s$	πrs
Cylinder	Radius $= r$, height $= h$	$2\pi rh$
Sphere	Radius $= r$	$4\pi r^2$
Zone	Height $= h$ Radius of its sphere $= r$	$2\pi rh$
Volume of		
Prism or cylinder	Area of base $= b$ Height $= h$	bh
Pyramid or cone	Area of base $= b$ Height $= h$	$\frac{bh}{3}$
Frustum of		
Pyramid or cone	Area of bases $= b$ and b' Height $= h$	$\frac{h}{3}(b + b' + \sqrt{bb'}$
Sphere	Radius $= r$	$\frac{4}{3}\pi r^3$

FORM A.

CHAIN SURVEY: CALCULATION OF AREA.

$$A = \sqrt{s(s-a)(s-b)(s-c)}. \qquad s = \frac{a+b+c}{2}.$$

Triangle.	1	2	3	4	5	6	7	8	Summation of Partial Areas.
a b c	503.2 174.5 542.7	491.9 580.4 542.7	650.4 580.4 626.2	844.8 689.7 626.2	705.2 689.7 557.5	546.9 603.5 557.5	232.2 603.5 523.0	546.9 689.2 446.5	43820.0 123800.0
$2s$	1220.4	1615.0	1866.0	2160.7	1952.4	1707.9	1358.7	1682.6	166180.0 212540.0
s	610.2	807.5	933.0	1080.4	976.2	854.0	679.4	841.3	178140.0 139570.0
$s - a$	107.0	315.6	273.6	235.6	271.0	307.1	447.2	294.4	60056.0 121960.0
$s - b$	435.7	227.1	352.6	390.7	286.5	250.5	75.9	152.1	1046066.0
$s - c$	67.5	264.8	306.8	454.2	418.7	296.5	156.4	394.8	
$\log s$ $\log (s-a)$ $\log (s-b)$ $\log (s-c)$	2.78547 2.02938 2.63919 1.82930	2.90714 2.49914 2.35622 2.42292	2.96988 2.43712 2.54728 2.48686	3.03358 2.37218 2.59184 2.65725	2.98954 2.43297 2.45712 2.62190	2.93146 2.48728 2.39881 2.47202	2.83213 2.65050 1.88024 2.19424	2.92495 2.46894 2.18213 2.59638	
	9.28334	10.18542	10.44114	10.65485	10.50153	10.28957	9.55711	10.17240	
$\log A_p$	4.64167	5.09271	5.22057	5.32743	5.25077	5.14479	4.77856	5.08620	
A_p	43820.0	123800.0	166180.0	212540.0	178140.0	139570.0	60056.0	121960.0	

FORM B.

CALCULATION OF LATITUDES AND LONGITUDES.

$$L_O = L \sin\theta. \qquad L_A = L \cos\theta.$$

Line.	A–B	B–C	C–D	D–E	E–F
Length Bearing Longitude	1913.0 S 5° 00′ W 166.7	1015.6 N 84° 55′ W 1001.7	1474.2 S 89° 00′ W 1474.0	522.6 S 61° 00′ W 457.1	803.4 N 8° 00′ W 111.8
Log longitude	2.22201	3.00072	3.16849	2.65999	2.04849
Log sin bearing Log length Log cos bearing	8.94030 3.28171 9.99834	9.99829 3.00243 8.94746	9.99993 3.16856 8.24186	9.94182 2.71817 9.68557	9.14356 2.90493 9.99575
Log latitude	3.28005	1.94989	1.41042	2.40374	2.90068
Latitude	1905.7	89.1	25.7	253.4	795.6

Line.	F–G	G–H	H–I	I–J	J–A
Length Bearing Longitude	90.0 N 13° 00′ W 20.2	1097.0 N 7° 00′ W 133.8	2834.0 S 84° 55′ E 2822.9	440.1 N 6° 00′ E 46.0	424.8 S 84° 40′ E 423.0
Log longitude	1.30633	2.12645	3.45069	1.66278	2.62630
Log sin bearing Log length Log cos bearing	9.35209 1.95424 9.98872	9.08589 3.04056 9.99675	9.99829 3.45240 8.94746	9.01923 2.64355 9.99761	9.99812 2.62818 8.96825
Log latitude	1.94296	3.03731	2.39986	2.64116	1.59643
Latitude	87.7	1089.7	251.1	437.7	39.5

FORM C.

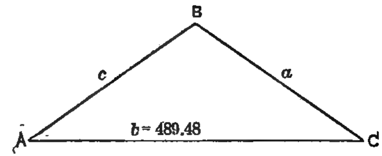
BALANCING OF LATITUDES AND LONGITUDES AND CALCULATION OF AREA.

Line.	Length.	Bearing.	N.	S.	E.	W.	Balanced. N.	Balanced. S.	Balanced. E.	Balanced. W.	D. M. D.	N. A.	S. A.
A B	1913.0	S 5° 00′ W		1905.7		166.7		1910.2		153.3	6482.3		12382000.0
B C	1005.6	N 84° 55′ W	89.1			1001.7	86.7			994.7	5334.3	462490.0	
C D	1474 2	S 89° 00′ W		25.7		1474.0		29.2		1463.7	2875.9		83977.0
D E	522.6	S 61° 00′ W		253.4		457.1		254.6		453.5	958.7		244080.0
E F	803.4	N 8° 00′ W	795.6			111.8	793.7			106.2	399.0	316690.0	
F G	90.0	N 13° 00′ W	87.7			20.2	87.7			20.2	272.6	23907.0	
G H	1097.9	N 7° 00′ W	1089.7			133 8	1087.4			126.2	126.2	137230.0	
H I	2834.0	S 84° 55′ E		251.1	2822.9			257.7	2842.8		2842.8		732580.0
I J	440.1	N 6° 00′ E	437.7		46.0		436.7		49.1		5734.7	2504300.0	
J A	424.8	S 84° 40′ E		39.5	423.0			40.5	425.9		6209.7		251490.0
	10605.6		2499.8	2475.4	3291.9	3365.3	2492.2	2492.2	3317.8	3317.8		3444617.0	13694127.0
			2475.4			3291.9							3444617.0
			24.4			73.4					2		10249510.0
											Area =		5124755.0
												= 117.65 Ac.	

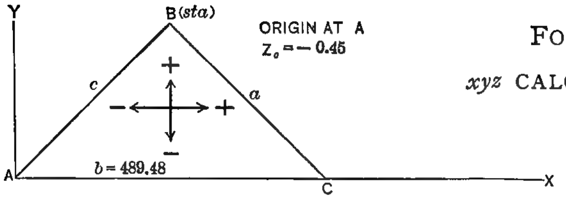
Form D.

CALCULATION OF TRIANGLES.

$$a = \frac{b \sin A}{\sin B} \qquad c = \frac{b \sin C}{\sin B}$$



Triangle.	ABC	ADC	AEC	AFC	AGC	AHC	AIC
A	117° 21′	101° 48′	83° 48′	69° 19′	59° 15′	38° 55′	33° 33′
B	6 04	6 27	4 32	3 24	5 07	3 57	3 09
C	56 35	71 45	91 40	107 17	115 38	137 08	143 18
Check	180° 00′	180° 00′	180° 00′	180° 00′	180° 00′	180° 00′	180° c0′
a	4113.7	4265.1	6156.6	7721.5	4716.7	4463.7	4922.9
Log a	3.61423	3.62993	3.78934	3.88770	3.67364	3.64969	3.69222
Log sin A	9.94852	9.99072	9.99745	9.97107	9.93420	9.79809	9.74246
Log b	2.68973	2.68973	2.68973	2.68973	2.68973	2.68973	2.68973
Colog sin B	0.97598	0.94948	1.10216	1.22690	1.04971	1.16187	1.26003
Log sin C	9.92152	9.97759	9.99982	9.97993	9.95500	9.83270	9.77643
Log c	3.58723	3.61680	3.79171	3.89656	3.69444	3.68430	3.72619
c	3865.7	4138.1	6190.3	7880.6	4948.1	4833.9	5323.4



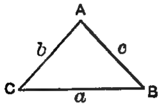
FORM E.

xyz CALCULATION.

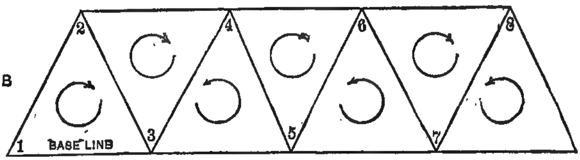
Station.	*B*	*D*	*E*
x	− 1776.0	− 846.22	668.54
Log x	3.24944	2.92748	2.82513
Log cos A	9.66221	9.31068	9.03342
Log c	3.58725	3.61680	3.79171
Log sin A	9.94852	9.99072	9.99745
Log y	3.53575	3.60752	3.78916
y	3433.6	4050.6	6154.0
x	− 1776.0	− 846.22	668.54
b	489.48	489.48	489.48
a cos C	2265.5	1335.7	179.06
Log (a cos C)	3.35516	3.12570	2.25300
Log cos C	9.74093	9.49577	8.46366
Log a	3.61423	3.62993	3.78934
Log sin C	9.92152	9.97759	9.99982
Log y	3.53575	3.60752	3.78916
y	3433.6	4050.6	6154.0
Log c	3.58723	3.61680	3.79171
Log tan E_a	8.06581	8.00781	8.16273
Log (c tan E_a)	1.65304	1.62461	1.95444
c tan E_a	44.98	42.13	90.04
(H of I)$_a$	5.00	5.00	5.00
z	49.98	47.13	95.04
Log a	3.61423	3.62993	3.78934
Log (tan E_c)	8.05481	8.00781	8.16273
Log (a tan E_c)	1.66904	1.63774	1.95207
a tan E_c	46.67	43.43	89.55
(H of I)$_c$	4.70	4.70	4.70
	51.37	48.13	94.25
z_c	.45	.45	.45
z	50.92	47.68	93.80

Form F.

COMPUTATION OF LINES IN SIMPLE TRIANGULATION.



$$c = \frac{a \sin C}{\sin A}$$



	Logs	Antilogs		Logs	Antilogs
1–3 sin 1–3–2			4–5 sin 4–5–6		
sin 1–2–3			sin 4–6–5		
1–2 sin 2–1–3			4–6 sin 5–4–6		
sin 1–3–2			sin 4–5–6		
2–3 sin 2–3–4			5–6 sin 5–6–7		
sin 2–4–3			sin 5–7–6		
2–4 sin 3–2–4			5–7 sin 6–5–7		
sin 2–3–4			sin 5–6–7		
3–4 sin 3–4–5			6–7 sin 6–7–8		
sin 3–5–4			sin 6–8–7		
3–5 sin 4–3–5			6–8 sin 7–6–8		
sin 3–4–5			sin 6–7–8		
4–5			7–8		

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